ECE-2025 Spring-2004

Lecture 13
Digital Filtering of Analog Signals
23-Feb-04

Info: Web-CT, Lab, HW

- Quiz #2 on 5-March (Friday)
  - Coverage: HW #4, #5, #6, and #7
- Lab #7 is posted
- Avoid printing "mostly black" images
  - Convert black to white, and white to black
  - Use `colormap(1-gray(256))` for "negative"

Perseverance

- **A** lowly virtue whereby mediocrity achieves a glorious success…A. Bierce
- **B**ear in mind, if you are going to amount to anything, that your success does not depend upon the brilliance and the impetuosity with which you take hold, but upon the ever lasting and sanctified bull doggedness with which you hang on after you have taken hold…Dr. A. B. Meldrum

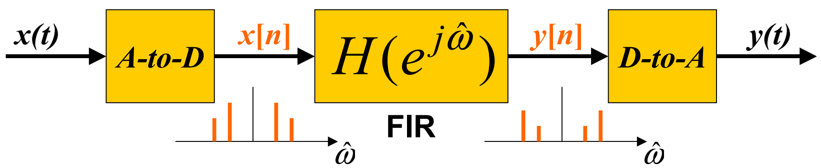
Lecture

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

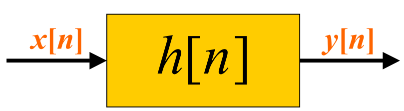
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$

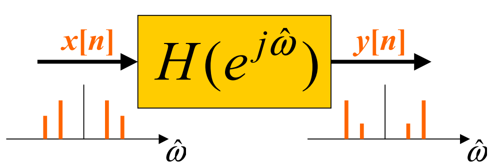
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



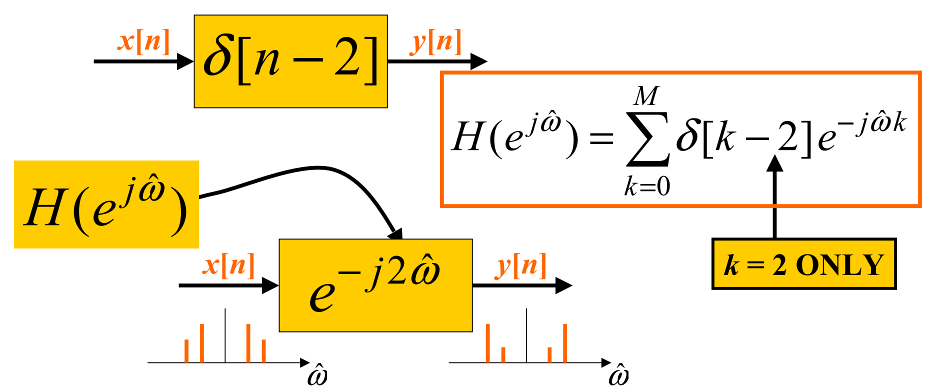
$$b_k = \{\, 0, 0, 1 \,\}$$

$$h[n] = \delta[n-2]$$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-n_d]$

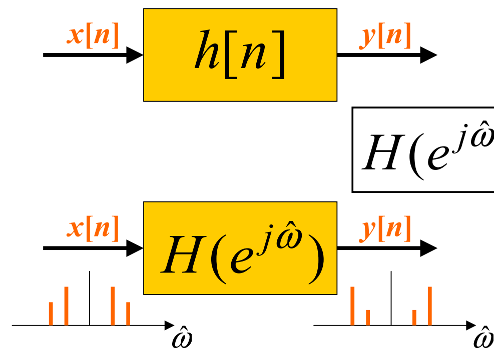
$$h[n] = \delta[n-n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} \delta[k-n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at $k = n_d$

FREQ DOMAIN --> TIME ??

- START with $H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$h[n] = ?$

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

EULER's Formula

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING

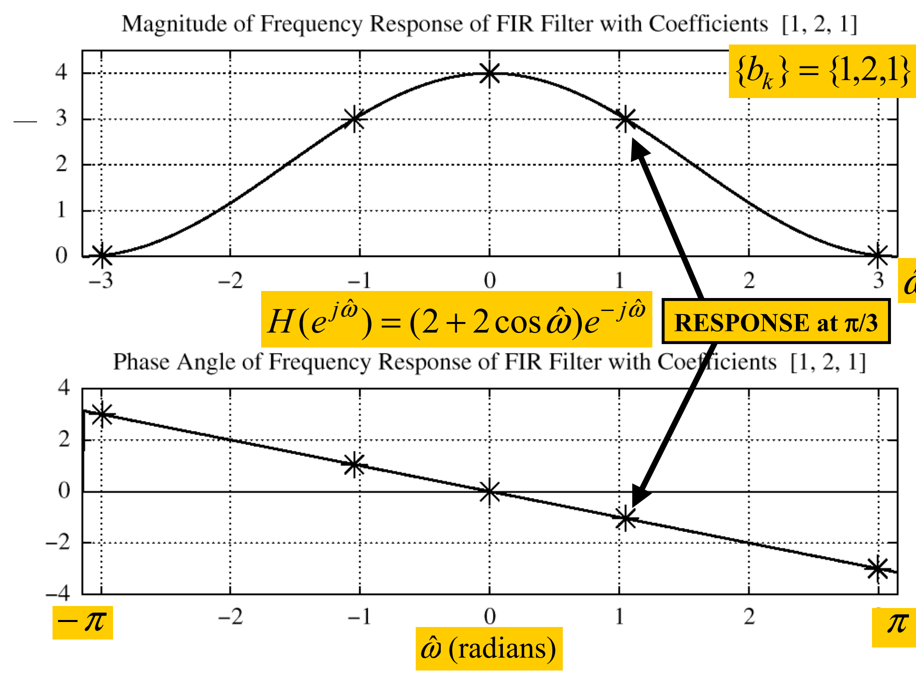
MAG **PHASE**

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

FREQ. RESPONSE PLOTS

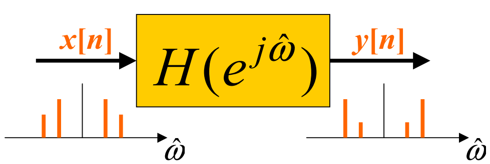
- DENSE GRID **(ww)** from -π to +π
  - `ww = -pi:(pi/100):pi;`
- `HH = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `HH = freekz(bb,1,ww)`

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

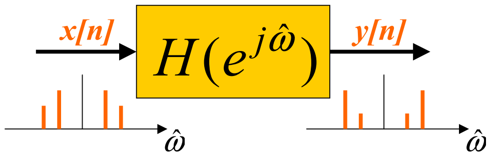
$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\, \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EX: COSINE INPUT (ans-1)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$
$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$
$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$
$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$
$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

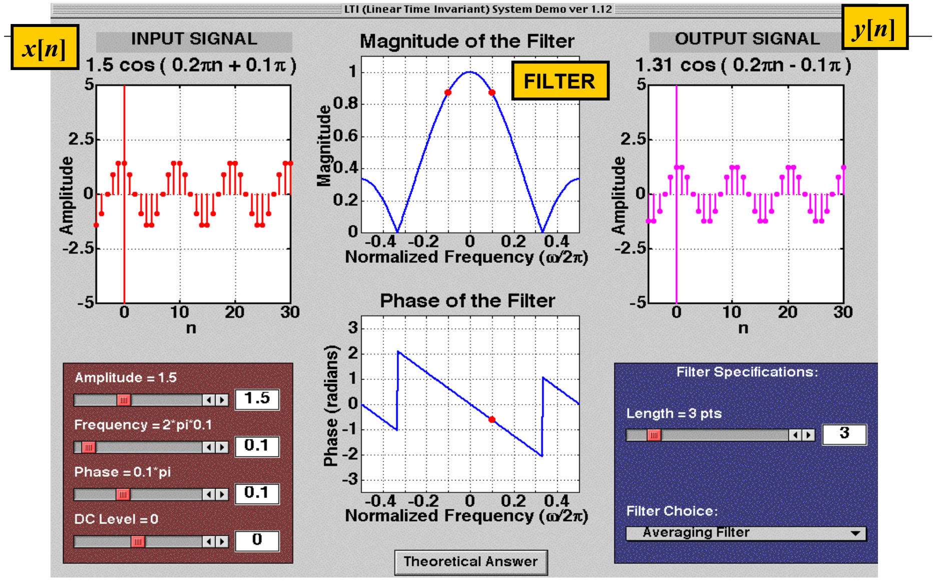
$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$
$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

SINUSOID thru FIR

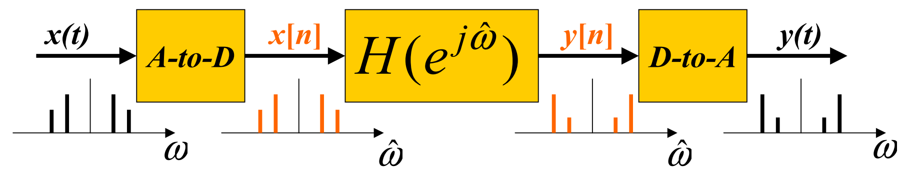
- IF $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A\cos(\hat{\omega}_1 n + \phi)$$
$$\Rightarrow y[n] = A\left|H(e^{j\hat{\omega}_1})\right|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

LTI Demo with Sinusoids

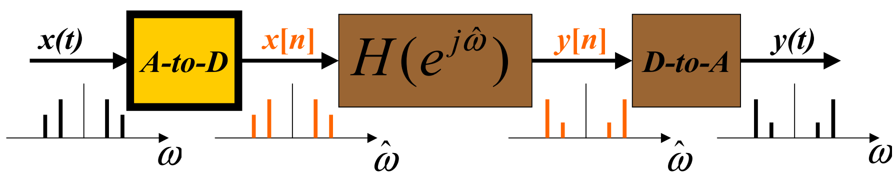


DIGITAL "FILTERING"



- (ω) SPECTRUM of x(t) (SUM of SINUSOIDS)
- ($\hat{\omega}$) SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- SPECTRUM y[n] (FIR Gain or Nulls)
- (ω) Then, OUTPUT y(t) = SUM of SINUSOIDS

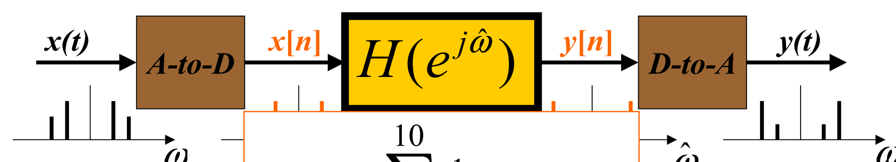
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} \tfrac{1}{11} x[n-k]$$

250 Hz

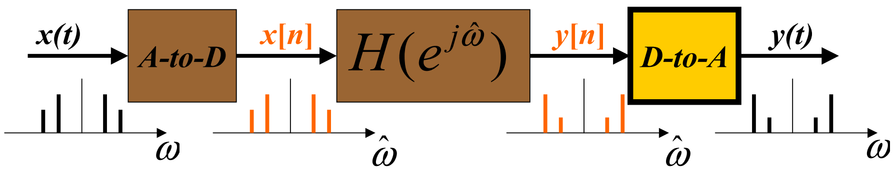
25 Hz

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

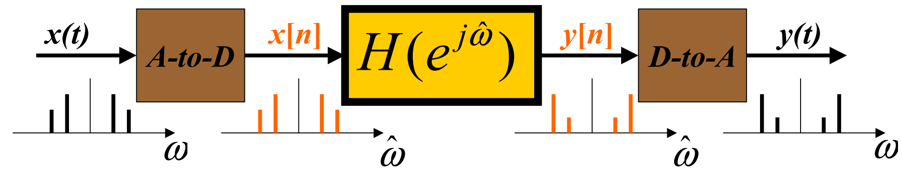
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES

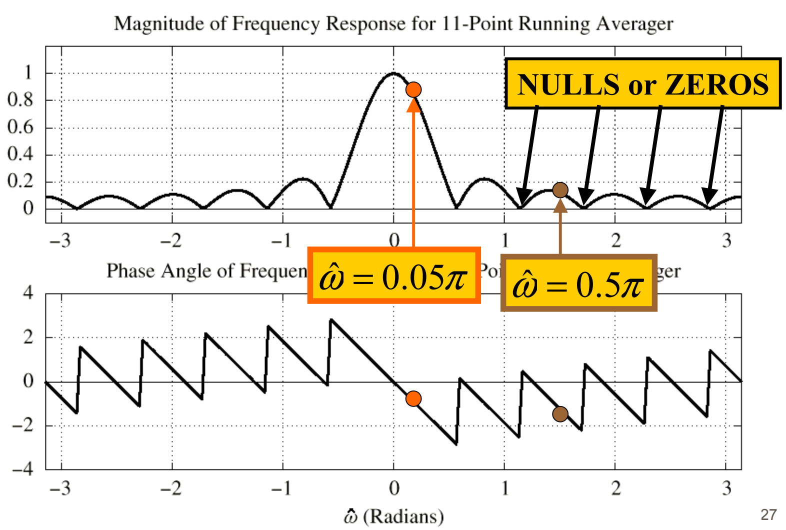


250 Hz	0.5π	$H(e^{j0.5\pi})$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(e^{j0.05\pi})$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

f_s = 1000

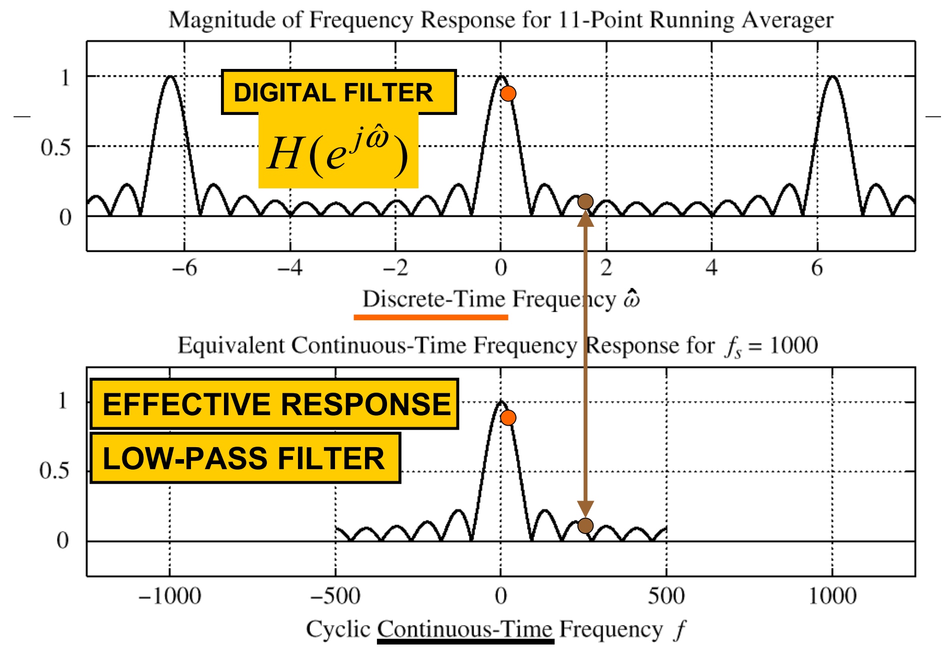
$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000} = 0.8811 e^{-j\pi/4}$$

MAG SCALE

PHASE CHANGE

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000} = 0.0909 e^{-j\pi/2}$$

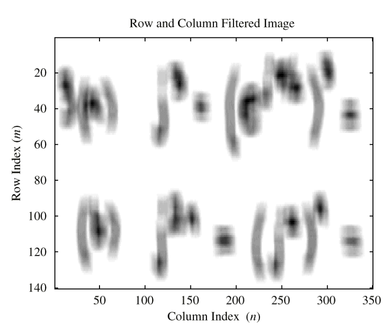
$$y(t) = \underline{0.8811}\cos(2\pi(25)t \underline{- \pi/4}) + \underline{0.0909}\sin(2\pi(250)t \underline{- \pi/2})$$



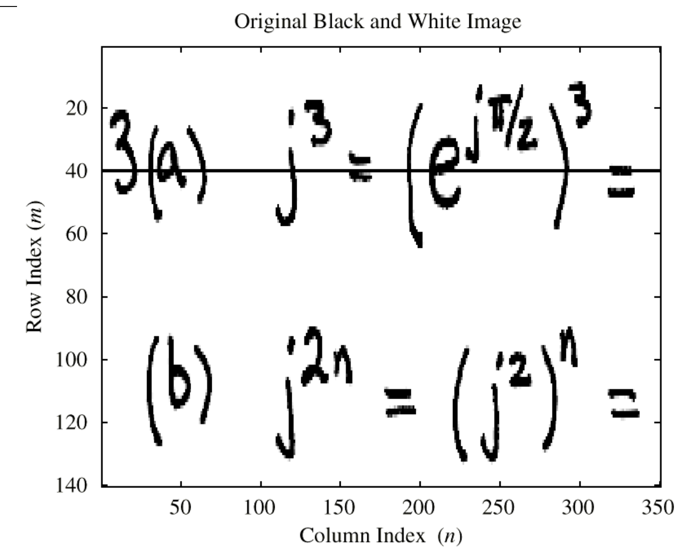
FILTER TYPES

- LOW-PASS FILTER (**LPF**)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BANDPASS FILTER

  (**BPF**)

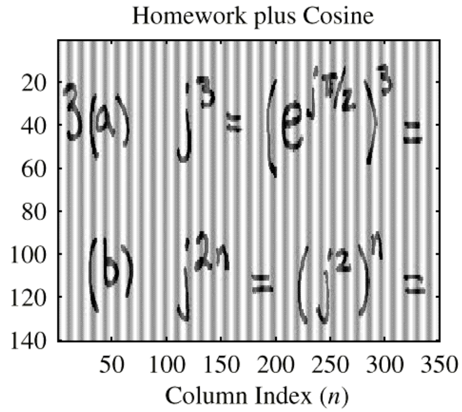


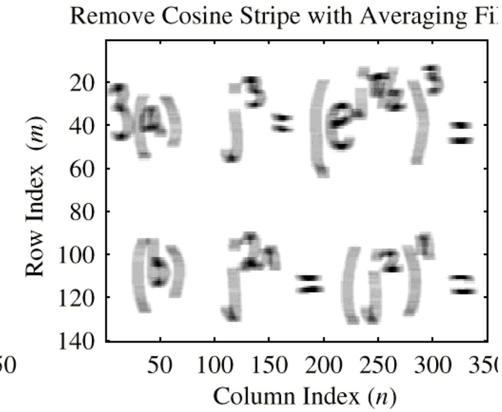
B & W IMAGE



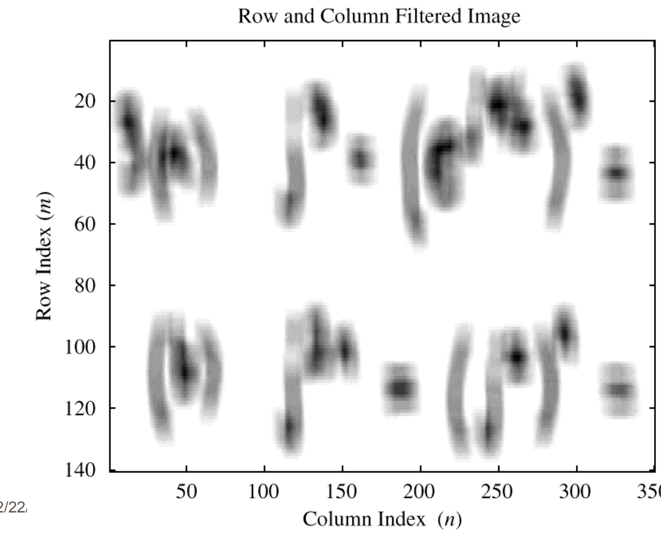
B&W IMAGE with COSINE

FILTERED: 11-pt AVG





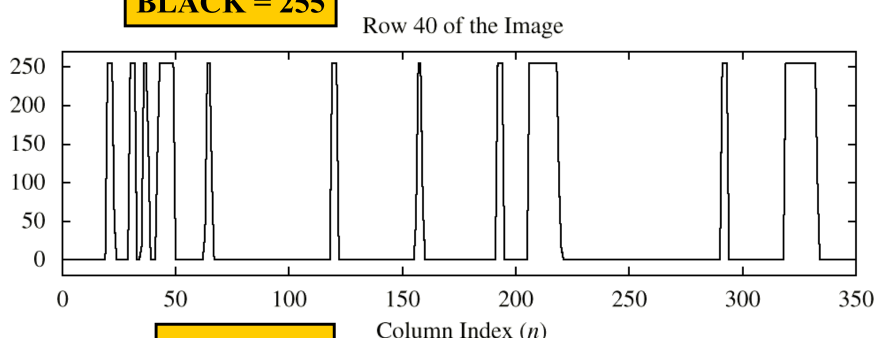
FILTERED B&W IMAGE



LPF: BLUR

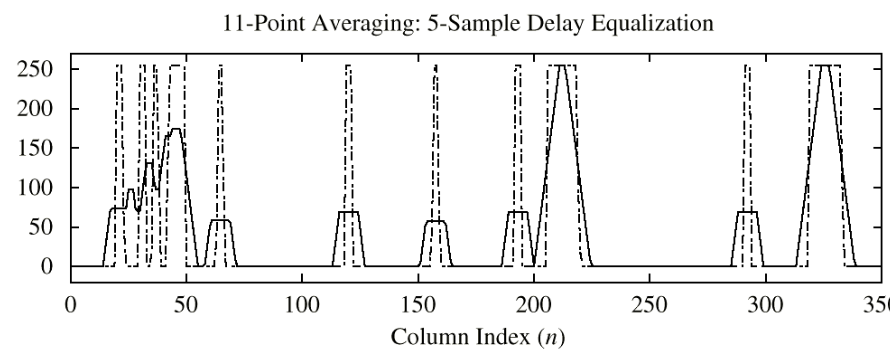
ROW of B&W IMAGE

BLACK = 255



WHITE = 0

FILTERED ROW of IMAGE



ADJUSTED DELAY by 5 samples