GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #8

Assigned: 1-October-04
Due Date: Week of 11-October-04

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters*

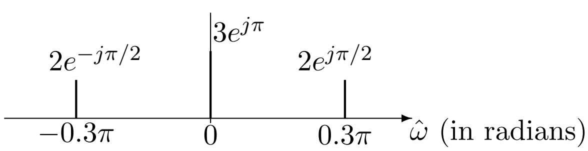
$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

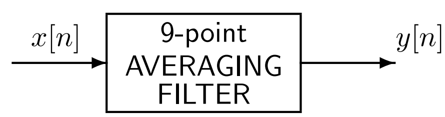
ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 8.1:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 8.2*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

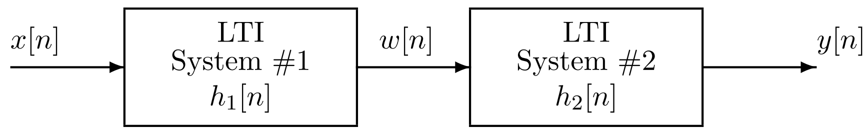


Figure 1: Cascade connection of two LTI systems.

Suppose that the two systems are described by the impulse responses

$$h_1[n] = \delta[n] + \delta[n-7] \qquad \text{and} \qquad h_2[n] = u[n] - u[n-7].$$

(a) Determine $H_1(e^{j\hat{\omega}})$, the frequency response of the first system.

(b) Determine $H_2(e^{j\hat{\omega}})$, the frequency response of the second system.

(c) By using numerical convolution, show that $h[n] = h_1[n] * h_2[n] = u[n] - u[n-14]$.

(d) From $h[n]$ determine $H(e^{j\hat{\omega}})$ the frequency response of the overall system (from $x[n]$ to $y[n]$).

(e) Show that your result in part (d) is the product of the results in parts (a) and (b); i.e., $H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}})$.

PROBLEM 8.3:

A discrete-time system is defined by the input/output relation

$$y[n] = 3x[n] + 6x[n-2] + 3x[n-4]. \qquad (1)$$

(a) Obtain an expression for the frequency response of this system.

(b) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

(c) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 10 - 10\cos(0.5\pi(n-1))$$

Hint: Use the frequency response and superposition to solve this problem.

PROBLEM 8.4*:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\begin{aligned} \mathcal{S}_1: &\quad y_1[n] = -x_1[n] + x_1[n-2] \\ \mathcal{S}_2: &\quad h_2[n] = 5\delta[n-3] + 5\delta[n-4] \\ \mathcal{S}_3: &\quad H_3(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}} \end{aligned}$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

The objective in this problem is to determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

(a) Determine the difference equation for $\mathcal{S}_3$.

(b) Determine the frequency response of the first two systems: $H_i(e^{j\hat{\omega}})$ for $i = 1, 2$.

(c) Determine the frequency response of the overall cascaded system.

(d) Write *one difference equation* that defines the overall system in terms of $x[n]$ and $y[n]$ only.

PROBLEM 8.5*:

The intention of the following MATLAB program is to filter a sinusoid via the `conv` function. However, the cosine signal has a starting point at $n = 0$; so we assume that it is zero for $n < 0$.

```
omegahat = pi/6;
nn = 0:4000;
xn = 4*cos(omegahat*nn + pi/2);
bb = ones(1,5)/5;
yn = conv( bb, xn );
```

(a) Determine a formula for $H(e^{j\hat{\omega}})$ for this FIR filter.

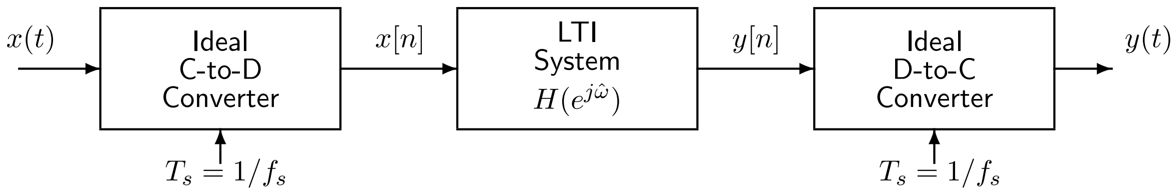
(b) Make a plot of the magnitude of $H(e^{j\hat{\omega}})$ and label *all* the frequencies where $|H(e^{j\hat{\omega}})|$ is zero. Use `freqz(bb, 1, ww)` in MATLAB, where `ww` is a vector of frequencies that defines a dense grid for $\hat{\omega}$.

(c) Use convolution to determine $y[n]$, the signal contained in the vector `yn`. Give the individual values for $n = 0, 1, 2, 3, 4$, and then provide a general formula for $y[n]$ that is correct for $4 \le n \le 4000$. This formula should give numerical values for the amplitude, phase and frequency of $y[n]$. *Hint:* the formula is a sinusoid for $n \ge 5$.

(d) It is possible to choose `omegahat` so that almost all of the output is zero. Give at least one value of `omegahat` such that the output is guaranteed to be zero, for $n \ge 4$ and $n \le 4000$.

PROBLEM 8.6*:

Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the frequency response of the discrete-time system is

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-j3\hat{\omega}}$$

(a) Make a plot of the frequency response magnitude for $H(e^{j\hat{\omega}})$ over the frequency range $-\pi < \hat{\omega} \le \pi$.

(b) For a sampling rate of $f_s = 500$ samples/sec, determine the frequency of an input sinusoid of the form $x(t) = \cos(\omega t)$ such that the resulting output will be zero.

(c) In this part, assume that the input is

$$x(t) = 5 + 3\cos(250\pi t + \pi/4) \quad \text{for} \quad -\infty < t < \infty$$

For a sampling rate of $f_s = 500$ samples/sec, determine the output $y(t)$ for $-\infty < t < \infty$.

PROBLEM 8.7*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 + e^{-j\pi/2}e^{-j\hat{\omega}})(1 + e^{j\pi/2}e^{-j\hat{\omega}}). \tag{2}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint:* Multiply out the factors to obtain a sum of powers of $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?

(d) Use superposition to determine the output of this system when the input is

$$x[n] = 1 + 2\delta[n-3] + 7\cos(0.5\pi n) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results.