GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #4

Assigned: 2-Sept-04
Due Date: Week of 13-Sept-04

Quiz #1 will be held in lecture on Monday 13-Sept-04. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, **both sides)**

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* all.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

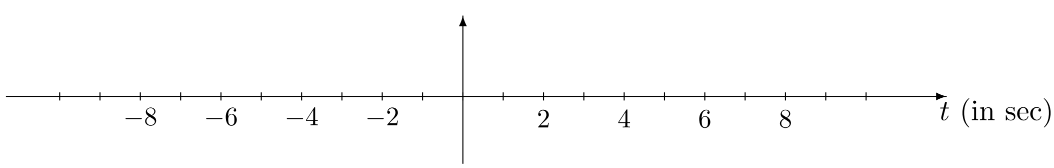
ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 4.1*:

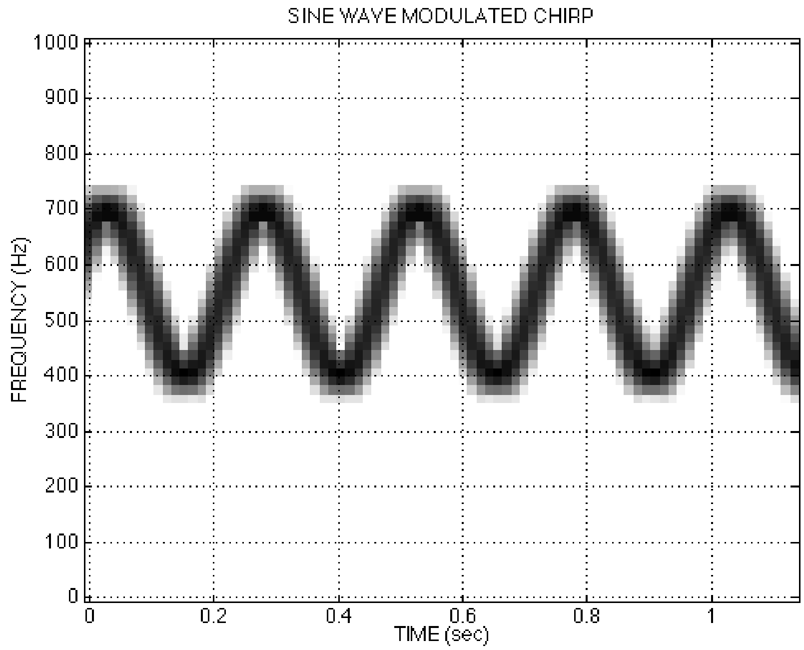
Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 1 - t^2 & \text{for } -1 \le t \le 1 \\ 0 & \text{for } 1 < t < 3 \end{cases}$

(a) Assume that the period of $x(t)$ is 4 sec. Draw a plot of $x(t)$ over the range $-4 \le t \le 4$ sec.



(b) Determine the DC value of $x(t)$ from the Fourier series integral.

PROBLEM 4.2*:



The spectrogram above was produced from a signal of the form:

$$x(t) = A\cos(B\cos[Ct + D] + Et + F),$$

for some choice of the parameters A, B, C, D, E, and F. The dominant sinusoidal variation that is visible in that plot shows the variation of the instantaneous frequency as a function of time.

(a) Determine one possible signal that could have produced the above spectrogram.

(b) Which of the six parameters above cannot be determined uniquely from the graph? (There may be more than one.)

PROBLEM 4.3*:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since the G above middle C is 2 tones below A440, its frequency is approximately $(440)2^{-2/12} \approx 392$ Hz. In musical notation the tones are called notes; the names of the notes in the octave starting with the G above middle-C and ending with the next G are:

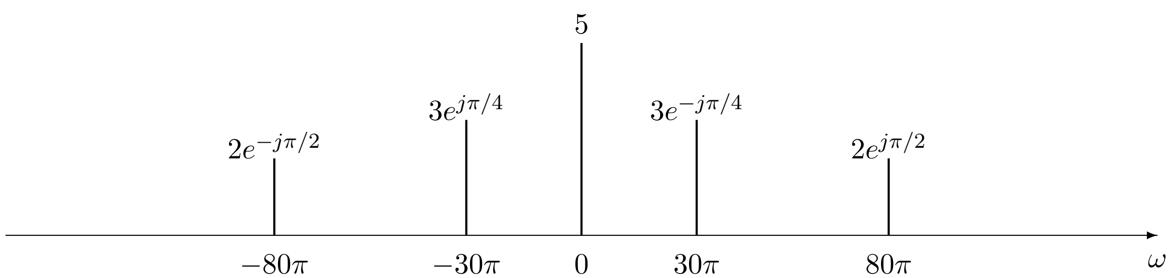
note name	G	$G\#$	A	$B^\flat$	B	C	$C\#$	D	$E^\flat$	E	F	$F\#$	G
note number	47	48	49	50	51	52	53	54	55	56	57	58	59
frequency													

(a) Make a table of the frequencies of the tones in the octave beginning with the G above middle-C assuming that the A above middle C is tuned to 440 Hz.

(b) The above notes on a piano are numbered 47 through 59. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

PROBLEM 4.4*:

A real periodic signal $x(t)$ has the following two-sided spectrum:



(a) Determine the period of this signal in seconds.

(b) Determine the DC value of this signal.

PROBLEM 4.5*:

Let $x(t)$ be a periodic signal with Fourier series coefficients, a_k.

(a) Consider the signal $y(t)$ which is related to $x(t)$ by

$$y(t) = 3x(t) - 2$$

Express the Fourier series coefficients for this signal, b_k, in terms of the coefficients a_k for $x(t)$. Hint: You might begin by reading the discussion in Problem P-3.14 on page 67 of the text. We are looking for a simple relationship, and finding it should not require that you compute any coefficients explicitly.

(b) Now consider the signal $z(t)$, which is related to $x(t)$ by

$$z(t) = x(t+1)$$

Express the Fourier series coefficients for this signal, c_k, in terms of the coefficients a_k for $x(t)$. Hint: Again, this is a simple relationship, and finding it should not require that you compute any coefficients explicitly.