GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #11

Assigned: 29-Oct-04
Due Date: Week of 8-Nov-04

Quiz #3 will be given on 19-November. One page ($8\frac{1}{2} \times 11''$) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 10: *Frequency Response*
Chapter 11: *Continuous-Time Fourier Transforms.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 11.1*:

A continuous-time system is defined by the impulse response:

$$h(t) = \frac{5}{3}[ae^{-at}u(t) - be^{-bt}]u(t)$$

(a) Determine a simple expression for the frequency response of this system.

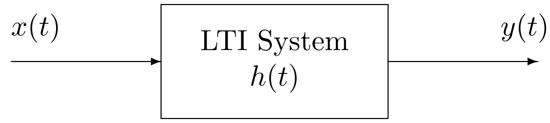
(b) Make a plot of the frequency response (magnitude only) when $a = 40\pi$, $b = 10\pi$.

(c) Describe the type of filter in the plot of part (b), e.g., LPF, HPF, BPF, or something else.

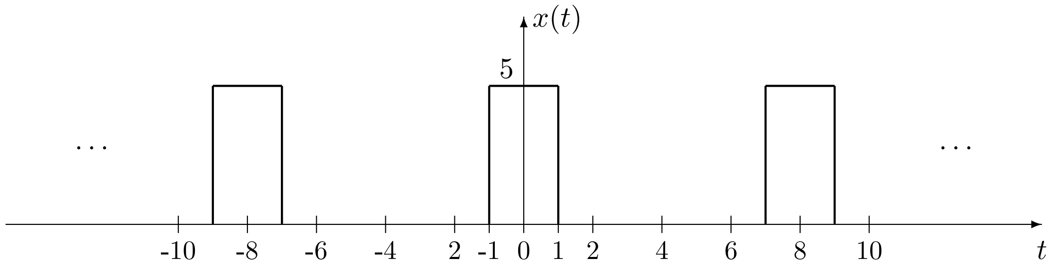
(d) Find the output $y(t)$ when the input signal is $x(t) = 40 + 10\cos(20\pi t) + 30\cos(100\pi t)$, and the parameters a and b are as in part (b).

PROBLEM 11.2*:

Consider the LTI system below:



The input to this system is the periodic pulse wave $x(t)$ depicted below:



The input can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{where} \quad a_k = \frac{5\sin(\pi k/4)}{\pi k}.$$

(a) Determine ω_0 in the Fourier series representation of $x(t)$. Also, write down the integral that must be evaluated to obtain the Fourier coefficients a_k.

(b) Plot the spectrum of the signal $x(t)$; i.e., make a plot showing the a_k's plotted at the frequencies $k\omega_0$ for $-4\omega_0 \le \omega \le 4\omega_0$.

(c) If the frequency response of the system is an ideal *lowpass* filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where ω_c is the *cutoff frequency*, for what values of ω_c will the output of the system have the form

$$y(t) = A + B\cos(\omega_0 t + \phi)$$

where A and B are nonzero?

(d) If the frequency response of the system is the ideal *highpass* filter

$$H(j\omega) = \begin{cases} 0 & |\omega| < \pi/8 \\ 1 & |\omega| > \pi/8 \end{cases}$$

plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: First determine what frequency is removed by the filter, and then determine what effect this will have on the waveform.*

(e) If the frequency response of the LTI system is $H(j\omega) = e^{-j2\omega} - e^{-j4\omega}$, plot the output of the system, $y(t)$, when the input is $x(t)$ as plotted above. *Hint: In this case it will be easiest to determine the impulse response $h(t)$ corresponding to $H(j\omega)$ and from $h(t)$ you can easily find an equation that relates $y(t)$ to $x(t)$. This will allow you to plot $y(t)$.*

PROBLEM 11.3*:

The delay property of Fourier transform states that if $X(j\omega)$ is the Fourier transform of $x(t)$, the the Fourier transform of $x(t-t_d)$ is $e^{-j\omega t_d}X(j\omega)$, i.e.,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d}X(j\omega).$$

Use this property to find the Fourier transforms of the following signals. Simplify your answer as much as possible.

(a) $x(t) = -\delta(t) + 2\delta(t-1) - \delta(t-2)$

(b) $x(t) = 10\dfrac{\sin(5\pi(t-4))}{\pi(t-4)}$

(c) $x(t) = 4e^{-2t}u(t) - 4e^{-2t}u(t-1)$

PROBLEM 11.4*:

For each of the following cases, use the table of known Fourier transform pairs to complete the following Fourier transform pair relationships.

(a) Find $x(t)$ if $X(j\omega) = \dfrac{j\omega}{100+j\omega}e^{-j0.02\omega}$.

(b) Find $x(t)$ if $X(j\omega) = 15 - 15\cos(\omega)$.

(c) Find $x(t)$ if $X(j\omega) = \dfrac{1}{1+j\omega} - \dfrac{1}{1+j2\omega}$.

(d) Find $x(t)$ if $X(j\omega) = e^{-j\omega/2}[j\delta(\omega-150\pi) - j\delta(\omega+150\pi)]$.

PROBLEM 11.5*:

A continuous-time LTI system is defined by the following input/output relation:

$$y(t) = x(t) - 3x(t-T) + 3x(t-2T) - x(t-3T). \qquad (1)$$

(a) Find the impulse response $h(t)$ of the system; i.e., determine the output when the input is an impulse.

(b) Substitute your answer for $h(t)$ into the integral formula

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

to find the frequency response. Express your answer in the form $H(j\omega) = jAe^{-j\omega t_d}\sin^3(B\omega)$ where A, B, and t_d are real numbers.

(c) Apply the system definition given in Eq. (1) directly to the input $x(t) = e^{j\omega t}$ for $-\infty < t < \infty$ and show that $y(t) = H(j\omega)e^{j\omega t}$, where $H(j\omega)$ is as determined in part (b); i.e., just substitute $x(t) = e^{j\omega t}$ into Eq. (1).

(d) Sketch the magnitude $|H(j\omega)|$ and phase $\angle H(j\omega)$ as functions of ωT.