GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #10

Assigned: 22-Oct-04
Due Date: Week of 1-Nov-04

Quiz #3 will be given on Friday, 19-November. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, all Chapter 9: *Continuous-Time Signals & Systems.*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 10.1:

Signal Processing First, Chapter **9**, Problem **1**, page 279. (Steps and Pulse-like Signals)

PROBLEM 10.2*:

Try your hand at expressing each of the following ***continuous-time*** signals into a simpler form:

(a) $[e^{-t/2}u(t-1.5) - 2\sin(7\pi t)u(t)]\delta(t+0.1) =$

(b) $[\cos(1.5\pi(t-1)) + u(t-1)] * \delta(t-3) =$

(c) $\displaystyle\int_{-\infty}^{t+0.002} \delta(\tau - 0.001)\cos\frac{\pi}{2}\tau u(\tau)d\tau =$

(d) $\displaystyle\int_{-\infty}^{2} e^{\tau}\delta(\tau+1)\,d\tau$

(e) $\displaystyle\frac{d}{dt}\left\{3\cos(\frac{3}{2}\pi t)[u(t) - u(t-2)]\right\} =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \quad u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution. Convolution is denoted by a "star", as in $x(t) * \delta(t-2) = x(t-2)$ and multiplication is usually indicated as in $x(t)\delta(t-2) = x(2)\delta(t-2)$.

PROBLEM 10.3*:

Signal Processing First, Chapter **9**, Problem **2**, page 279. (Linearity and Time-Invariance)

PROBLEM 10.4*:

A linear time-invariant system has impulse response:

$$h(t) = e^{-2t}\{u(t+2) - u(t-2)\} = \begin{cases} e^{-2t} & -2 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t-\tau)$ as a function of τ for $t = 0$, 2, and 10.

(b) Find the output $y(t)$ when the input is $x(t) = \delta(t-3)$, and make a sketch of $y(t)$.

(c) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = e^{-0.5t}\{u(t+1) - u(t-6)\} = \begin{cases} e^{-0.5t} & -1 \leq t < 6 \\ 0 & \text{otherwise} \end{cases}$$

PROBLEM 10.5*:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{t-3}^{t+1} x(\tau - 1)\, d\tau$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof (if true) or counter-example (if false).

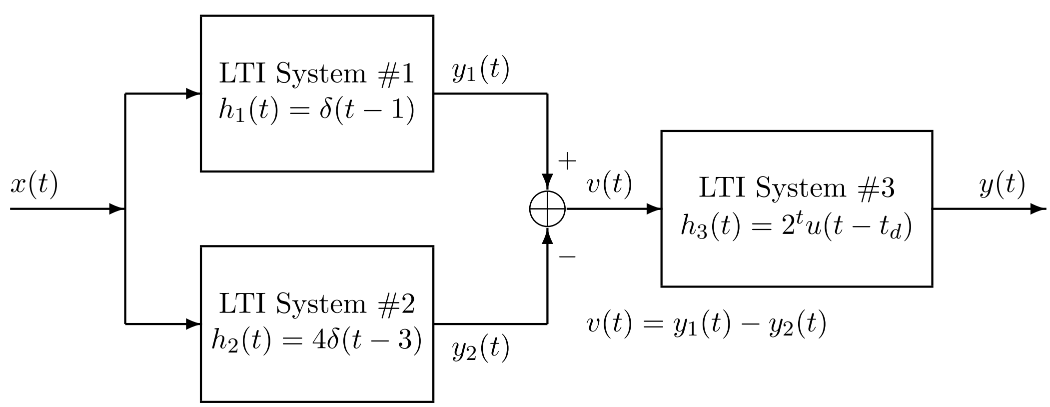
(c) Is it a causal system? Explain with a proof (if true) or counter-example (if false).

(d) Use the convolution integral to determine the output of the system when the input is the pulse

$$x(t) = u(t-1) - u(t+1)$$

(e) (*Optional:*) Use the GUI `cconvdemo` to verify your answer to part (d).

PROBLEM 10.6*:

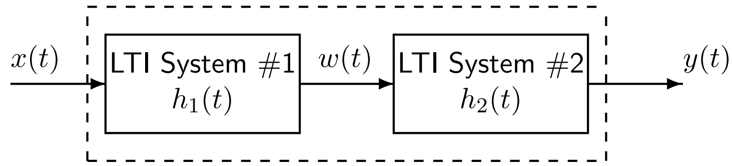


(a) If $t_d = 4$, what is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer <u>both</u> as an equation and as a carefully labeled sketch.

(b) How should the time delay t_d be chosen so that the overall system is causal?

(c) Which systems (#1, #2, #3) are stable? Is the overall system a stable system? Explain to receive credit.

PROBLEM 10.7:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = e^{-t}u(t - \tfrac{1}{2})$$

and the second system is described by the input/output relation

$$y(t) = \frac{d}{dt}w(t-2) - \int_{-\infty}^{t-2} w(\tau)d\tau$$

(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

(b) Find the output when the input signal is $x(t) = e^{t}u(t)$.