GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #5

Assigned: 10-Sep-04
Due Date: Week of 20-Sep-04

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Sections 3-4, 3-5 and 3-6.
Start reading Chapter 4: *Sampling and Aliasing.*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 5.1*:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j30\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4+j2k} & \text{for } k = -3,-2,-1,0,1,2,3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

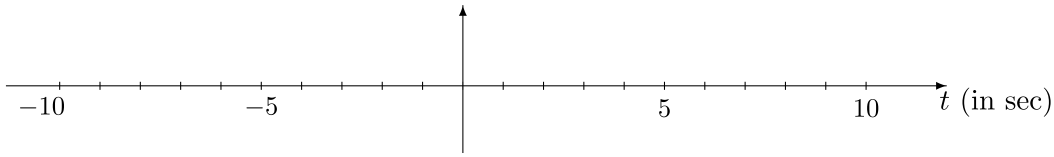
(a) Sketch the two-sided spectrum of this signal. Label all complex amplitudes in **polar form.**

(b) Determine the fundamental frequency (in Hz) and the fundamental period (in secs.) of this signal.

PROBLEM 5.2*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 15 & \text{for } 0 \le t \le 3 \\ -9 & \text{for } 3 < t < 8 \end{cases}$

(a) Assume that the period of $x(t)$ is 8 sec. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ sec.



(b) Determine the DC value of $x(t)$ from the Fourier series integral.

(c) Determine a general expression for the Fourier series coefficients a_k.

(d) Make a spectrum plot of this signal showing the frequency range $-\frac{1}{2} < f < \frac{1}{2}$ Hz.

PROBLEM 5.3*:

A signal $x(t)$ is periodic with period $T_0 = 8$. Therefore, it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{8}\int_{-2}^{2} \cos(\frac{\pi}{4}t)e^{-j(2\pi/8)kt}dt$$

(a) In the expression for a_k above, the integral and its limits effectively define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-8 \le t \le 8$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$ found in part (a).

PROBLEM 5.4*:

A periodic signal $x(t)$ is described over one period $-3 \le t < 3$ by the equation

$$x(t) = e^{-|t|/3} \qquad \text{for } -3 \le t < 3$$

The period of this signal is $T_0 = 6$ sec.

(a) Sketch the periodic function $x(t)$ for $-9 \le t < 9$.

(b) Determine a_0, the DC coefficient for the Fourier series.

(c) Set up the *Fourier analysis* integral for determining a_k for $k \neq 0$. (Insert proper limits and integrand.)

(d) Evaluate the integral in part (c) and obtain an expression for a_k that is valid for all $k \neq 0$.

(e) Make a plot of the spectrum over the range $-3\omega_0 \le \omega \le 3\omega_0$, where ω_0 is the fundamental frequency of the signal in rad/s. Use MATLAB or a calculator to determine the complex numerical values (in polar form) for each of the Fourier coefficients corresponding to this range of frequencies.

PROBLEM 5.5*:

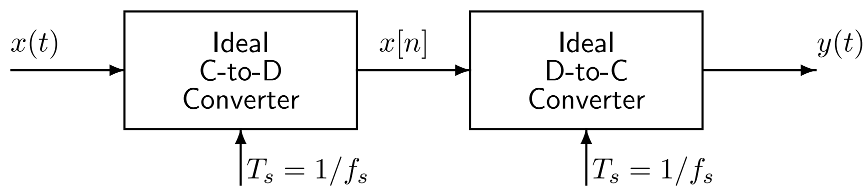


Figure 1: Ideal sampling and reconstruction system.

Two signals, $x_1(t)$ and $x_2(t)$, are described by the formulas

$$\begin{aligned} x_1(t) &= 5\cos(15\pi t + \frac{\pi}{4}) \\ x_2(t) &= 3\cos(20\pi t - \frac{\pi}{3}) \end{aligned}$$

For each of the signals $x(t)$ below, constructed from $x_1(t)$ and $x_2(t)$, determine the *minimum* sampling rate f_s that can be used with the above system so that $y(t) = x(t)$. Give a reason to justify your answer. *Hint:* apply the Sampling Theorem in Chapter 4.

(a) $x(t) = x_1(t) + x_2(t)$

(b) $x(t) = x_1^3(t)$

(c) $x(t) = x_1(t)x_2(t)$

(d) $x(t) = 2 - 3x_2(t-2)$