9.1 (a)

Impulse Response

$$h[n] = 1/2\,(u[n-1] - u[n-5])$$

Difference equation:

$$y[n] = 1/2\,x[n-1] + 1/2\,x[n-2] + 1/2\,x[n-3] + 1/2\,x[n-4]$$

Frequency Response:

$$H(e^{j\hat{\omega}}) = 1/2 \sum_{k=1}^{4} e^{-jk\hat{\omega}}$$

System Function

$$H(z) = 1/2 \sum_{k=1}^{4} z^{-k}$$

(b) Frequency Response:

$$H(e^{j\hat{\omega}}) = \sum_{k=2}^{5} e^{-jk\hat{\omega}}$$

Impulse Response:

$$h[n] = \sum_{k=2}^{5} \delta[n-k]$$

Difference Equation:

$$y[n] = x[n-2] + x[n-3] + x[n-4] + x[n-5]$$

System Function:

$$H(z) = \sum_{k=2}^{5} z^{-k}$$

(c) Difference Equation:

$$y[n] = \tfrac{1}{2}x[n-1] + 2x[n-4] + \tfrac{1}{2}x(n-7)$$

Impulse Response:

$$h[n] = \tfrac{1}{2}\delta[n-1] + 2\delta[n-4] + \tfrac{1}{2}\delta(n-7)$$

Frequency Response:

$$H(e^{j\hat{\omega}}) = \tfrac{1}{2}e^{-j\hat{\omega}} + 2e^{-j4\hat{\omega}} + \tfrac{1}{2}e^{-j7\hat{\omega}}$$

System Function:

$$H(z) = (\tfrac{1}{2})z^{-1} + 2z^{-4} + (\tfrac{1}{2})z^{-7}$$

$\boxed{\text{Problem 9.2}}$

(a) Given $H(z) = z^{-2}$

Difference Equation

$$y(n] = x[n-2]$$

Impulse Response

$$h[n] = \delta[n-2]$$

9.2 continued

Frequency Response:

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}$$

(b) Given $H(z) = 2(2 - 3z^{-4} - z^{-5})$

$$= 4 - 6z^{-4} - 2z^{-5}$$

Difference Equation

$$y[n] = 4x[n] - 6[x-4] - 2[x-5]$$

Impulse Response

$$h[n] = 4\delta[n] - 6\delta[x-4] - 2\delta[x-5]$$

Frequency Response:

$$H(e^{j\hat{\omega}}) = 4 - 6e^{-j4\hat{\omega}} - 2e^{-j5\hat{\omega}}$$

(c) Given $H(z) = \dfrac{1 - z^{-4}}{1 - z^{-1}} = 1 + z^{-1} + z^{-2} + z^{-3}$

Difference Equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

Impulse Response:

$$h[n] = \sum_{k=0}^{3} \delta[n-k]$$

Frequency Response:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{3} e^{-jk\hat{\omega}}$$

(d) Given

$$H(z) = (1-z^{-1})^2 \left(1-\tfrac{1}{\sqrt{2}} e^{j\pi/4} z^{-1}\right)\left(1-\tfrac{1}{\sqrt{2}} e^{-j\pi/4} z^{-1}\right)$$

$$= (1-z^{-1})^2\left[1 - z^{-1}\left(1/\sqrt{2}\, e^{-j\pi/4} + 1/\sqrt{2}\, e^{j\pi/4}\right) + \left(\tfrac{1}{\sqrt{2}} e^{j\pi/4)} z^{-1}\right)\left(\tfrac{1}{\sqrt{2}} e^{-j\pi/4} z^{-1}\right)\right]$$

$$= (1-z^{-1})^2\left[1 - z^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) + 1/2\, z^{-2}\right]$$

$$= (1-2z^{-1}+z^{-2})(1-z^{-1}+0.5z^{-2})$$

$$= 1 - 3z^{-1} + 3.5z^{-2} - 2z^{-3} + 0.5z^{-4}$$

Difference Equation:

$$y[n] = x[n] - 3x[n-1] + 3.5x[n-2] - 2x[n-3] + 0.5x[n-4]$$

Impulse Response:

$$h[n] = \delta[n] - 3\delta[n-1] + 3.5\delta[n-2] - 2\delta[n-3] + \tfrac{1}{2}\delta[n-4]$$

Frequency Response:

$$H(e^{j\hat{\omega}}) = 1 - 3e^{-j1\pi} + 3.5e^{-j2\pi} - 2e^{-j3\pi} + \tfrac{1}{2}e^{-j4\pi}$$

Problem 9.3

(a) $H(z)$ is already nicely factored for you. To find the difference equation we need the coefficients b_k. You can just multiply out the factors of $H(z)$, or you can use the Matlab "Poly" function to find the coefficients, or use your calculator if it has such a function. We will use "Poly".

It is helpful to start by multiplying $H(z)$ by $\frac{z^5}{z^5}$ giving:

$$H(z) = \frac{(z-1)\left(z-e^{j\pi/2}\right)\left(z-e^{-j\pi/2}\right)\left(z-0.9e^{j\pi/3}\right)\left(z-0.9e^{-j\pi/3}\right)}{z^5}$$

9.3 continued

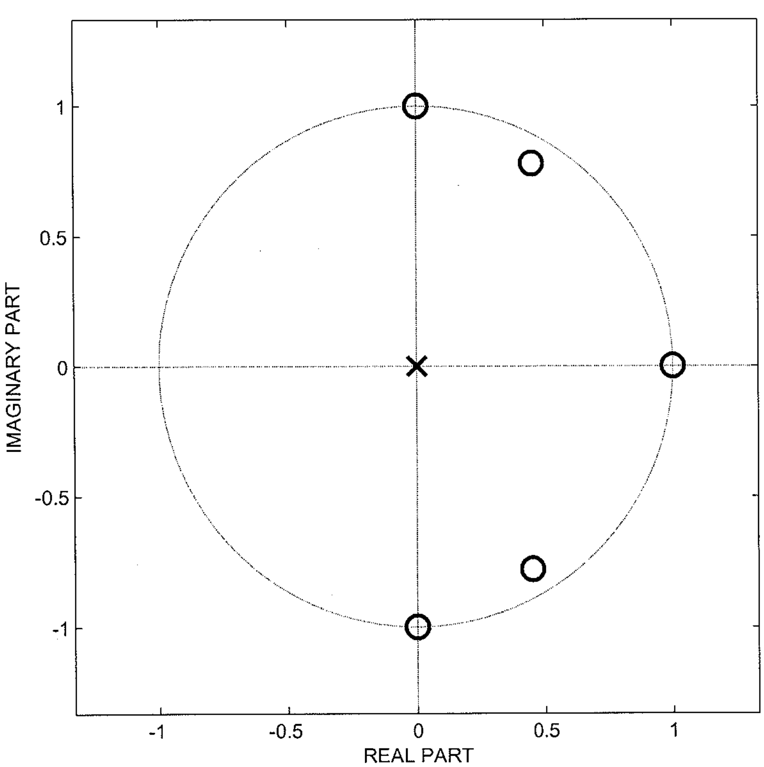
The zeros in the numerator are easily found as $z=1$, $z=e^{j\pi/2}$, $z=e^{-j\pi/2}$, $z=0.9e^{j\pi/3}$, $z=0.9e^{-j\pi/3}$

Applying this vector to Matlab's "poly" function gives $b_k = [1, -1.9, 2.71, -2.71, 1.71, -0.81]$

From this, the difference equation is:

$$y[n] = x[n] - 1.9x[n-1] + 2.71x[n-2] - 2.71x[n-3] + 1.71x[n-4] - 0.81x[n-5]$$

(b) From our expression for $H(z)$ in part (a), there are 5 poles at $z=0$. The zeros we know. Use Matlab's zplot of zzplane from the SP-first collection.



(c) A zero on the unit circle will force the frequency response to be zero at the frequency corresponding to the angular position of the zero. The zeros <u>on</u> the unit circle are $z=1$, $z=e^{j\pi/2}$ and $z=e^{-j\pi/2}$. This corresponds to angular positions of these zeros, $\hat{\omega}$, equal to $\boxed{\hat{\omega}=0,\ \hat{\omega}=\pi/2 \text{ and } \hat{\omega}=-\pi/2.}$ Note that the zeros at $z=0.9e^{j\pi/3}$ and $z=0.9e^{-j\pi/3}$ do <u>not</u> lie on the unit circle, so the frequency response corresponding to those angulare positions are not zero.

Problem 9.4

You can factor $P(z)$ using "roots" in MATlab:

```
roots=([1, 0.5, 0.5, 1])
```

This will find 3 roots at:

$z=-1$

$z=e^{j1.318}$

$z=e^{-j1.318}$

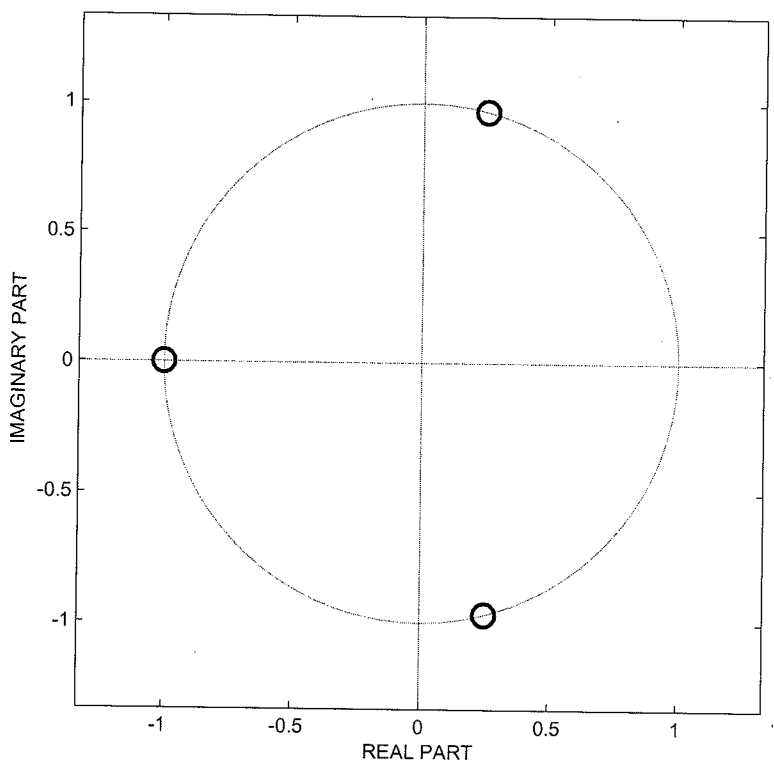
9.4 continued

Plot the roots using Matlab's zplane or zzplane from SP-first:

```
zzplane([-1, exp(j*1.318), exp(-j*1.318)],[])
```

or, have one call the other:

```
zzplane(roots([1,0.5,0.5,1]))
```



Problem 9.5

a) $$H(e^{j\hat{\omega}}) = \sum_{k=0}^{3} e^{-j\hat{\omega}k}$$

This follows from the general expression for the frequency response $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$

where $M=3$ & $h[0]=h[1]=h[2]=h[3]=1$

b) The difference equation $y[n] = \sum_{k=0}^{3} x[n-k]$ is a L-point running sum. Its related to a L-point running averager, except you don't divide by L. It is:

$$y[n] = \sum_{k=0}^{3} x[n-k] = \sum_{k=0}^{L-1} x[n-k]$$

using the result from section a)

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} e^{-j\hat{\omega}k} = \sum_{k=0}^{L-1} e^{-j\hat{\omega}k}$$

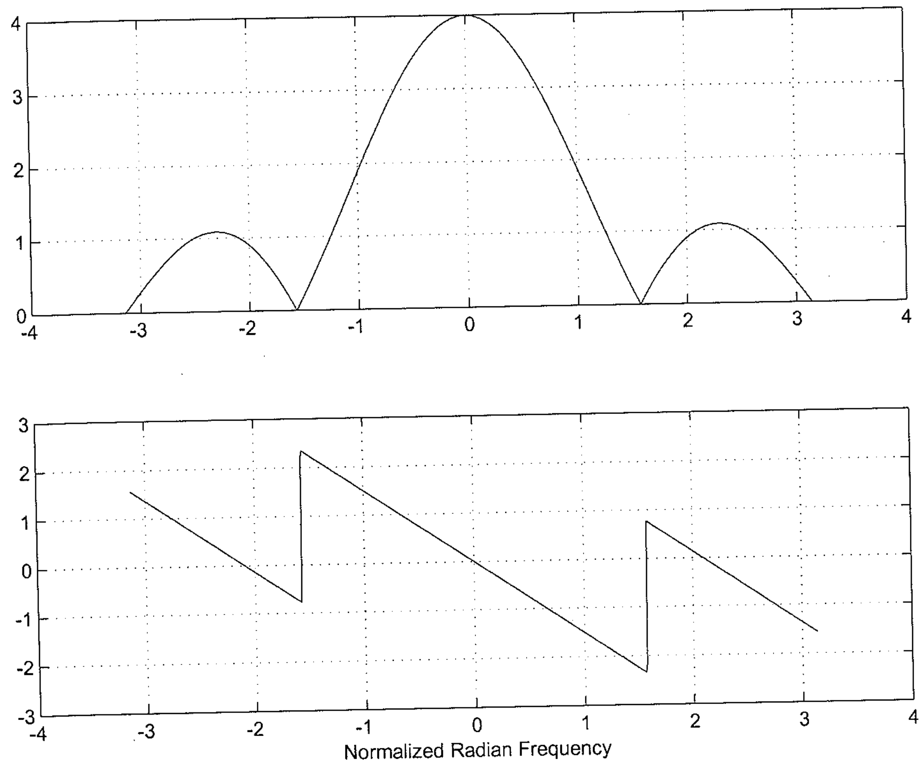
$$= \frac{1-e^{-j\hat{\omega}L}}{1-e^{-j\hat{\omega}}} = \frac{e^{-j\hat{\omega}L/2}\left(e^{j\hat{\omega}L/2} - e^{-j\hat{\omega}L/2}\right)}{e^{-j\hat{\omega}/2}\left(e^{j\hat{\omega}/2} - e^{-j\hat{\omega}/2}\right)}$$

$$= \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}(L-1/2)}$$

$\boxed{\text{If } L=4}$ $H(e^{j\hat{\omega}}) = \dfrac{\sin(4\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j\hat{\omega}3/2}$

prob 9.5 continued

c)



d) From the plot in part c), the amplidue of the frequency response is zero at $\hat{\omega} = \pm \pi/2$ and $\hat{\omega} = \pm \pi$. Therefore a non-zero frequency where $y[n]$ is a constant for all n is $\hat{\omega} = \pi/2$

ie $y[n] = 1\, H(e^{j0}) + 2\cos(\pi/2\, n)\, H(e^{j\pi/2})$

where $H(e^{j\pi/2}) = \emptyset$

$\therefore$ $y[n] = 1$ for all n @ $\hat{\omega} = \pi/2$

Problem 9.6

After ideal C-D conversion, we have

$$x[n] = 4 + \cos\left(\frac{250\pi}{1000}n - \pi/4\right) - 3\cos\left(\frac{2000\pi}{3000}n\right)$$

$$= 4 + \cos\left(\frac{\pi n}{4} - \pi/4\right) - 3\cos\left(\frac{2\pi}{3}n\right)$$

Now, to find $y[n]$ we need to evaluate $H(z)$ at frequencies 0, $\pi/4$ & $\frac{2\pi}{3}$

$$H(z) = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right) = \frac{1}{3}\sum_{k=0}^{2} z^{-k}$$

The relation between the z-domain and the $\hat{\omega}$-domain is based on $z = e^{j\hat{\omega}}$ or

$$H(z) = \frac{1}{3}\sum_{k=0}^{2} z^{-k} \longrightarrow \frac{1}{3}\sum_{k=0}^{2} e^{-j\hat{\omega}k} = H(e^{j\hat{\omega}})$$

This is a running-averager of length $= 3$

We recall from equation 6.22 in the text that the general equation for a L-point running averager is:

$$H(e^{j\hat{\omega}}) = \frac{1}{L}\sum_{k=0}^{L-1} e^{-j\hat{\omega}k}$$

In this case $L = 3$

Prob 9.6 continued.

Using equation 6.25 in the text then gives for $L=3$

$$H(e^{j\hat{\omega}}) = \left(\frac{\sin \hat{\omega}\, 3/2}{3 \sin \hat{\omega}/2}\right) e^{-j\hat{\omega}}$$

To get the output signal $y[n]$ we need to evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0,\ \pi/4\ \&\ 2\pi/3$ as determined earlier in the problem.

$$H(e^{j0}) = 1$$

$$H(e^{j\pi/4}) = .804\, e^{-j\pi/4}$$

$$H(e^{j2\pi/3}) = 0$$

Giving for $y[n]$:

$$y[n] = 4 + .804 \cos\left(\frac{\pi n}{4} - \pi/4 - \pi/4\right) - 0 \cos\left(\frac{2\pi}{3} n\right)$$

$$= 4 + .804 \cos\left(\frac{\pi n}{4} - \pi/2\right)$$

Prob 9.6 CONT.

Converting back to continuous time using $f_s = 1000\ Hz$

$$y(t) = 4 + .804\cos\left(\frac{\pi n}{4}1000 - \pi/2\right)$$

$$y(t) = 4 + .804\cos\left(250\pi t - \pi/2\right)$$

$\boxed{\text{Problem 9.7}}$

a) The system function, $H(z)$, used in the conv() statement is:

$$H(z) = 1/4\left(1 - z^{-4}\right)$$

b) Using the relationship $z = e^{j\hat{\omega}}$, the frequency response is

$$H(e^{j\hat{\omega}}) = 1/4\left(1 - e^{-j\hat{\omega}4}\right)$$

Prob 9.7 cont.

c) Using the results of section b), we evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0, \frac{3\pi}{4}$ and $(1.5\pi - 2\pi = \underline{0.5\pi})$

subtract 2π from 1.5π to get $-\pi \le \hat{\omega} \le \pi$.

$H(e^{j0}) = 0$

$H(e^{j\frac{3\pi}{4}}) = 1/2$

$H(e^{j\pi/2}) = 0$

This gives

$y[n] = 0 + 2/2 \cos\left(\frac{3}{4}\pi n - \pi/3\right) + 0$

Fs for reconstruction is 11025 Hz. Thus

$y(t) = \cos\left(\frac{3\pi}{4}(11025)n - \pi/3\right)$

$= \cos\left(8269\pi t - \pi/3\right)$