GEORGIA INSTITUTE OF TECHNOLOGY
EE 2025: Introduction to Signal Processing

Solutions to Problem Set #1

Problem 1: Here we are to convert numbers in rectangular form to polar form.

(a) $z = -20 = 20e^{j\pi}$

(b) $z = j10 = 10e^{j\pi/2}$

(c) $z = (4, -4) = 4 - 4j = \sqrt{32}e^{j\arctan(-4/4)} = 4\sqrt{2}e^{-j\pi/4}$

(d) $z = -2 - j2 = 2\sqrt{2}e^{j\arctan(-2/2)} = 2\sqrt{2}e^{-j3\pi/4}$

(e) $z = -3 + j4 = 5e^{j\arctan(4/-3)} = 5e^{j(0.7048\pi)}$

(f) $z = -j8 = 8e^{-j\pi/2}$

Problem 2: In this problem, we convert complex numbers in polar form into rectangular form using Euler's formula.

(a) $z = 3\sqrt{2}e^{-j(5\pi/4)} = 3\sqrt{2}\cos(5\pi/4) - 3\sqrt{2}\sin(5\pi/4) = -3 + j3$

(b) $z = 8e^{-j(\pi/2)} = 8\cos(\pi/2) - j8\sin(\pi/2) = -j8$

(c) $z = 10\angle(-\pi/3) = 10e^{-j\pi/3} = 10\cos(\pi/3) - j10\sin(\pi/3) = 5 - j5\sqrt{3}$

(d) $z = \sqrt{3}\angle(43\pi) = \sqrt{3}e^{j43\pi} = \sqrt{3}e^{j\pi} = -\sqrt{3}$

Problem 3: Here, we are given

$$z_1 = 3 - j3 = 3\sqrt{2}e^{-j\pi/4} \quad ; \quad z_2 = 5\sqrt{2}e^{j3\pi/2} = -j5\sqrt{2}$$

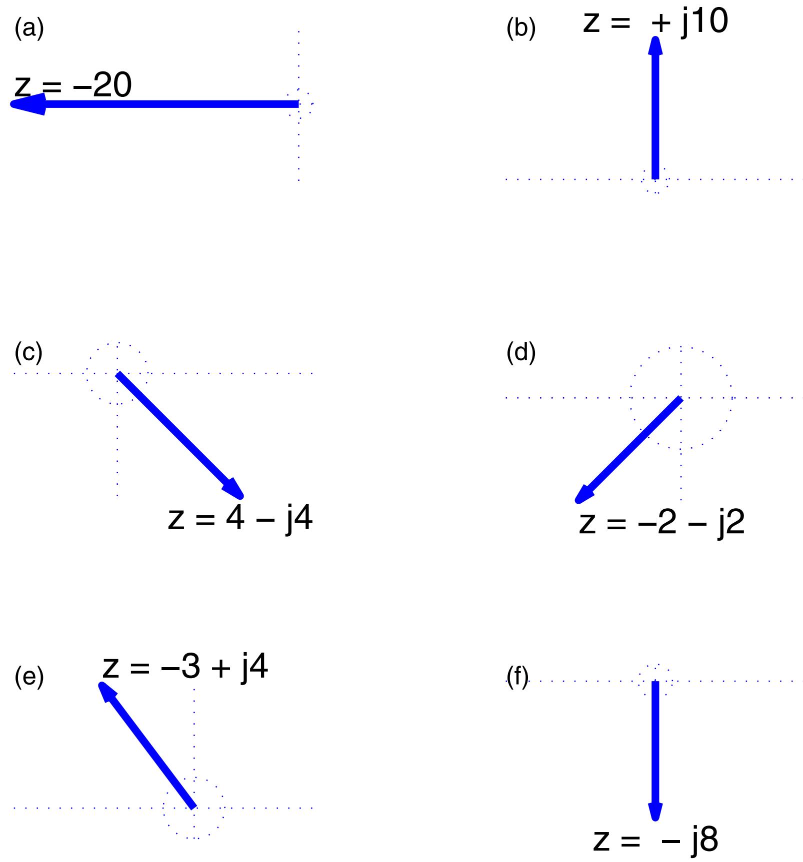
and we are asked to perform some operations on these complex numbers and express the answer in both polar and rectangular form.

(a) $z_1^* = 3 + j3 = 3\sqrt{2}e^{j\pi/4}$

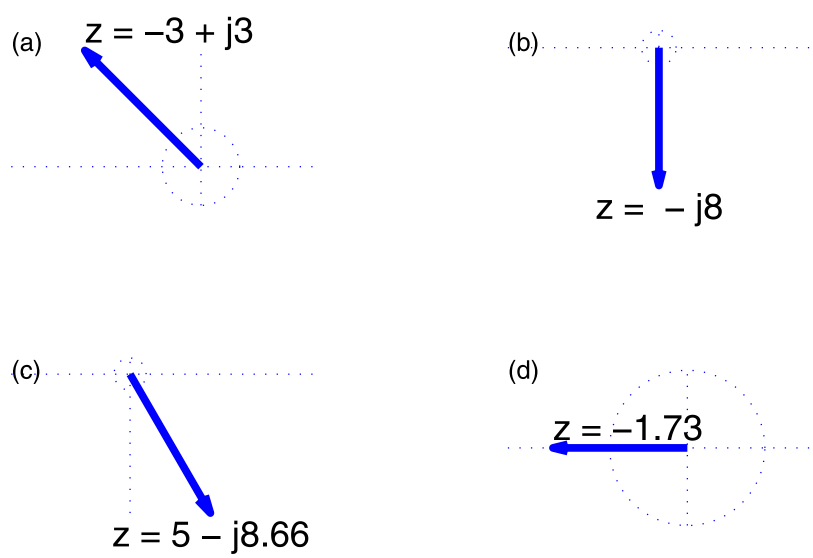
(b) $jz_2 = e^{j\pi/2}5\sqrt{2}e^{j3\pi/2} = 5\sqrt{2}e^{j2\pi} = 5\sqrt{2}$

(c) $z_2/z_1 = \frac{-j5\sqrt{2}}{3-j3} = \frac{-j5\sqrt{2}(3+3j)}{18} = \frac{5\sqrt{2}}{18}(-j)(3+j3) = \frac{5}{6}\sqrt{2}(1-j) = \frac{5}{3}e^{-j\pi/4}$

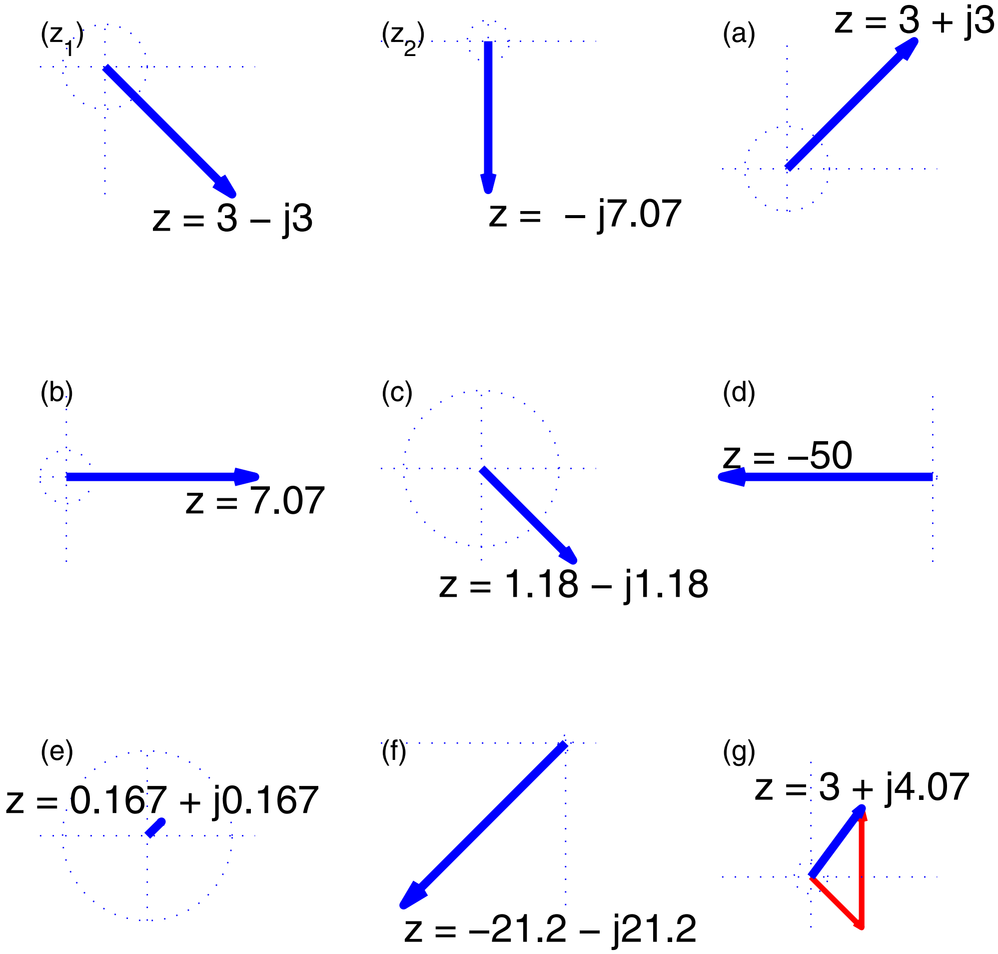
Problem 1.1 (plots):



Problem 1.2 (plots):



Problem 1.3 (plots):



(d) $z_2^2 = (-j5\sqrt{2})^2 = -50 = 50e^{j\pi}$

(e) $z_1^{-1} = \frac{1}{3\sqrt{2}e^{-j\pi/4}} = \frac{1}{3\sqrt{2}}e^{j\pi/4} = \frac{1}{3\sqrt{2}}(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}) = \frac{1}{6}(1+j)$

(f) $z_1 z_2 = (3-j3)(-j5\sqrt{2}) = 30(-1-j) = -15\sqrt{2}(1+j) = 30e^{-j3\pi/4}$

(g) $z_1 + z_2^* = 3 + j(5\sqrt{2}-3) = \sqrt{9+(5\sqrt{2}-3)^2}e^{j\arctan(5\sqrt{2}-3,3)} = 5.05e^{j0.2979\pi}$

(h) $|z_2|^2 = 50$

(i) $z_2 + z_2^* = 2\text{Re}(z_2) = 0$

Problem 4: In this problem we are to plot two periods of a sinusoid with $t = 0$ in the middle.

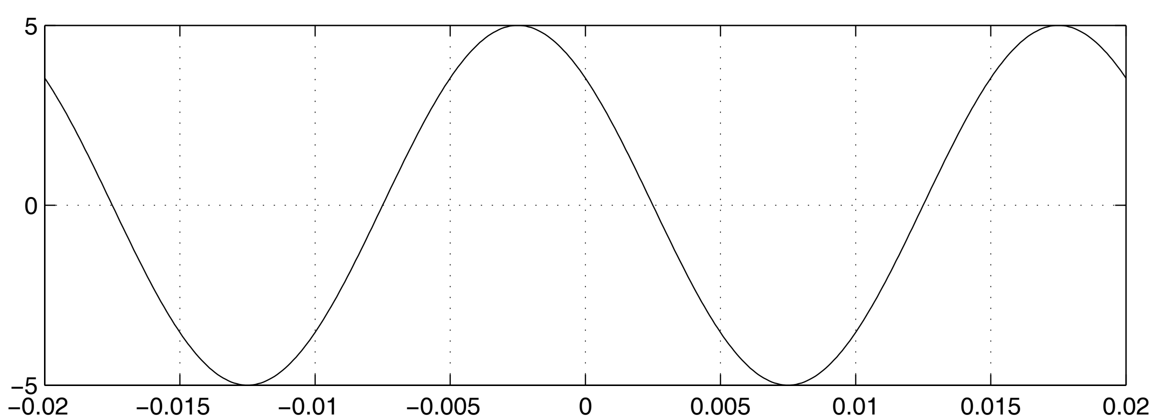
(a) The first sinusoid is given by

$$x(t) = 5\cos(100\pi t + \pi/4) = 5\cos((50)2\pi(t + 1/400))$$

the first thing we should note is that the amplitude of the sinusoid is $A = 5$, the frequency is $f_0 = 50$, and the period is $T_0 = 1/f_0 = 1/50$. Therefore, we want to make a plot for values of t in the interval $[-1/50, 1/50]$. Since this sinusoid is delayed by $t_d = -1/400$ (shifted to the left), one way to make the plot is to plot a sinusoid with an amplitude of five and a frequency of 1/50 Hz, and then label the the time axis consistent with a delay of $-1/400$. To label the axes, note that at

$$t = -1/400 = 0.0025$$

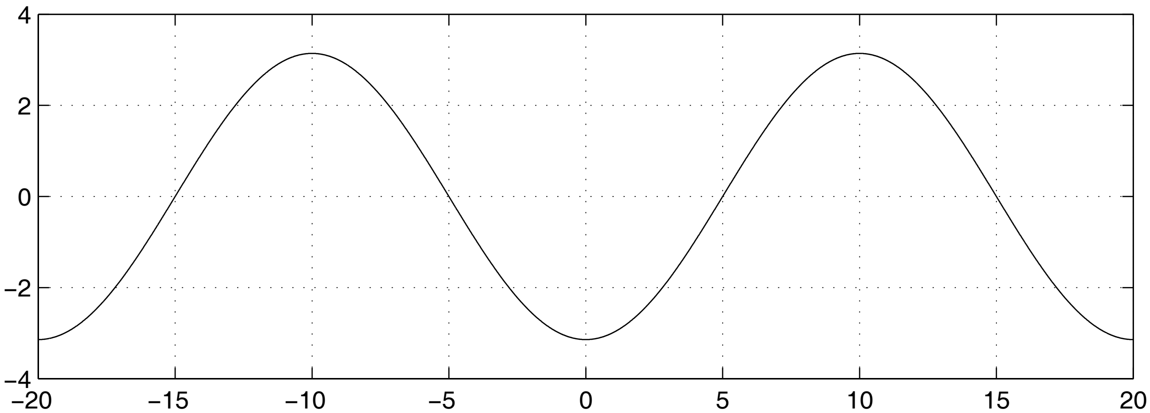
the sinusoid reaches its peak value. Shown in the figure below is a plot of the given sinusoid.



(b) In this part, the sinusoid that we want to plot is given by

$$x(t) = \pi \cos((\pi/10)t + \pi) = -\pi \cos((1/20)2\pi t)$$

Therefore, we have a sinusoid with an amplitude of $A = \pi$, a frequency of $f_0 = 1/20$ and a period of $T_0 = 20$. So, we want to make a plot of this sinusoid for values of t in the interval $[-20, 20]$. Since the delay shifts the cosine by a half a period, the plot is easy to make, as shown in the following figure



Problem 5: In the given plot, note that we have a sinusoid with a period $T_0 = 2 \cdot 10^{-3}$. Therefore, the frequency is

$$f_0 = \frac{1}{T_0} = 500\text{Hz}$$

We also note that the amplitude of the sinusoid is $A = 1$. Therefore, we have

$$x(t) = \sin(2\pi(500)t) = \cos(1000\pi t - \pi/2) = \cos\Big(1000\pi(t - \frac{1}{2000})\Big)$$

Thus, $\omega_0 = 1000\pi$, $f_0 = 500$, $A = 1$, $\phi = -\pi/2$, and $t_d = 1/2000$.