GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #3

Assigned: 27-Aug-04
Due Date: Week of 6-Sep-04

Quiz #1 will be held in lecture on Monday 13-Sep-04. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2, and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$**, both sides)**

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Sections 3-1, 3-2 and 3-3.

There is a web site for *SP First* text: `www.ece.gatech.edu/~spfirst`

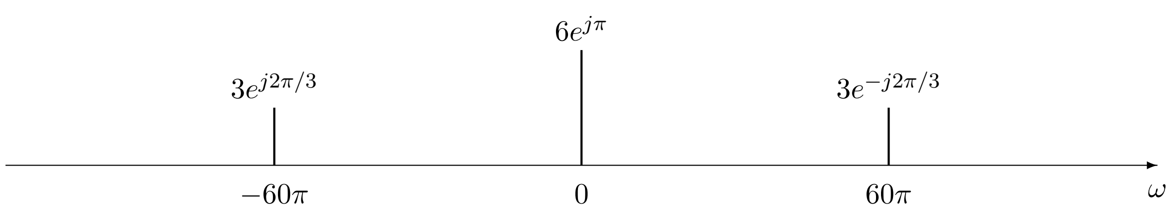
$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 3.1*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal: $y(t) = x^3(t)$.

(c) Plot the spectrum of the *real-valued* signal: $z(t) = 2x(t)\cos(200\pi t - \frac{\pi}{4})$.

PROBLEM 3.2*:

Consider the signal

$$x(t) = 3[\cos(150\pi t)][\sin(200\pi t)].$$

(a) Using the inverse Euler relation for the sine and cosine functions, express $x(t)$ as a sum of complex exponential signals with positive and negative frequencies.

(b) Use your result in part (a) to express $x(t)$ in the form $x(t) = A_1 \cos(\omega_0 t + \phi_1) + A_2 \cos(\omega_1 t + \phi_2)$.

(c) Determine the period T_0 of $x(t)$ and sketch its waveform over the interval $-T_0 \le t \le 2T_0$. Carefully label the graph.

(d) Plot the spectrum of $x(t)$.

PROBLEM 3.3*:

A piano derives some of the richness of its sounds from multiple strings being hit by the same hammer for a particular note. Usually three strings are used for each note. Piano strings produce sounds which are not perfect sinusoids, but let's pretend they produce cosine waves.

The note C-262 (Middle C) on a piano should be 262 Hz. Suppose that the three strings for C-262 are tuned to 259 Hz, 262 Hz and 265 Hz, and that all three strings produce exactly the same volume when the C-262 key is struck. The resulting sound that we hear will be the sum of three sinusoids:

$$x(t) = \cos(2\pi(259)t + \phi_1) + \cos(2\pi(262)t + \phi_2) + \cos(2\pi(265)t + \phi_3)$$

where the phases ϕ_1, ϕ_2, and ϕ_3 depend on how the three strings are struck with the hammer.

(a) Consider the case where the phases are $\phi_1 = \phi_3 = \pi/4$, $\phi_2 = -3\pi/4$. Show that the signal $x(t)$ can be written as a sinusoid at the desired frequency of 262 Hz, multiplied by another function. In other words,

$$x(t) = e(t)\cos(2\pi(262)t + \phi)$$

Find $e(t)$ as a simple real-valued function.
Hint: Use a derivation that writes $x(t)$ as the real part of the sum of three complex exponentials.

(b) The signal $e(t)$ is usually called the *envelope* because its frequency is low and and it causes the amplitude of $x(t)$ to go up and down slowly. Determine the time interval between the maximal peak locations of the low-frequency envelope.

PROBLEM 3.4*:

In FM stereo radio transmissions, the left, $x_L(t)$, and right $x_R(t)$ channel signals are initially combined into a sum, $x_s(t)$, and difference, $x_d(t)$, signal using the relations

$$x_s(t) = x_L(t) + x_R(t)$$
$$x_d(t) = x_L(t) - x_R(t).$$

These two signals are then combined by amplitude modulation to produce an intermediate signal, $x_i(t)$, which then passes to the FM transmitter. In the intermediate signal, the sum and difference waveforms are mixed with a *carrier signal:*

$$x_i(t) = x_s(t) + x_d(t)\cos(2\pi(38,000)t).$$

(a) Suppose that $x_L(t)$ is an 8,000 Hz sinusoid, $x_L(t) = 2\cos(2\pi(8000)t+0.5\pi)$ and that $x_R(t) = 0$. Draw the spectrum for $x_L(t)$.

(b) Now draw the spectrum for $x_i(t)$. Use $x_L(t)$ and $x_R(t)$ from part (a).
Hint: Express $x_s(t)$, $x_d(t)$, and the cosine as a sum of complex exponentials, and then multiply.

(c) Now draw the spectrum for $x_i(t)$ if $x_L(t) = 2\cos(2\pi(8000)t+0.5\pi)$ and $x_R(t) = 2\cos(2\pi(8000)t)$

PROBLEM 3.5*:

Signal Processing First, Chapter **3**, Problem **P-3.19**, page 69.

PROBLEM 3.6:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly.*

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{ e^{j2\pi(20t - 20t^2)} \right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.