

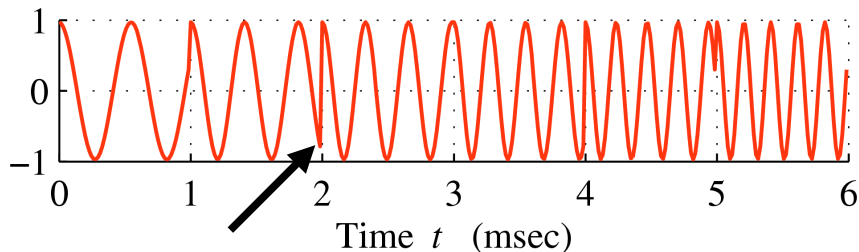
Lecture 9
D-to-A Conversion
9-Feb-2004

Lab #4 Announcement

- **NO “regular” Lab on Tuesday, 10-Feb**
 - **Instead, 12-3 Open lab for help on Lab #4**
- Lab #4 listening starts on Wed
 - **Formal Lab Report:** will be worth 150 pts
 - Make the song sound right
 - Get the Correct frequencies and durations
 - Then tweak
 - Eliminate “clicks” with envelope
- Lab #5 is posted
 - Practice image display and down-sampling

Concatenating Sinusoids

- How do you prevent JUMPS?
 - Envelope: taper the beginning & end
- Example



Education

- “Education is an admirable thing, but it is well to remember from time to time that **nothing that is worth knowing can be taught.**”Oscar Wilde
- So, Labs are one way to approximate knowledge acquisition in the “real world”

Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 4: Sections 4-4, 4-5
- Other Reading:
 - Recitation: Section 4-3 (Strobe Demo)
 - Next Lecture: Chapter 5 (beginning)

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LECTURE OBJECTIVES

- FOLDING: a type of ALIASING
- DIGITAL-to-ANALOG CONVERSION is
 - Reconstruction from samples
 - SAMPLING THEOREM applies
 - Smooth **Interpolation**
- Mathematical Model of D-to-A
 - **SUM of SHIFTED PULSES**
 - Linear Interpolation example

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SIGNAL TYPES



- A-to-D
 - Convert $x(t)$ to **numbers** stored in memory
- D-to-A
 - Convert $y[n]$ back to a “continuous-time” signal, $y(t)$
 - $y[n]$ is called a “**discrete-time**” signal

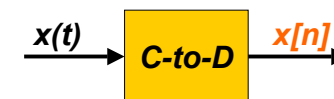
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SAMPLING $x(t)$

- UNIFORM SAMPLING at $t = nT_s$
 - IDEAL: $x[n] = x(nT_s)$



Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{\max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

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NYQUIST RATE

- “Nyquist Rate” Sampling
 - $f_s > \text{TWICE}$ the HIGHEST Frequency in $x(t)$
 - “Sampling above the Nyquist rate”
- **BANDLIMITED SIGNALS**
 - DEF: $x(t)$ has a HIGHEST FREQUENCY COMPONENT in its SPECTRUM
 - NON-BANDLIMITED EXAMPLE
 - TRIANGLE WAVE is **NOT** BANDLIMITED

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SPECTRUM for $x[n]$

- INCLUDE **ALL** SPECTRUM LINES
 - ALIASES
 - ADD INTEGER MULTIPLES of 2π and -2π
 - FOLDED ALIASES
 - ALIASES of NEGATIVE FREQS
- PLOT versus **NORMALIZED** FREQUENCY
 - i.e., DIVIDE f_0 by f_s

$$\hat{\omega} = 2\pi \frac{f}{f_s} + 2\pi l$$

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EXAMPLE: SPECTRUM

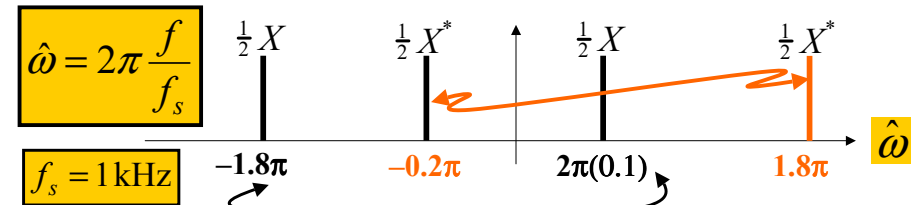
- $x[n] = A \cos(0.2\pi n + \phi)$
- FREQS @ 0.2π and -0.2π
- ALIASES:
 - $\{2.2\pi, 4.2\pi, 6.2\pi, \dots\}$ & $\{-1.8\pi, -3.8\pi, \dots\}$
 - EX: $x[n] = A \cos(4.2\pi n + \phi)$
- ALIASES of **NEGATIVE** FREQ:
 - $\{1.8\pi, 3.8\pi, 5.8\pi, \dots\}$ & $\{-2.2\pi, -4.2\pi \dots\}$

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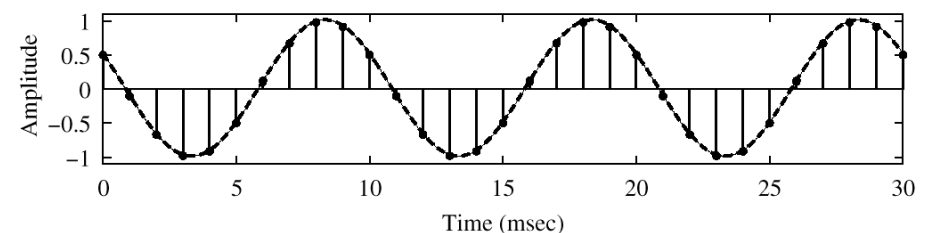
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SPECTRUM (MORE LINES)

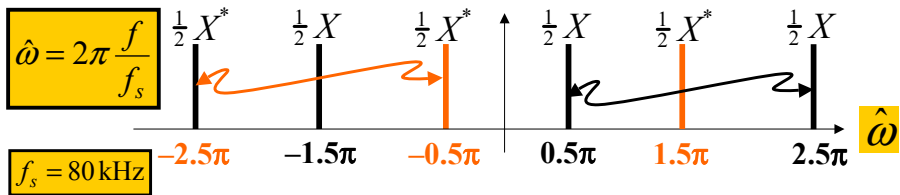


$$x[n] = A \cos(2\pi(100)(n/1000) + \phi)$$

100-Hz Cosine Wave: Sampled with $T_s = 1$ msec (1000 Hz)

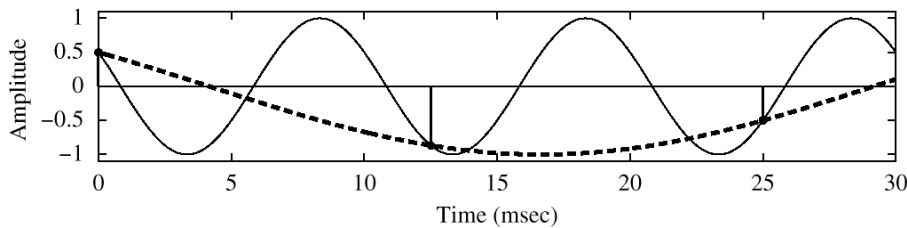


SPECTRUM (ALIASING CASE)



$$x[n] = A \cos(2\pi(100)(n/80) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 12.5$ msec (80 Hz)



FOLDING (a type of ALIASING)

- EXAMPLE: 3 different $x(t)$; same $x[n]$

$$f_s = 1000$$

$$\cos(2\pi(100)t) \rightarrow \cos[2\pi(0.1)n]$$

$$\cos(2\pi(1100)t) \rightarrow \cos[2\pi(1.1)n] = \cos[2\pi(0.1)n]$$

$$\cos(2\pi(900)t) \rightarrow \cos[2\pi(0.9)n]$$

$$= \cos[2\pi(0.9)n - 2\pi n] = \cos[2\pi(-0.1)n] = \cos[2\pi(0.1)n]$$

$$\hat{\omega} = 2\pi \frac{100}{1000} = 2\pi(0.1)$$

- 900 Hz “folds” to 100 Hz when $f_s = 1$ kHz

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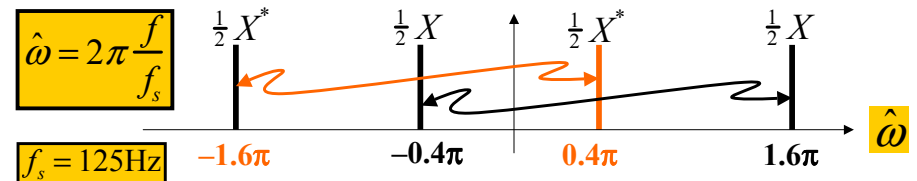
DIGITAL FREQ $\hat{\omega}$ AGAIN

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi \ell \quad \text{ALIASING}$$

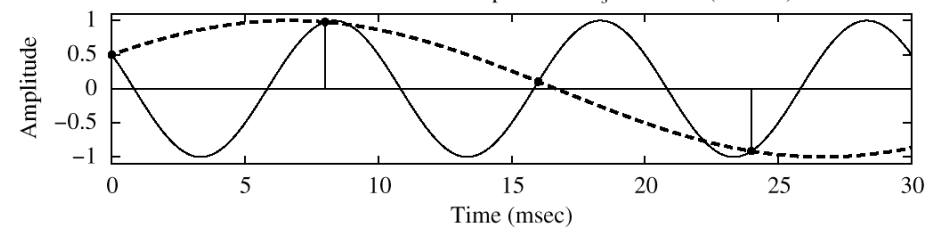
$$\hat{\omega} = \omega T_s = -\frac{2\pi f}{f_s} + 2\pi \ell \quad \text{FOLDED ALIAS}$$

SPECTRUM (FOLDING CASE)



$$x[n] = A \cos(2\pi(100)(n/125) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 8$ msec (125 Hz)

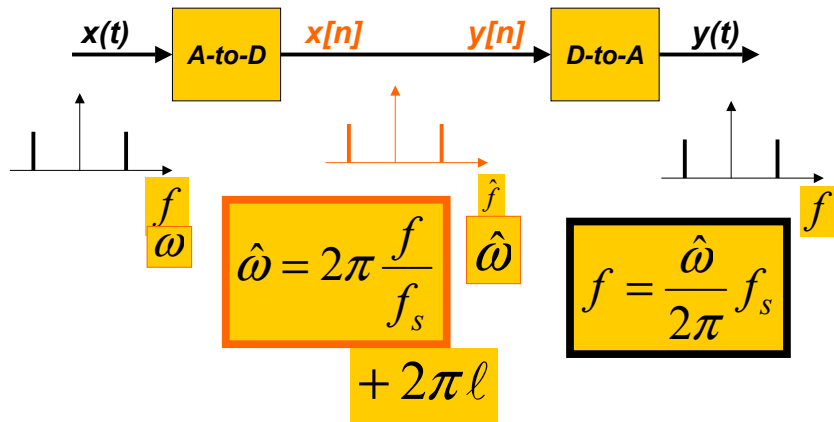


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FREQUENCY DOMAINS



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DEMOS from CHAPTER 4

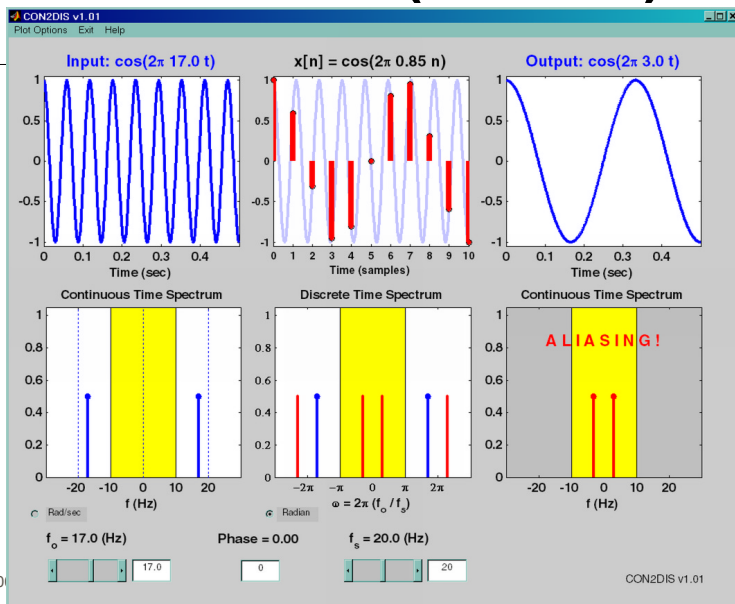
- CD-ROM DEMOS
- SAMPLING DEMO (con2dis GUI)
 - Different Sampling Rates
 - Aliasing of a Sinusoid
- STROBE DEMO
 - Synthetic vs. Real
 - Television **SAMPLES** at 30 fps
- Sampling & Reconstruction

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SAMPLING GUI (con2dis)

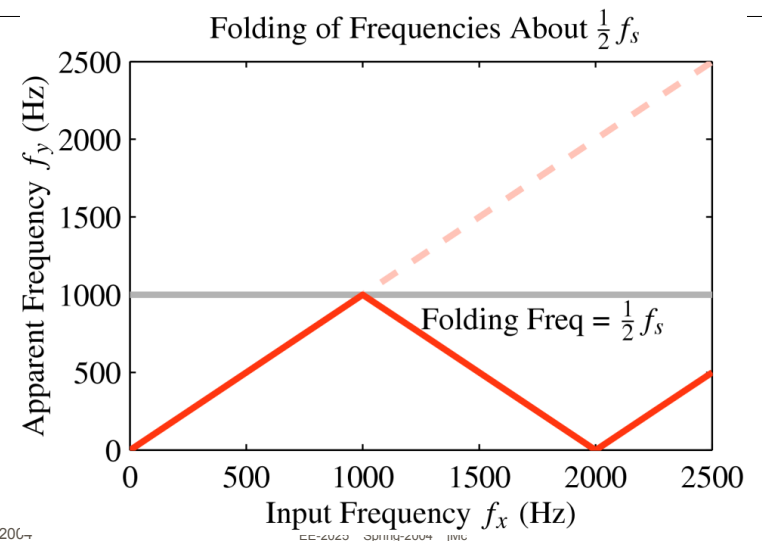


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CON2DIS v1.01

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FOLDING DIAGRAM



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D-to-A Reconstruction



- Create continuous $y(t)$ from $y[n]$
 - **IDEAL**
 - If you have formula for $y[n]$
 - Replace n in $y[n]$ with $f_s t$
 - $y[n] = A\cos(0.2\pi n + \phi)$ with $f_s = 8000$ Hz
 - $y(t) = A\cos(2\pi(800)t + \phi)$

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D-to-A is AMBIGUOUS !

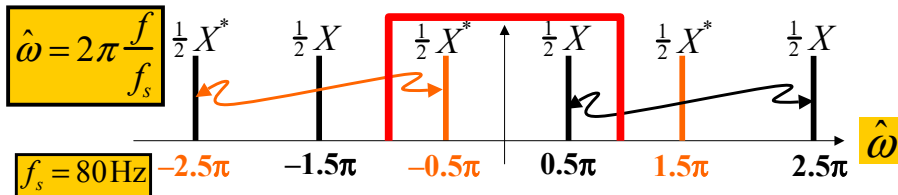
- ALIASING
 - Given $y[n]$, which $y(t)$ do we pick ???
 - INFINITE NUMBER of $y(t)$
 - PASSING THRU THE SAMPLES, $y[n]$
 - D-to-A RECONSTRUCTION MUST CHOOSE ONE OUTPUT
- RECONSTRUCT THE **SMOOTHEST ONE**
 - THE **LOWEST** FREQ, if $y[n] = \text{sinusoid}$

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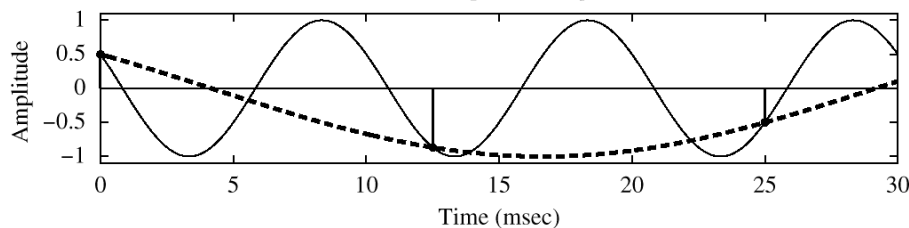
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SPECTRUM (ALIASING CASE)



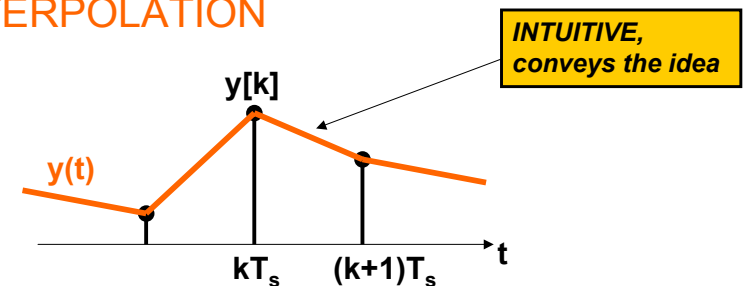
$$x[n] = A\cos(2\pi(100)(n/80) + \phi)$$

100-Hz Cosine Wave: Sampled with $T_s = 12.5$ msec (80 Hz)



Reconstruction (D-to-A)

- CONVERT STREAM of NUMBERS to $x(t)$
- “CONNECT THE DOTS”
- INTERPOLATION



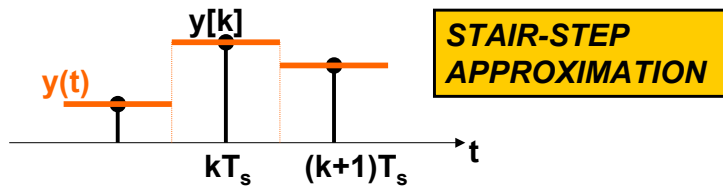
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SAMPLE & HOLD DEVICE

- CONVERT $y[n]$ to $y(t)$
 - $y[k]$ should be the value of $y(t)$ at $t = kT_s$
 - Make $y(t)$ equal to $y[k]$ for
 - $kT_s - 0.5T_s < t < kT_s + 0.5T_s$



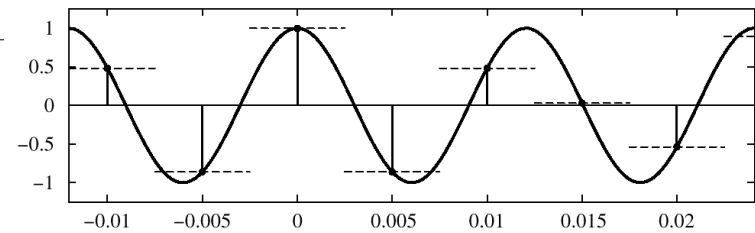
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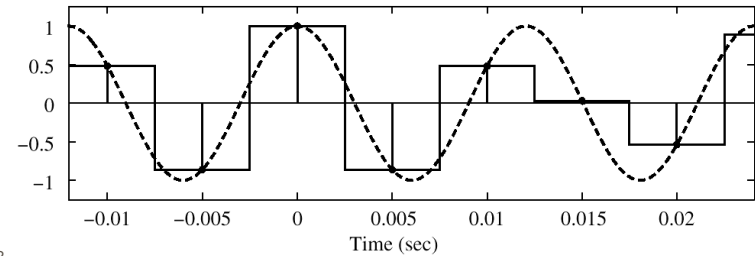
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SQUARE PULSE CASE

Sampling and Zero-Order Reconstruction: $f_0 = 83$ $f_s = 200$



Original and Reconstructed Waveforms

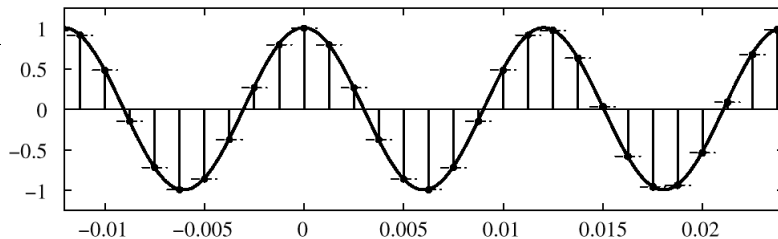


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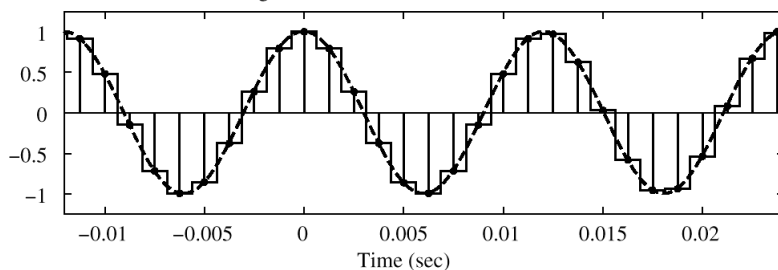
OVER-SAMPLING CASE

Sampling and Zero-Order Reconstruction: $f_0 = 83$ $f_s = 800$



EASIER TO RECONSTRUCT

Original and Reconstructed Waveforms



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MATH MODEL for D-to-A

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

SQUARE PULSE:

$$p(t) = \begin{cases} 1 & -\frac{1}{2}T_s < t \leq \frac{1}{2}T_s \\ 0 & \text{otherwise} \end{cases}$$

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EXPAND the SUMMATION

$$\sum_{n=-\infty}^{\infty} y[n]p(t - nT_s) =$$

$$\dots + y[0]p(t) + y[1]p(t - T_s) + y[2]p(t - 2T_s) + \dots$$

- SUM of SHIFTED PULSES $p(t-nT_s)$
 - “WEIGHTED” by $y[n]$
 - CENTERED at $t=nT_s$
 - SPACED by T_s
 - RESTORES “REAL TIME”

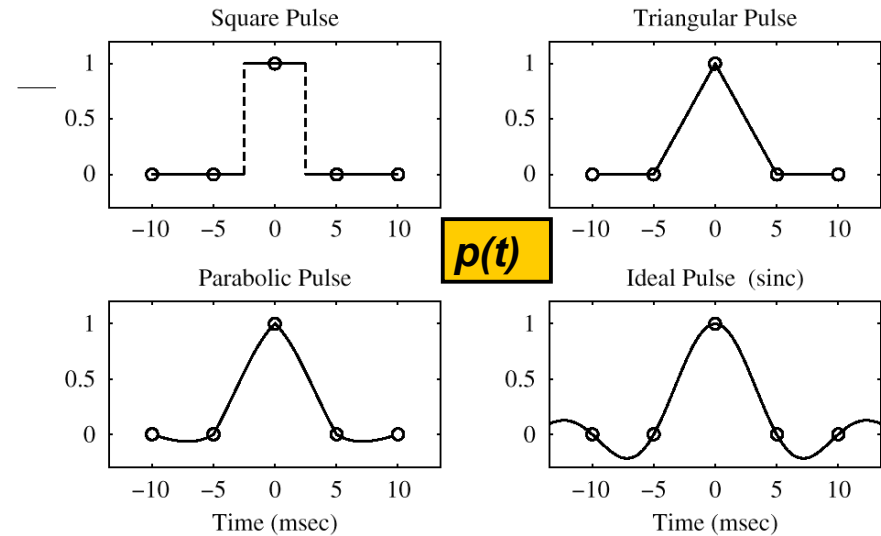
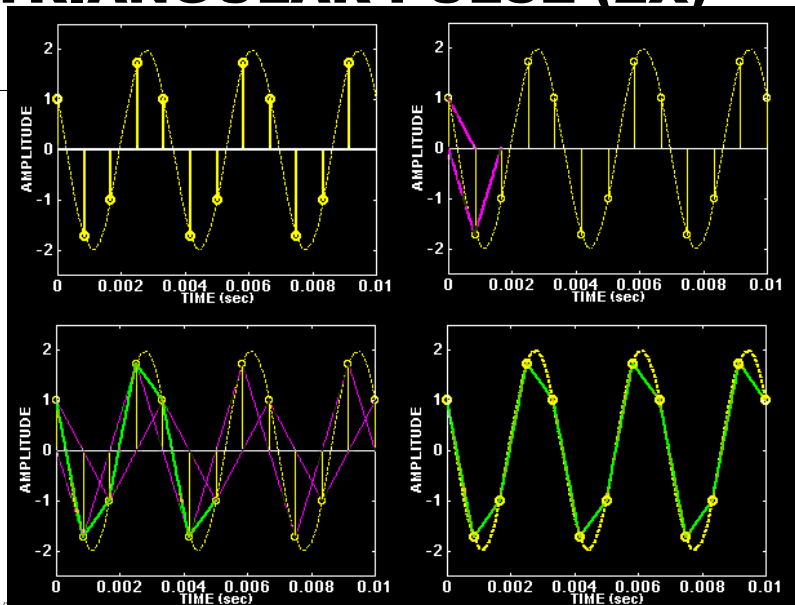


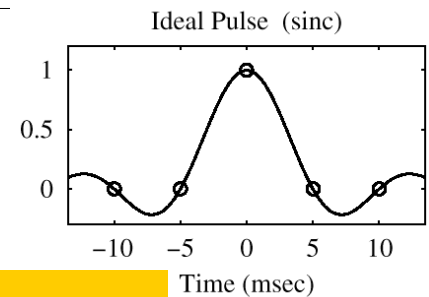
Figure 4.17 Four different pulses for D-to-C conversion. The sampling period is $T_s = 0.005$, i.e., $f_s = 200$ Hz. Note that the duration of each pulse is approximately one or two times T_s .

TRIANGULAR PULSE (2X)



OPTIMAL PULSE ?

**CALLED
“BANDLIMITED
INTERPOLATION”**



$$p(t) = \frac{\sin \frac{\pi t}{T_s}}{\frac{\pi t}{T_s}} \quad \text{for } -\infty < t < \infty$$

$$p(t) = 0 \quad \text{for } t = \pm T_s, \pm 2T_s, \dots$$