GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004
Problem Set #12

Assigned: 5-Nov-04
Due Date: Week of 15-Nov-04

Quiz #3 will be given on Friday, 19-November. One page ($8\frac{1}{2} \times 11''$) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 11: *Continuous-Time Fourier Transforms.*
Chapter 12: *Filtering, Modulation, and Sampling (Applications of the Fourier Transform.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 12.1*:

Signal Processing First, Chapter **11**, Problem **7**, page 343.

PROBLEM 12.2*:

Signal Processing First, Chapter **11**, Problem **13**, page 344.

PROBLEM 12.3*:

An ideal bandpass filter has a passband from ω_{co1} to ω_{co2} ($\omega_{co2} > \omega_{c01}$), as depicted in Fig. 12-8(b) on p. 353 of the text.

(a) Show that its impulse response can be represented as the difference of two ideal lowpass filter impulse responses

$$h_{bp1}(t) = \frac{\sin(\omega_{co2}t)}{\pi t} - \frac{\sin(\omega_{co1}t)}{\pi t}.$$

(b) Show that the filter

$$h_{bp2}(t) = \left(\frac{2\sin(Wt)}{\pi t}\right)\cos(\omega_0 t)$$

is also a bandpass filter and draw its frequency response $H_{bp2}(j\omega)$.

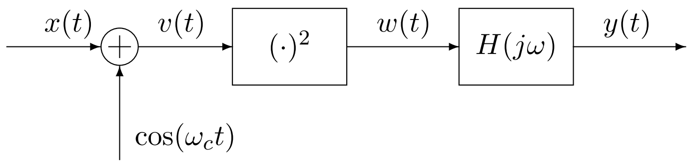
(c) Determine the values of W and ω_0 so that the frequency response of the filter in part (b) will be the same as the frequency response of the filter in part (a).

PROBLEM 12.4*:

In lecture we implemented a DSB modulator by multiplying the modulating signal $x(t)$ by the carrier $\cos(\omega_c t)$ to produce the modulated signal $y(t)$

$$y(t) = x(t)\cos(\omega_c t).$$

Building the modulator this way requires that we build circuitry to actually multiply two signals together. An alternative ***adds*** the modulating signal to the carrier and then passes the sum through a nonlinearity and a filter as shown in the figure below.



For this circuit we assume that the nonlinearity is a circuit whose output signal is the square of the input signal

$$w(t) = v^2(t).$$

Let $x(t)$ be a bandlimited signal with a bandwidth ω_b. For the sake of the drawings requested, assume that it has a "typical" shape that will be represented by a triangle with the correct bandwidth.

(a) Sketch the Fourier transform of the signal $v(t)$.

(b) Sketch the Fourier transform of the signal $w(t)$ which comes out of the squaring device.

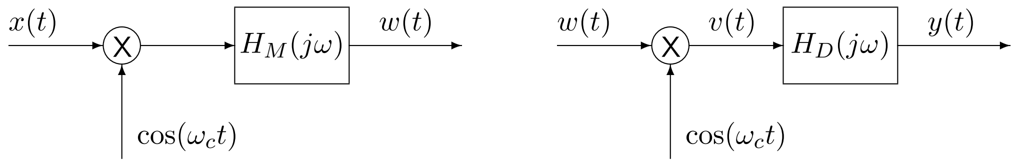
(c) Design a filter $H(j\omega)$ so that the filter output will be

$$y(t) = x(t)\cos(\omega_c t).$$

Discuss any limitations that must be placed on the value of ω_c.

PROBLEM 12.5*:

In lecture we explored double-sideband amplitude modulation. In this problem, we will explore another strategy called single-sideband modulation (SSB). If is similar to the modulation used in amateur radio and in the video channel of television systems. Block diagrams of an SSB modulator and an SSB demodulator are given below.



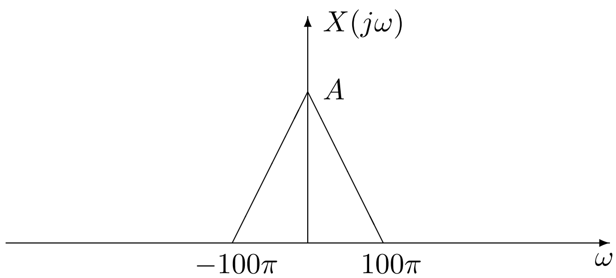
H_M is a *lowpass filter*, here specified by:

$$H_M(j\omega) = \left\{ \begin{array}{ll} 1, & |\omega| \leq 500\pi \\ 0, & \text{otherwise} \end{array} \right\}$$

and H_D is another *lowpass filter*, specified by

$$H_D(j\omega) = B[u(\omega + 200\pi) - u(\omega - 200\pi).]$$

Suppose that $\omega_c = 500\pi$ and the input signal $x(t)$ has the "typical" Fourier transform $X(j\omega)$



We will now trace the input signal through the modulation and demodulation stages to the output by analyzing it in the Fourier domain. In your sketches be sure to label the amplitudes and frequencies correctly. You will find it easiest to think *graphically*; you will not need to write any complicated equations.

(a) Sketch $W(j\omega)$, the Fourier transform of $w(t)$.

(b) Sketch $V(j\omega)$, the Fourier transform of $v(t)$.

(c) How should B, the gain of the lowpass filter in the demodulator, be chosen so that the output exactly equals the input?