GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #6

Assigned: 17-Sep-04
Due Date: Week of 27-Sep-04

Reading: In *SP First*, Chapter 4: *Sampling*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

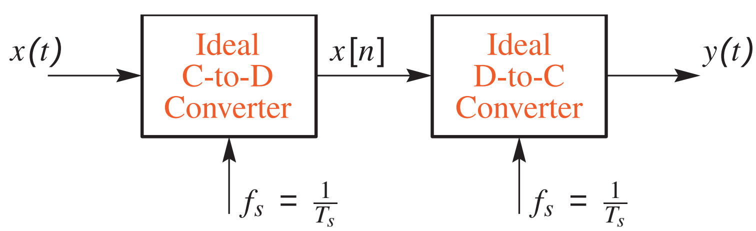


Figure 1: Ideal sampling and reconstruction systems. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

PROBLEM 6.1*:

Consider the ideal sampling and reconstruction system shown in Fig. 1.

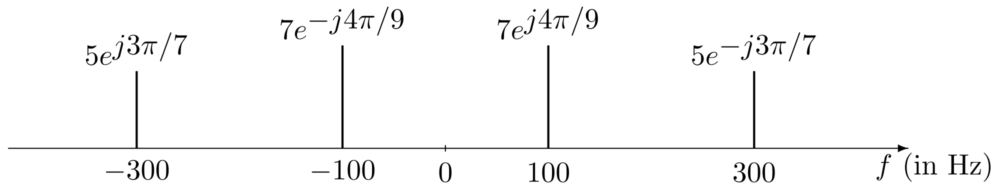
(a) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = 2\cos(0.7\pi n - 3\pi/4)$$

If the sampling rate of the C-to-D converter is $f_s = 9000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency between 36000 and 45000 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_{\rm s}) = x_2(nT_{\rm s})$ if $T_{\rm s} = 1/9000$ secs.

(b) Now if the input $x(t)$ to the system in Fig. 1 has the two-sided spectrum representation shown below, what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input

$x(t)$?

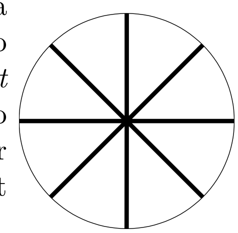


(c) Using the signal $x(t)$ from part (b), determine the spectrum for $x[n]$ when $f_s = 8000$ samples/sec. Simplify your as much as possible and make a plot for your answer, but label the frequency, and complex amplitude (magnitude and phase) of each spectral component.

PROBLEM 6.2*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the car's hubcaps have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.

In the figure to the right, an eight-spoked wheel is shown. Assume that the diameter of this wheel is 0.6 meters, which is nearly the tire diameter of a typical automobile. In addition, assume that the wheel is rotating clockwise, so that if attached to a car, the car would be traveling to the right *at a constant speed.* However, when seen on TV the spoke pattern of the car wheel appears to rotate backwards (i.e., CCW) at 1.5 revolutions per second. How fast is the car traveling (in kilometers per hour)? Derive a general equation that will make it easy to give all possible answers.



PROBLEM 6.3:

Signal Processing First, Chapter **4**, Problem **12**, page 98.

PROBLEM 6.4*:

Chirps are very useful signals for probing the behavior of sampling operations and illustrating the "folding" type of aliasing.

(a) If the input to the ideal C/D converter is $x(t) = 7\cos(1800\pi t + \pi/4)$, and the sampling frequency is 1000 Hz, then the output $y(t)$ is a sinusoid. Determine the formula for the output signal.

(b) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(2000\pi t - 400\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 1000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.** Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the SP-First function `spectgr()`.

PROBLEM 6.5*:

Suppose that a MATLAB function has been written to calculate a sum of discrete-time sinusoids. Here is the actual function:

```
function  xn = makedcos(omegahat,XX,Length)
%MAKEDCOS  make a discrete-time sinusoid for x[n]
%
xn = real( exp( j*(0:Length-1)'*omegahat(:)' ) * XX(:));
```

(a) Write an equation for $x[n]$, the discrete-time signal that is created by this MATLAB function, when the following function call is used:

```
xn = makedcos(pi*[0,0.35,0.70,1.75],[1,1+1i,-7i,2i],200001)
```

Your equation should be in terms of cosine functions. To do this you must figure out how the matrix multiplications and `exp( )` in the MATLAB statement defining `xn` work.[1]

(b) Draw a plot of the discrete-time spectrum (vs. $\hat{\omega}$) of the discrete-time signal defined by this MATLAB operation. Make sure that you include all the spectrum components in the $-\pi$ to $+\pi$ interval.

PROBLEM 6.6*:

This is a direct continuation of Problem 6.5*. Use your results from Problem 5.5(a) and (b) in this problem. The following MATLAB commands are used to make an output sound:

```
xn = makedcos(pi*[0,0.35,0.70,1.75],[1,1+1i,-7i,2i],200001);
soundsc(xn,10000)
```

Since we can listen to the sound produced by the `soundsc()` function, we can regard the `soundsc()` function as a D-to-C converter whose input is `xn`, and whose output is the analog signal that we hear.

(a) Draw a plot of the (idealized) continuous-time spectrum (vs. f in Hz) of the continuous-time signal that would be created at the output of an ideal D-to-C converter (approximately realized by the `soundsc()` function).

(b) Write an equation for $x(t)$, the continuous-time signal that is created at the output of the ideal D-to-C converter.

(c) What is the duration (in seconds) of the continuous-time signal $x(t)$?

[1] For this part, ignore the fact that the total length of the signal `xn` is finite.