GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004

Lab #12: (A) PeZ - The z, n, and $\hat{\omega}$ Domains

Date: 16–22 Nov. 2004

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time. You **MUST** complete the online Pre-Post-Lab exercise on Web-CT at the beginning of your scheduled lab session. You can use MATLAB and also consult your lab report or any notes you might have, but you cannot discuss the exercises with any other students. You will have approximately 20 minutes at the beginning of your lab session to complete the online Pre-Post-Lab exercise. The Pre-Post-Lab exercise for this lab includes some questions about concepts from the previous Lab report as well as questions on the Pre-Lab section of this lab.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

NO lab report is required for this lab. However, there will be a second warm-up for Lab #12 during the week of 29-Nov – 3-Dec.

This part of Lab #12 will be worth 50 points; the second part of Lab #12 (next week) will also be 50 pts.

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1 PeZ: Introduction

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain), A graphical user interface (GUI) called **PeZ** was written in MATLAB for doing interactive explorations of the three domains.[1] **PeZ** is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\,\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{\ell=1}^{N}(1 - p_\ell z^{-1})} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{\ell=1}^{N} a_\ell z^{-\ell}} \tag{1}$$

There are two version of the **PeZ** GUI: the original one written for versions 4 and 5 of MATLAB; and a newer one for version 6. Both versions are contained in the *SP-First* toolbox. To run **PeZ**, type `pezdemo` at the command prompt and you will see the GUI shown in Fig. 1.[2]

1.1 Controls for PeZ using `pezdemo`

The **PeZ** GUI is controlled by the `Pole-Zero Plot` where the user can add (or delete) poles and zeros, as well as move them around with the pointing device. For example, Fig. 1 shows a case where two (complex-

[1] The original **PeZ** was written by Craig Ulmer; a later version by Koon Kong is the one that we will use in this lab.

[2] The command `pez` will invoke the older version of **PeZ** which is distinguished by a black background in all the plot regions.

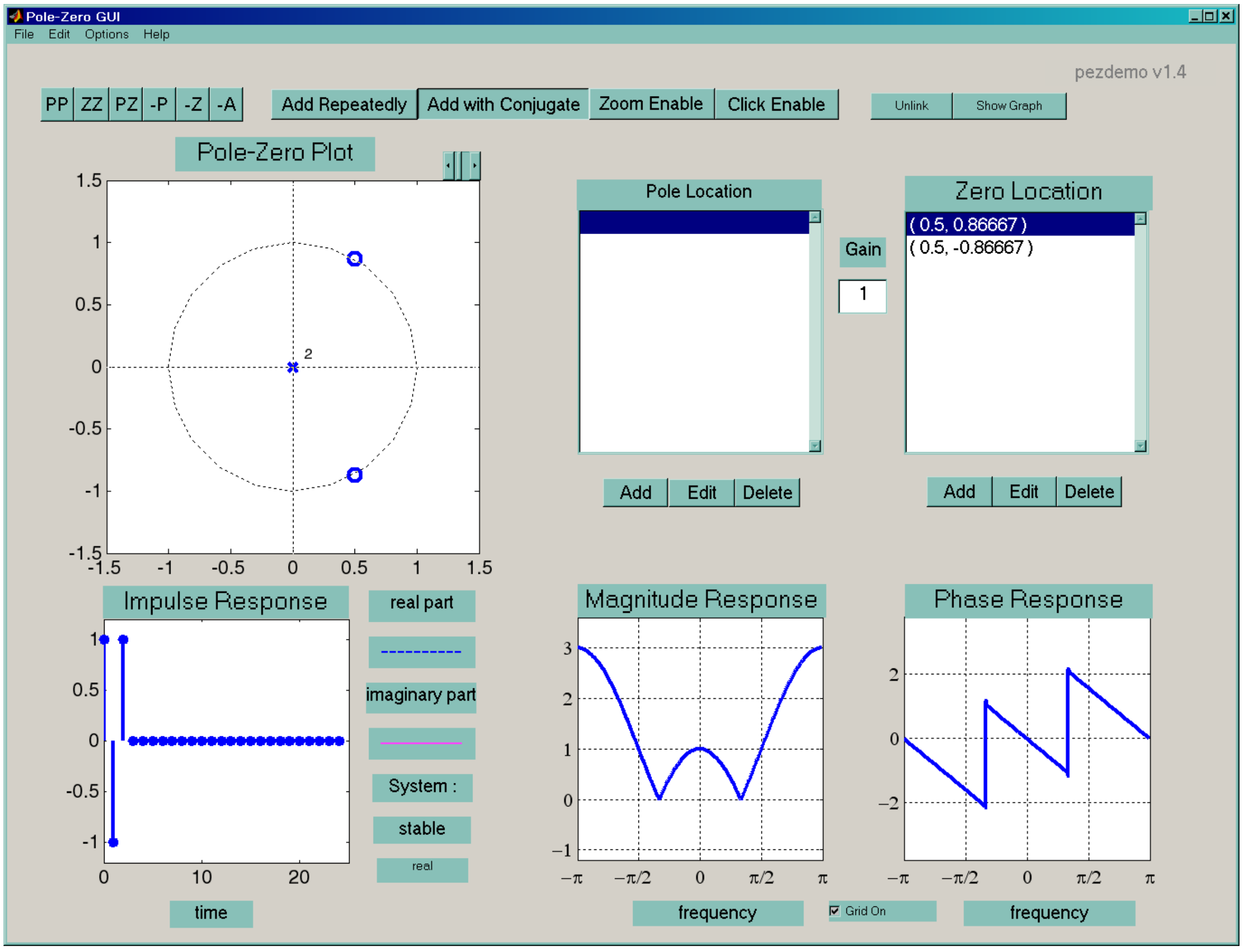

Figure 1: GUI interface for `pezdemo` running in MATLAB version 6. A length-3 FIR filter is shown. Zero locations are given in rectangular coordinates.

conjugate) zeros have been added, and **PeZ** has automatically included two poles at $z = 0$ so that the system is causal. The buttons named `PP` and `ZZ` were used to add these poles and zeros. By default, the `Add with Conjugate` property is turned on, so poles and zeros are typically added in pairs to satisfy the complex-conjugate property:

A polynomial with real coefficients has roots that are real, or occur in complex-conjugate pairs.

To learn about the other controls in `pezdemo`, access the menu item called "Help" for extensive information about all the **PeZ** controls and menus.

Here are a few things to try. In the `Pole-Zero Plot` panel you can selectively place poles and zeros in the z-plane, and then observe (in the other plotting panels) how their placement affects the impulse and frequency responses. In **PeZ** an individual pole/zero pair can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero). Since exact placement of poles and zeros with the mouse is difficult, an `Edit` button is provided for numerical entry of the real and imaginary parts. Before you can edit a pole or zero, however, you must first select it in the list of `Pole Locations` or `Zero Locations`. Removal of individual poles or zeros can also be performed by using the `-P` or `-Z` buttons, or with the `Delete` button. Note that all poles and/or zeros can be easily cleared by clicking on the `-A` button.

2 Pre-Lab

Try various operations with **PeZ** to gain some familiarity with the interface. Here are two suggested filters that you can create.

2.1 Create an FIR Filter with PeZ

Implement the following FIR system:

$$H(z) = 1 - z^{-1} + z^{-2}$$

by factoring the polynomial and placing the two zeros correctly. Observe the following two facts:

- The impulse response $h[n]$ values are equal to the polynomial coefficients of $H(z)$.
- The frequency response has nulls because the zeros of $H(z)$ are exactly on the unit circle. Compare the frequencies of the nulls to the angles of the zeros

Move the zero-pair around the unit circle and observe that the location of the null also moves.

2.2 Create an IIR Filter with PeZ

Implement the following first-order IIR system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its pole and zero at the correct locations in the z-plane. First try placing the pole and zero with the mouse, and then use the `Edit` feature to get exact locations. Since **PeZ** wants to add complex-conjugate pairs, you might have to delete one of the poles/zeros that were added; or you can turn off the `Add with Conjugate` feature. Look at the frequency response and determine what kind of filter you have.

Now, use the mouse to "grab" the pole and move it from $z = -0.9$ to $z = +0.8$. Observe how the frequency response changes. Describe the type of filter that you have created (i.e., HPF, LPF, or BPF).

3 Warm-up

The lab verification requires that you write down your observations on the verification sheet when using the PeZ GUI. These written observations will be graded.

3.1 Relationships between z, n, and $\hat{\omega}$ domains

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

- Is the length of $h[n]$ finite or infinite? For the FIR case, what is the total length of the filter?
- How does $h[n]$ change with respect to its rate of decay for IIR filters? For example, when $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when a is smaller.
- If $h[n]$ exhibits an oscillating component, what is the period of oscillation? Also, estimate the decay rate of the "envelope" that overlays the oscillation.
- How does $H(e^{j\hat{\omega}})$ change with respect to null location, peak location or peak width?

Note: review the "Three-Domains - FIR" under the Demos link for chapter 7 and "Three-Domains - IIR" under the Demos link for chapter 8 for movies and examples of these relationships.

3.1.1 Real Poles

(a) Use **PeZ** to place a single pole at $z = \frac{1}{2}$. You may have to use the `Edit` button to get the location exactly right. Use the plots in **PeZ** for this case as the reference for answering the next five parts.

(b) Move the pole slowly from a location close to the origin out to $z = \frac{1}{2}$, and then to out to $z = 0.999$ (stay on the real axis). Observe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$. Record your observations on the Verification Sheet.

When you move poles and zeros, the impulse response and frequency response plots are updated continually in **PeZ**. Select the pole you want to move and start to drag it slowly. At the same time, watch for the update of the plots in the impulse response and frequency response panels.

(c) Place the pole exactly on the unit circle (or maybe just inside at a radius of 0.99999999). Describe the changes in $h[n]$ and $H(e^{j\hat{\omega}})$. What do you expect to see for $H(e^{j\hat{\omega}})$?

(d) Move the pole outside the unit circle. Describe the changes in $h[n]$. Explain how the appearance of $h[n]$ validates the statement that the system is not stable. In this case, the frequency response $H(e^{j\hat{\omega}})$ is not legitimate because the system is no longer stable.

(e) In general, where should poles be placed to guarantee system stability? By stability we mean that the system's output does not blow up.

Instructor Verification (separate page)

3.1.2 Zeros

The 8-point running-sum FIR digital filter has the following system function:

$$H(z) = \sum_{n=0}^{7} z^{-n} = \frac{1 - z^{-8}}{1 - z^{-1}}$$

(a) Use the `roots` function in MATLAB to determine the zeros of $H(z)$ for this running-sum filter.

(b) Use **PeZ** to place the seven zeros of the running-sum filter at the correct locations. Observe that the impulse response will be all ones when you have the correct zero locations. Describe the frequency response of the filter: low pass versus high pass.

(c) List the frequencies that are nulled by the 8-point running-sum filter.

(d) One of the zeros will be at $z = -1$. Take that zero and move it from $z = -1$ to $z = +1$. Write down the system function $H(z)$ for the new filter.
Hint: The GUI contains the answer if you know where to look.

(e) Describe the frequency response of the filter created in part (d).

Instructor Verification (separate page)

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WORKSHEET & VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ____ Date of Lab: __________

Part	Observations
3.1.1(a)	$h[n]$ decays exponentially with no oscillations, $H(e^{j\hat{\omega}})$ has a hump at $\hat{\omega} = 0$
3.1.1(b)	
3.1.1(c)	
3.1.1(d)	
3.1.1(e)	
3.1.2(a)	
3.1.2(b)	
3.1.2(c)	
3.1.2(d)	$H(z) =$
3.1.2(e)	

Verified:____________________ Date/Time:__________