GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #7

Assigned: 24-Sep-04
Due Date: Week of 4-Oct-04

Reading: In *SP First*, Chapter 5 of *FIR Filters* and Chapter 6 on *Frequency Response.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 7.1*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] + 2x[n-1] - 3x[n-2] - x[n-3] + x[n-5]$$

(a) Determine the impulse response $h[n]$ for this system.

(b) Determine the filter coefficients b_k in the causal FIR representation: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

(c) Determine the *order* of the filter (M), and the *length* of the filter (L).

(d) Use convolution to determine the output due to the input

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] = \begin{cases} 1, & n = 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \le n \le 12$.

PROBLEM 7.2:

Signal Processing First, Chapter **5**, Problem **15**, page 129.

PROBLEM 7.3:

Signal Processing First, Chapter **5**, Problem **18**, page 129.

PROBLEM 7.4*:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n-3] - 2x[n] + x[n+3]$

(b) $y[n] = (x[3-n])^2$

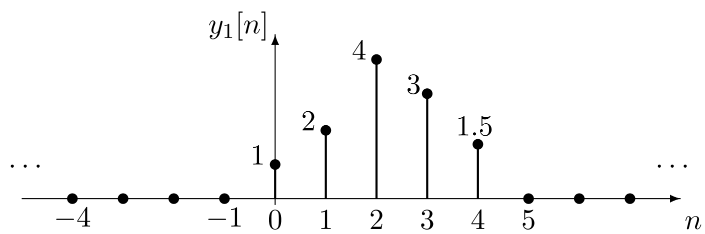
(c) $y[n] = x[n]\cos(0.3\pi n)$

PROBLEM 7.5*:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

(a) When tested with an input signal that is a shifted impulse, $x_1[n] = \delta[n+3]$, the observed output from the filter is the signal $y_1[n]$ shown below:



Use linearity and time-invariance to solve the following problem. Determine the output when the input to the LTI system is $x_2[n] = \delta[n+1] + 2\delta[n-2]$. Give your answer as a plot of $y_2[n]$ versus n, or a list of values for $-\infty < n < \infty$.

(b) State the property of *causality* found in the text. Is this system *causal?*

PROBLEM 7.6*:

Consider a system defined by $\quad y[n] = \sum_{k=0}^{M} b_k x[n-k]$

(a) What is the filter length?

(b) Suppose that the input $x[n]$ is non-zero only for $8 \le n \le 17$ and $M = 7$. Where will the output $y[n]$ first become non-zero? What is the index of the last non-zero value in the output sequence $y[n]$? What is the total length of the input sequence (in samples).

(c) Suppose that the input $x[n]$ is non-zero only for $N_1 \le n \le N_2$. What is the length of the input sequence (in samples).

(d) For the input in (b) and the above system, show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \le n \le N_4$ and determine N_3 and N_4.

(e) What is the length of the output sequence (in samples)?

PROBLEM 7.7*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

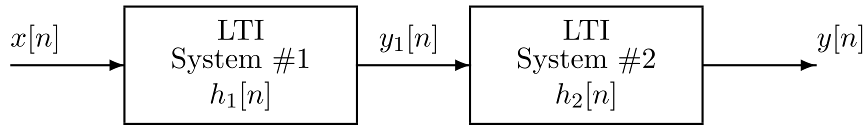


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a "predistortion" filter described by the difference equation

$$y_1[n] = x[n-2] + \beta x[n-3]$$

and System #2 is a distortion filter described by the impulse response:

$$h_2[n] = \sum_{k=1}^{6} (-\beta)^k \delta[n-k].$$

(a) Determine the impulse response sequence, $h_2[n]$, of the second system. Plot $h_2[n]$ versus n.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\beta = 0.75$.