GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #9

Assigned: 8-Oct-04
Due Date: Week of 25-Oct-04

Reading: In *SP First*, Chapter 7: *z-Transform*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 9.1*:

We now have ***four ways*** of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $h[n] = \frac{1}{2}(u[n-1] - u[n-5])$

(b) $H(e^{j\hat{\omega}}) = \sum_{k=2}^{5} e^{-jk\hat{\omega}}$

(c) $y[n] = \frac{1}{2}x[n-1] + 2x[n-4] + \frac{1}{2}x[n-7]$

PROBLEM 9.2*:

In each of the following parts, you are given a system function $H(z)$ and you must find: (i) the difference equation, (ii) the impulse response $h[n]$, and (iii) the frequency response, $H(e^{j\hat{\omega}})$.

(a) $H(z) = z^{-2}$

(b) $H(z) = 2(2 - 3z^{-4} - z^{-5})$

(c) $H(z) = \dfrac{1 - z^{-4}}{1 - z^{-1}}$

(d) $H(z) = (1 - z^{-1})^2(1 - \frac{1}{\sqrt{2}}e^{j\pi/4}z^{-1})(1 - \frac{1}{\sqrt{2}}e^{-j\pi/4}z^{-1})$

PROBLEM 9.3*:

Work Problem P-7.5 on page 191 of *Signal Processing First*.

PROBLEM 9.4:

Work Problem P-7.7 on page 192 of *Signal Processing First*.

PROBLEM 9.5:

A linear time-invariant filter is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] = \sum_{k=0}^{3} x[n-k]$$

(a) Find an expression for the frequency response $H(e^{j\hat{\omega}})$ of the system.

(b) Show that your answer in (a) can be expressed in the form

$$H(e^{j\hat{\omega}}) = \frac{\sin(4\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j1.5\hat{\omega}}.$$

(c) Sketch the frequency response (magnitude and phase) as a function of frequency from the formula above (or plot it using `freqz( )`).

(d) Suppose that the input is

$$x[n] = 1 + 2\cos(n\hat{\omega}_0) \text{ for } -\infty < n < \infty$$

Find a non-zero frequency $0 < \hat{\omega}_0 < \pi$ for which the output $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the sinusoid is removed by the filter.)

PROBLEM 9.6*:

Work Problem P-7.15 on page 194 of *Signal Processing First*.

PROBLEM 9.7*:

Consider the following MATLAB program:

```
nn = 0:22050;
xx = 1 + 2*cos(0.75*pi*nn-pi/3) + 5*cos(1.5*pi*nn+pi/4);
yy = conv([1,0,0,0,-1]/4,xx);
soundsc(yy,11025)
```

(a) What is the system function $H(z)$ of the system that is implemented by the `conv( )` statement?

(b) What is the frequency response of the system?

(c) Neglecting the end effects in the convolution, determine $y(t)$ that describes the signal produced by the `soundsc( )` statement.