GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Lab #12: (B) PeZ - The z, n, and $\hat{\omega}$ Domains

Date: 29-Nov. – 2-Dec. 2004

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

NO lab report is required for this lab; it is a warm-up only.

This second part of Lab #12 is worth 50 points; the first part of Lab #12 was also 50 pts.

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1 Introduction

In this part of the lab, you will use **PeZ** to create filters with complex conjugate poles and zeros. These are called *second-order filters* because the denominator polynomial is second-order.

2 PreLab

2.1 PeZ: Introduction

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain), A graphical user interface (GUI) called **PeZ** was written in MATLAB for doing interactive explorations of the three domains.[1] **PeZ** is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\,\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{\ell=1}^{N}(1 - p_\ell z^{-1})} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{\ell=1}^{N} a_\ell z^{-\ell}} \tag{1}$$

There are two version of the **PeZ** GUI: the original one written for versions 4 and 5 of MATLAB; and a newer one for version 6. Both versions are contained in the *SP-First* toolbox. To run **PeZ**, type `pezdemo` at the command prompt and you will see the GUI shown in Fig. 1.[2]

2.1.1 Controls for PeZ using `pezdemo`

The **PeZ** GUI is controlled by the `Pole-Zero Plot` where the user can add (or delete) poles and zeros, as well as move them around with the pointing device. For example, Fig. 1 shows a case where two (complex-

[1] The original **PeZ** was written by Craig Ulmer; a later version by Koon Kong is the one that we will use in this lab. Recent modifications by Greg Krudysz have added new features such as movie-making capability.

[2] The command `pez` will invoke the older version of **PeZ** which is distinguished by a black background in all the plot regions.

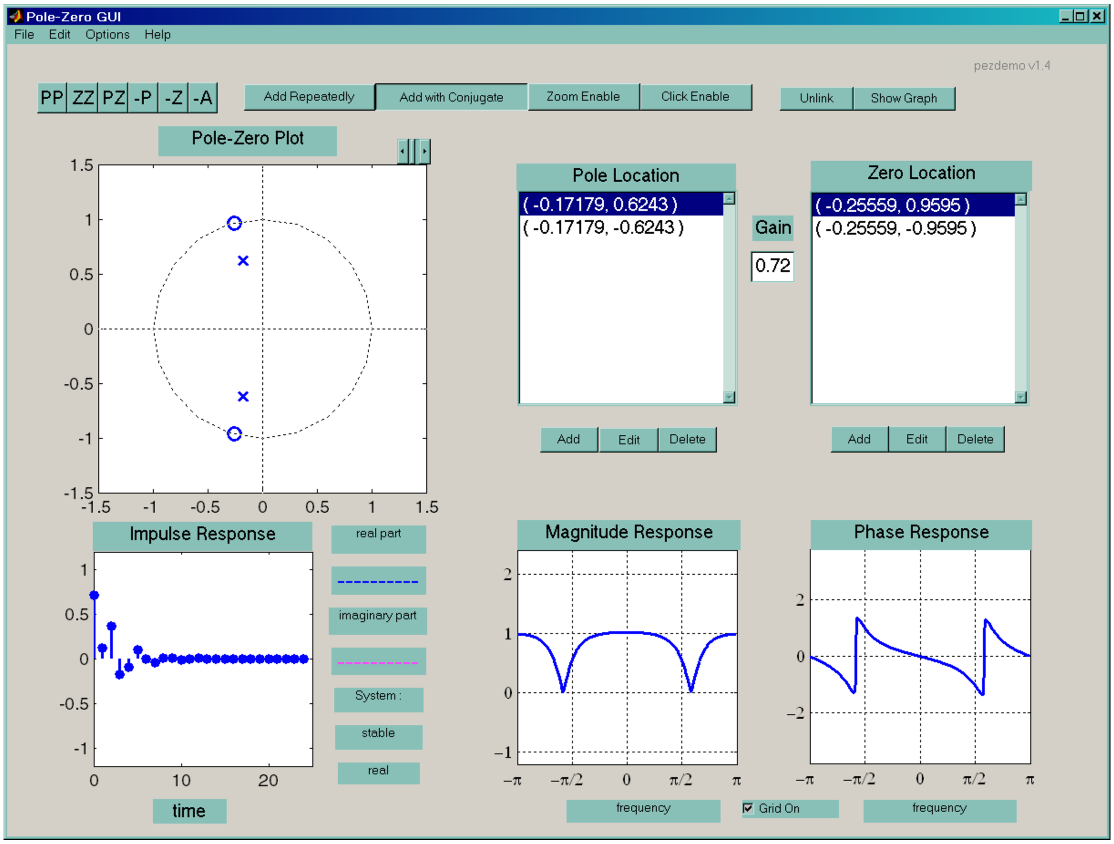

Figure 1: GUI interface for `pezdemo` running in MATLAB version 6. A second-order filter is shown. Pole and zero locations are given in rectangular coordinates.

conjugate) poles have been added, along with two (complex-conjugate) zeros on the unit circle. The buttons named **PP** and **ZZ** were used to add these poles and zeros. By default, the **Add with Conjugate** property is turned on, so poles and zeros are typically added in pairs to satisfy the complex-conjugate property:

A polynomial with real coefficients has roots that are real, or occur in complex-conjugate pairs.

To learn about the other controls in `pezdemo`, access the menu item called "Help" for extensive information about all the **PeZ** controls and menus.

Here are a few things to try. You can use the `Pole-Zero Plot` to selectively place poles and zeros in the z-plane, and then observe (in the other plots) how their placement affects the impulse and frequency responses. In **PeZ** an individual pole/zero pair can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero). Since exact placement of poles and zeros with the mouse is difficult, an **Edit** button is provided for numerical entry of the real and imaginary parts. Before you can edit a pole or zero, however, you must first select it in the list of **Pole Locations** or **Zero Locations**. Removal of individual poles or zeros can also be performed by using the **-P** or **-Z** buttons, or with the **Delete** button. Note that all poles and/or zeros can be easily cleared by clicking on the **-A** button.

2.1.2 Create an IIR Filter with PeZ

Use the **PeZ** interface to implement the following second-order system:

$$H(z) = \frac{1 - z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the z-plane. First try placing the poles and zeros with the mouse, and then use the **Edit** feature to get exact locations. Since **PeZ** wants to add complex-conjugate pairs, you should only have to add one of the poles; for the zeros, the `Add with Conjugate` feature should be turned off because you will be adding two real-valued zeros.

Look at the frequency response and determine what kind of filter you have.

2.2 Not Bandpass Filters

It is tempting to think that with two poles the frequency response ends up always having a peak, but there are two interesting cases where that doesn't happen: (1) all-pass filters where $|H(e^{j\hat{\omega}})| = $ constant, and (2) IIR notch filters that null out one frequency, but are relatively flat across the rest of the frequency band.

• Implement the following second-order system:

$$H(z) = \frac{64 + 80z^{-1} + 100z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the z-plane.
⇒ Look at the frequency response and determine what kind of filter you have.[3]

• Now, use the mouse to "grab" the zero-pair and move the zeros to be exactly on the unit-circle at the same angle as the poles. Observe how the frequency response changes. In addition, determine the $H(z)$ for this filter.

$$H(z) = \frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

⇒ Describe the type of filter that you have now created.

3 Warm-up

3.1 Relationships between z, n, and $\hat{\omega}$ domains

The lab verification requires that you write down your observations on the verification sheet when using the PeZ GUI. These written observations will be graded.

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

- How does $h[n]$ change with respect to its rate of decay? For example, when $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when a is smaller.
- If $h[n]$ exhibits an oscillating component, what is the period of oscillation? Also, estimate the decay rate of the "envelope" that overlays the oscillation. How are the period and decay rate related to the pole location?
- How does $|H(e^{j\hat{\omega}})|$ change with respect to peak location and peak width?
- If $|H(e^{j\hat{\omega}})|$ doesn't have a peak, is it a notch filter or an all-pass filter ?

Note: review the "Three-Domains - FIR" under the Demos link for Chapter 7 and "Three-Domains - IIR" under the Demos link for Chapter 8 for movies and examples of these relationships.

[3]The relationship between the poles and zeros of an all-pass filter is zero = 1/(pole)*; this situation where two poles and two zeros are linked together can be done with the `PZ` option in **PeZ**.

3.2 Complex Poles and Zeros

PeZ assumes real coefficients for the numerator and denominator polynomials when the `Add with Conjugate` mode is on (which it is by default). Therefore, if we enter a complex pole or zero, **PeZ** will automatically insert a second root at the conjugate location, i.e., $z = \frac{1}{3} + j\frac{1}{2}$ would be accompanied by $z = \frac{1}{3} - j\frac{1}{2}$.

(a) State the property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ that will guarantee that the two roots of $A(z)$ are either both real, or are complex conjugates of each other.

(b) Clear all the poles and zeros from **PeZ**. Now place a pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Determine the radius and angle of the poles. Note that **PeZ** automatically places a conjugate pole in the z-domain. The frequency response has a peak—record the frequency (location) of this peak.

(c) Change the angle of the pole and observe $h[n]$ and $H(e^{j\hat{\omega}})$: move the pole-pair angles to ($\pm 90°$), then to ($\pm 60°$) and ($\pm 45°$). Describe the changes in $|H(e^{j\hat{\omega}})|$. Concentrate on the height and location of the peak versus frequency $\hat{\omega}$.

(d) Start again with the pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Decrease the radial distance of the poles from the origin (by dragging), e.g., try $-0.2 \pm j0.533$, and then $-0.1 \pm j0.266$. If you use the `Pole Location` edit window to change the values, the two poles will be "unlinked" and you will have to edit them separately. Therefore, dragging is a more informative way to do this even though it's less precise. Describe the changes in both $h[n]$ and $|H(e^{j\hat{\omega}})|$, as you reduce the pole radius.

(e) Increase the magnitude of the poles by pushing them closer to the unit circle, and then move the poles outside the unit circle. When the pole-pair is outside the unit circle, describe what happens to $h[n]$.

(f) Clear all the poles and zeros from **PeZ**, using the `-A` button. Then place two poles at $-0.3 \pm j0.8$ again, and place two zeros exactly on the unit circle at the same angles as the poles. Judging from the frequency response what type of filter have you just implemented? Is the system IIR or FIR?

(g) Write out the expression for $H(z)$ created in part (f). *Hint:* use MATLAB's `poly` function.

(h) Once again clear all the zeros from **PeZ** keeping the two poles at $-0.3 \pm j0.8$. Now place zeros at $\frac{1}{-0.3 \pm j0.8}$. It would be helpful to get the polar representation of these complex-valued zeros. Judging from the frequency response what type of filter have you just implemented? Is the system IIR or FIR?

(i) Write out the expression for $H(z)$ created in part (h).

(j) One last question that relates to your understanding of sampling as well as digital filtering. If the notch filter that you created in one of the previous parts were used as the system $H(z)$ in Fig. 2, and the sampling rate is 1850 Hz, which analog frequencies would it remove from $x(t)$ (assuming no aliasing of the input signal)?

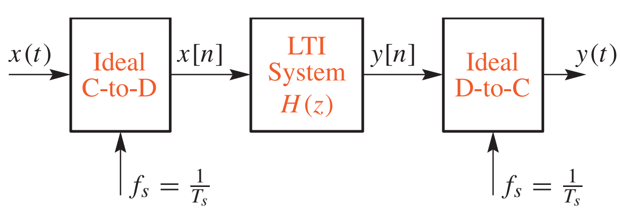

Figure 2: Filtering of analog signal with a digital filter.

Lab #12 (B)
ECE-2025 Fall-2004
WORKSHEET & VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ____________________ Date of Lab: __________

Evaluation: Completed the on-line survey in Web-CT, and the two GT surveys.

Verified:__________________ Date/Time:__________

Part	Observations from **PeZ**
3.2(a)	
3.2(b)	
3.2(c)	
3.2(d)	
3.2(e)	
3.2(f)	
3.2(g)	
3.2(h)	
3.2(i)	
3.2(j)	