11.1) $$h(t) = \frac{5}{3}\left(ae^{-at}u(t) - be^{-bt}\right)u(t)$$

(a) assuming $a>0$ and $b>0$,

$$H(j\omega) = \mathcal{F}\left\{\frac{5}{3}\left(ae^{-at}u(t) - be^{-bt}\right)u(t)\right\}$$

$$= \frac{5}{3}a\,\mathcal{F}\{e^{-at}u(t)\} - \frac{5}{3}b\,\mathcal{F}\{e^{-bt}u(t)\}$$

$$= \frac{5}{3}a\,\frac{1}{a+j\omega} - \frac{5}{3}b\,\frac{1}{b+j\omega}$$

$$= \frac{5}{3}(a-b)\,\frac{j\omega}{(a+j\omega)(b+j\omega)}$$

(b) $a = 40\pi$, $b = 10\pi$

$\Downarrow$

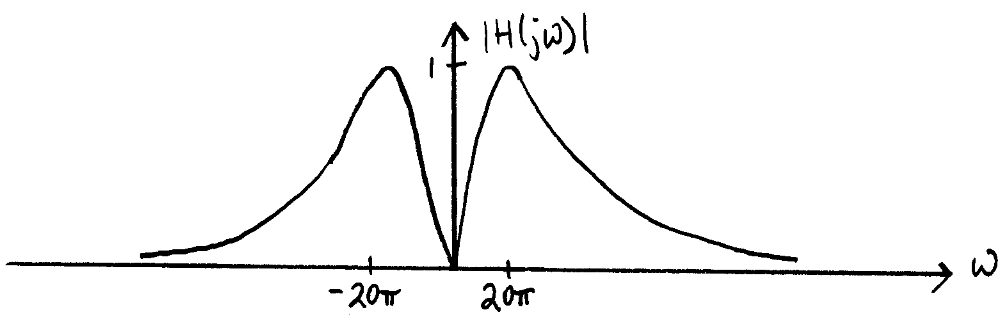
$$H(j\omega) = 50\pi\,\frac{j\omega}{(40\pi+j\omega)(10\pi+j\omega)}$$

$$|H(j\omega)| = 50\pi\,\frac{|\omega|}{\sqrt{(40\pi)^2+\omega^2}\cdot\sqrt{(10\pi)^2+\omega^2}}$$

by inspection, $|H(j\omega)| \to 0$ as $\omega \to 0$ and $\omega \to \pm\infty$ and the peak value of $|H(j\omega)|$ will occur between $\omega = 10\pi$ and $\omega = 40\pi$

to find location and value of peak, set

$$\frac{\partial |H(j\omega)|}{\partial \omega} = 0 \iff \omega^* = 20\pi, \quad |H(j\omega^*)| = 1$$



(c) the filter is a "bandpass" filter, BPF

(d)

$$x(t) = 40 + 10\cos(20\pi t) + 30\cos(100\pi t)$$

$\omega = 0)$ $\quad H(j0) = 0$

$\omega = 20\pi)$ $\quad H(j20\pi) = 1$

$\omega = 100\pi)$ $\quad H(j100\pi) = 0.4619\, e^{-j1.0906}$

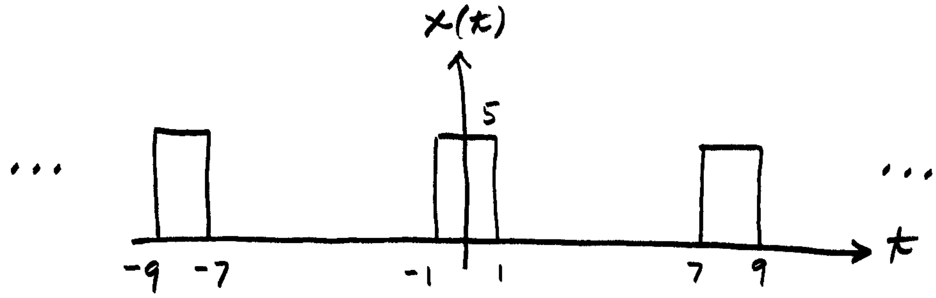
$$y(t) = 40\, H(j0) + 10\,|H(j20\pi)| \cos\left(20\pi t + \angle H(j20\pi)\right) + 30\,|H(j100\pi)| \cos\left(100\pi t + \angle H(j100\pi)\right)$$

$$= 10\cos(20\pi t) + 30(0.4619)\cos(100\pi t - 1.0906)$$

$$= 10\cos(20\pi t) + 13.858\cos(100\pi t - 1.0906)$$

11.2)

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$



(a)

$$a_k = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, e^{-jk\frac{2\pi}{T_0}t}\, dt \quad , \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$= \frac{1}{8}\int_{-1}^{1} 5e^{-jk\frac{\pi}{4}t}\, dt$$

$$= \frac{5}{8}\ \frac{-4}{jk\pi}\ e^{-jk\frac{\pi}{4}t}\Big|_{-1}^{1}$$

$$= \frac{5}{2jk\pi}\left(e^{jk\frac{\pi}{4}} - e^{-jk\frac{\pi}{4}}\right)$$

$$a_k = \frac{5\sin\left(k\frac{\pi}{4}\right)}{k\pi} \quad \Rightarrow \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{\pi}{4}t}$$

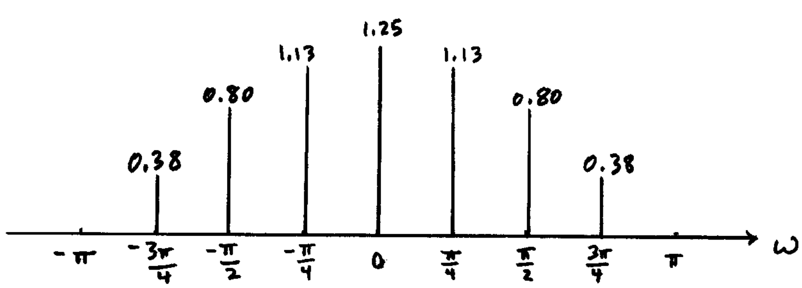
(b) $$a_0 = \lim_{k\to 0} \frac{5\sin\left(k\frac{\pi}{4}\right)}{k\pi} = \lim_{k\to 0} \frac{\frac{5\pi}{4}\cos\left(k\frac{\pi}{4}\right)}{\pi} = \frac{5}{4} = 1.25$$

$a_{\pm 1} = 1.1254$

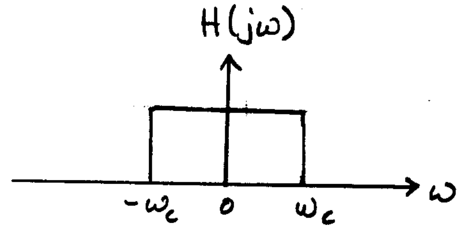
$a_{\pm 2} = 0.7958$

$a_{\pm 3} = 0.3751$

$a_{\pm 4} = 0$



(c) If $H(j\omega) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$



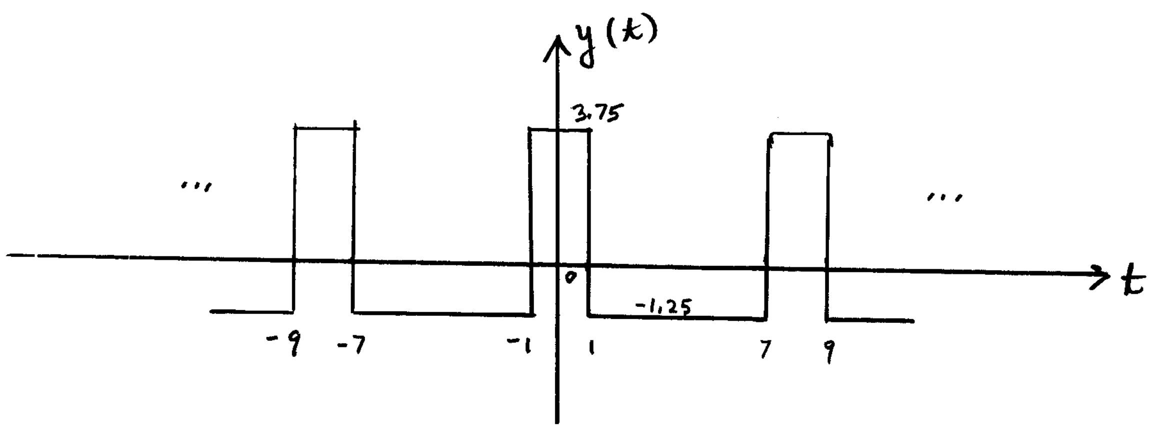
then $y(t) = A + B\cos(\omega_0 t + \phi)$
where $A \neq 0$ and $B \neq 0$, provided that

$$\frac{\pi}{4} < \omega_c < \frac{\pi}{2}$$

In that case, $A = a_0 = 1.25$, $B = 2a_1 = 2.25$

(d) $\omega_c = \frac{\pi}{8}$ $\Rightarrow$ all components of $x(t)$ will be passed except for the dc component.

$$\Rightarrow \quad y(t) = x(t) - a_0 = x(t) - \frac{5}{4}$$



(e) $H(j\omega) = e^{-j2\omega} - e^{-j4\omega}$

From Table 11-3, we know that

$$x(t - t_d) \longleftrightarrow e^{-j\omega t_d} X(j\omega)$$

and thus

$$h(t) = \mathcal{F}^{-1}\{H(j\omega)\} = \delta(t-2) - \delta(t-4)$$

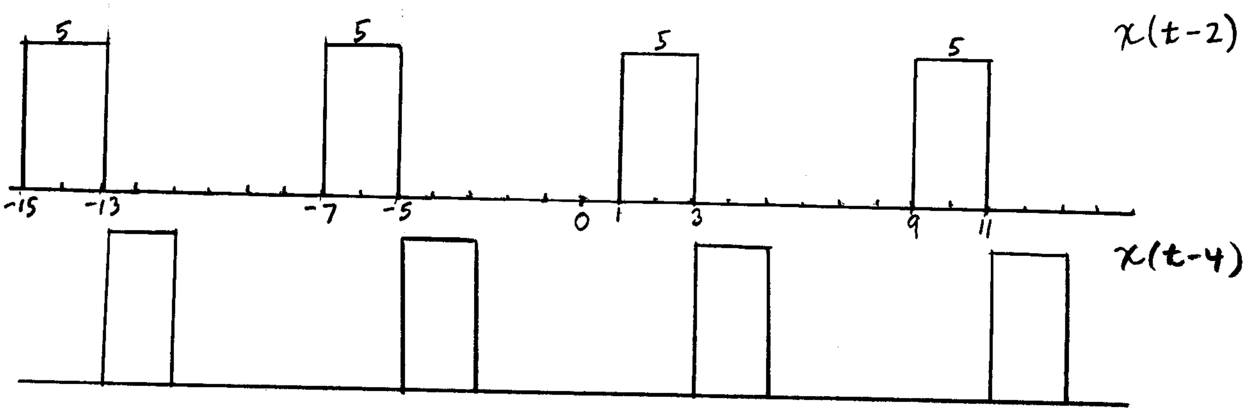
$$y(t) = h(t) * x(t)$$

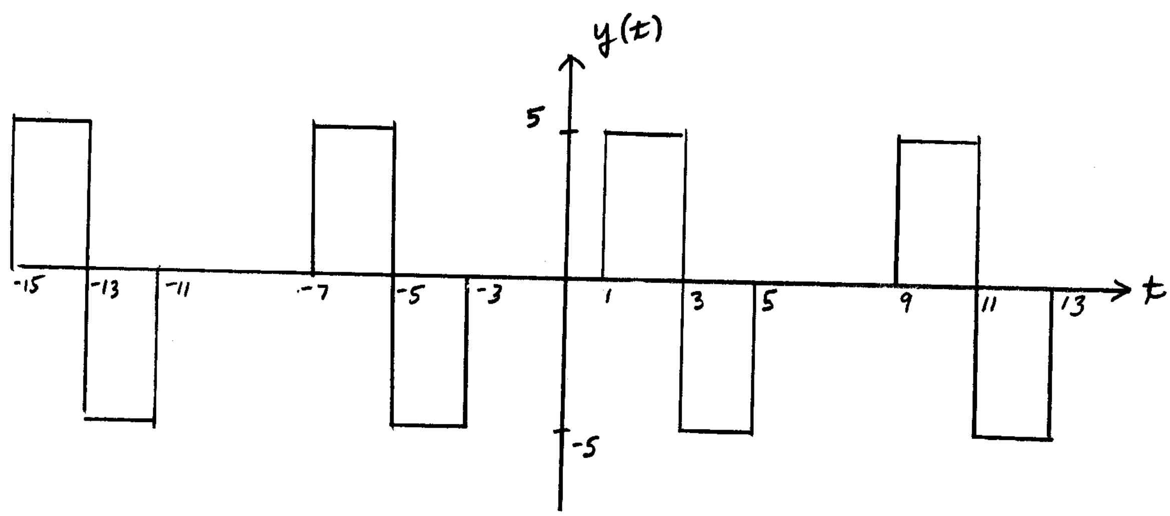
$$= \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left(\delta(t-\tau-2) - \delta(t-\tau-4)\right) x(\tau)\, d\tau$$

$$= x(t-2)\int_{-\infty}^{\infty} \delta(t-\tau-2)\, d\tau - x(t-4)\int_{-\infty}^{\infty} \delta(t-\tau-4)\, d\tau$$

$$= x(t-2) - x(t-4)$$





11.3)

$$x(t-t_d) \longleftrightarrow e^{-j\omega t_d} X(j\omega)$$

(a)
$$X(j\omega) = \mathcal{F}\{-\delta(t) + 2\delta(t-1) - \delta(t-2)\}$$
$$= \left(-1 + 2e^{-j\omega} - e^{-j2\omega}\right)\mathcal{F}\{\delta(t)\}$$
$$= -1 + 2e^{-j\omega} - e^{-j2\omega}$$
$$= e^{-j\omega}\left(-e^{j\omega} + 2 - e^{-j\omega}\right)$$
$$= 2(1-\cos\omega)\,e^{-j\omega}$$

(b)
$$X(j\omega) = \mathcal{F}\left\{10\,\frac{\sin(5\pi(t-4))}{\pi(t-4)}\right\}$$
$$= e^{-j4\omega}\,\mathcal{F}\left\{10\,\frac{\sin(5\pi t)}{\pi t}\right\}$$
$$= 10\Big(u(\omega+5\pi) - u(\omega-5\pi)\Big)e^{-j4\omega}$$
$$= \begin{cases} 10\,e^{-j4\omega}, & |\omega| < 5\pi \\ 0, & |\omega| > 5\pi \end{cases}$$

(c)
$$X(j\omega) = \mathcal{F}\{4e^{-2t}u(t) - 4e^{-2t}u(t-1)\}$$
$$= \mathcal{F}\{4e^{-2t}u(t) - 4e^{-2}e^{-2(t-1)}u(t-1)\}$$
$$= \left(4 - 4e^{-2}e^{-j\omega}\right)\mathcal{F}\{e^{-2t}u(t)\}$$
$$= 4\left(1 - e^{-(2+j\omega)}\right)\frac{1}{2+j\omega}$$

11.4)

(a) $$x(t) = \mathcal{F}^{-1}\left\{ \frac{j\omega}{100+j\omega} e^{-j0.02\omega} \right\}$$

$$= \mathcal{F}^{-1}\left\{ \frac{j\omega}{100+j\omega} \right\}\Bigg|_{t=t-0.02}$$

$$= \frac{d}{dt}\left(\mathcal{F}^{-1}\left\{ \frac{1}{100+j\omega} \right\} \right)\Bigg|_{t=t-0.02}$$

$$= \frac{d}{dt}\left(e^{-100t} u(t) \right)\Bigg|_{t=t-0.02}$$

$$= \left(e^{-100t}\delta(t) - 100e^{-100t} u(t) \right)\Bigg|_{t=t-0.02}$$

$$= e^{-100(t-0.02)}\delta(t-0.02) - 100e^{-100(t-0.02)}u(t-0.02)$$

$$= \delta(t-0.02) - 100e^{-100(t-0.02)}u(t-0.02)$$

(b) $$x(t) = \mathcal{F}^{-1}\{15 - 15\cos\omega\}$$

$$= \mathcal{F}^{-1}\left\{ 15 - \frac{15}{2}e^{j\omega} - \frac{15}{2}e^{-j\omega} \right\}$$

$$= 15\,\mathcal{F}^{-1}\{1\} - \frac{15}{2}\left(\mathcal{F}^{-1}\{1\}\Big|_{t=t+1} + \mathcal{F}^{-1}\{1\}\Big|_{t=t-1} \right)$$

$$= 15\,\delta(t) - \frac{15}{2}\left(\delta(t)\Big|_{t=t+1} + \delta(t)\Big|_{t=t-1} \right)$$

$$= 15\left(\delta(t) - \frac{1}{2}\delta(t+1) - \frac{1}{2}\delta(t-1) \right)$$

(c)

$$x(t) = \mathcal{F}^{-1}\left\{ \frac{1}{1+j\omega} - \frac{1}{1+j2\omega} \right\}$$

$$= \mathcal{F}^{-1}\left\{ \frac{1}{1+j\omega} - \frac{1/2}{1/2 + j\omega} \right\}$$

$$= e^{-t}u(t) - \frac{1}{2}e^{-\frac{1}{2}t}u(t)$$

(d)

$$x(t) = \mathcal{F}^{-1}\left\{ e^{-j\omega/2}\left(j\delta(\omega-150\pi) - j\delta(\omega+150\pi) \right) \right\}$$

$$= j\,\mathcal{F}^{-1}\left\{ \delta(\omega-150\pi)e^{-j\frac{1}{2}\omega} \right\} - j\,\mathcal{F}^{-1}\left\{ \delta(\omega+150\pi)e^{-j\frac{1}{2}\omega} \right\}$$

$$= j\,\mathcal{F}^{-1}\left\{ \delta(\omega-150\pi) \right\}\Big|_{t=t-\frac{1}{2}} - j\,\mathcal{F}^{-1}\left\{ \delta(\omega+150\pi) \right\}\Big|_{t=t-\frac{1}{2}}$$

$$= \frac{j}{2\pi}e^{j150\pi t}\Big|_{t=t-\frac{1}{2}} - \frac{j}{2\pi}e^{-j150\pi t}\Big|_{t=t-\frac{1}{2}}$$

$$= \frac{j^2}{\pi}\left(\frac{e^{j150\pi t} - e^{-j150\pi t}}{2j} \right)\Big|_{t=t-\frac{1}{2}}$$

$$= \frac{-1}{\pi}\sin(150\pi t)\Big|_{t=t-\frac{1}{2}}$$

$$= \frac{-1}{\pi}\sin\left(150\pi\left(t-\frac{1}{2}\right)\right)$$

$$= \frac{-1}{\pi}\sin(150\pi t - 75\pi)$$

$$= \frac{-1}{\pi}\sin(150\pi t - \pi)$$

$$= \frac{1}{\pi}\sin(150\pi t)$$

11.5)

$$y(t) = x(t) - 3x(t-T) + 3x(t-2T) - x(t-3T)$$

(a) $h(t) = y(t)\Big|_{x(t)=\delta(t)}$

$$= \delta(t) - 3\delta(t-T) + 3\delta(t-2T) - \delta(t-3T)$$

(b) $H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$

$$= \int_{-\infty}^{\infty} \left(\delta(t) - 3\delta(t-T) + 3\delta(t-2T) - \delta(t-3T) \right) e^{-j\omega t} dt$$

$$= e^{-j\omega 0} \int_{-\infty}^{\infty} \delta(t) dt - 3e^{-j\omega T} \int_{-\infty}^{\infty} \delta(t-T) dt$$

$$+ 3e^{-j\omega 2T} \int_{-\infty}^{\infty} \delta(t-2T) dt - e^{-j\omega 3T} \int_{-\infty}^{\infty} \delta(t-3T) dt$$

$$= 1 - 3e^{-j\omega T} + 3e^{-j\omega 2T} - e^{-j\omega 3T}$$

$$= e^{-j\omega \frac{3}{2}T} \left(e^{j\omega \frac{3}{2}T} - 3e^{j\omega \frac{1}{2}T} + 3e^{-j\omega \frac{1}{2}T} - e^{-j\omega \frac{3}{2}T} \right)$$

$$= -j8 e^{-j\omega \frac{3}{2}T} \left(\frac{e^{j\omega \frac{T}{2}} - e^{-j\omega \frac{T}{2}}}{2j} \right)^3$$

$$= jA e^{-j\omega t_d} \sin^3(B\omega)$$

where $A = -8$, $B = \frac{T}{2}$, $t_d = \frac{3T}{2}$

(c) $x(t) = e^{j\omega t}$

$$\Downarrow$$

$$y(t) = e^{j\omega t} - 3e^{j\omega(t-T)} + 3e^{j\omega(t-2T)} - e^{j\omega(t-3T)}$$

$$= \left(1 - 3e^{-j\omega T} + 3e^{-j\omega 2T} - e^{-j\omega 3T}\right) e^{j\omega t}$$

$$= H(j\omega)\, e^{j\omega t}$$

(d) $H(j\omega) = 0$ whenever $\sin\left(\frac{\omega T}{2}\right) = 0$

$$\frac{\omega T}{2} = k\pi, \quad k = 0, \pm 1, \pm 2, \ldots$$

$$\omega = \frac{k2\pi}{T}$$

$$|H(j\omega)| = 8\left|\sin^3\left(\frac{\omega T}{2}\right)\right|, \quad \angle H(j\omega) = \begin{cases} \vdots \\ -\frac{3T}{2}\omega + \frac{\pi}{2}, & -\frac{2\pi}{T} < \omega < 0 \\ -\frac{3T}{2}\omega - \frac{\pi}{2}, & 0 < \omega < \frac{2\pi}{T} \\ \vdots \end{cases}$$