GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2004

Problem Set #13

Assigned: 12-Nov-04
Due Date: 3-Dec-04

This Homework can be turned at the last lecture on ***Friday, 3-December before Noon,*** or earlier that week.

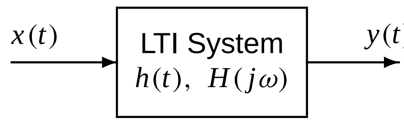
The Final Exam will be given on 6-December at 2:50 PM (for 10AM lecture) and 7-December at 2:50 PM (for 11AM lecture). One page ($8\frac{1}{2} \times 11''$) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 12: *Filtering, Modulation and Sampling,* (applications of the Fourier Transform) and Chapter 8: *IIR Filters*.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

PROBLEM 13.1*:



The impulse response of the above system is $h(t) = \dfrac{3\sin(\omega_{co}(t+0.2))}{\pi(t+0.2)}$,

and the input to this system is a periodic signal (with period $T_0 = 2$) given by the following equation:

$$x(t) = \sum_{k=-\infty}^{\infty} \tfrac{1}{2} e^{j\pi k t}$$

(a) Determine the Fourier transform $X(j\omega)$ of the input signal. [1] Plot $X(j\omega)$ over the range $|\omega| < 6\pi$.

(b) For the case $\omega_{co} = 1.5\pi$, determine $H(j\omega)$ and plot $|H(j\omega)|$ on the same graph as $X(j\omega)$

(c) For the case $\omega_{co} = 1.5\pi$, use the plot in (b) to determine $y(t)$, the output of the LTI system for the given input $x(t)$ above.

(d) Determine ω_{co} so that the output is a constant; i.e., $y(t) = C$ for all t, and determine the value of the constant C.

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[1]Recall that if $x(t) = \displaystyle\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, then the Fourier transform is $X(j\omega) = \displaystyle\sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$, where $\omega_0 = 2\pi/T_0$.

PROBLEM 13.2*:
Consider an LTI system with input/output relationship given by

$$y[n] = 0.3y[n-1] + x[n-3] + 0.6x[n-4]$$

(a) Find its system function $H(z)$.

(b) Plot the poles and zeros of this system.

(c) Use z-transform concepts to find the output $y[n]$ if the system is given an input $x[n] = (-0.6)^n u[n]$.

(d) Derive a formula for the frequency response of this system.

(e) Plot the magnitude of the frequency response versus $\hat{\omega}$.

PROBLEM 13.3:
Consider an LTI system with the system function

$$H(z) = \frac{1 - 0.2z^{-2} + 0.3z^{-5}}{1 + 0.5z^{-3} - 0.9z^{-7}}$$

(a) Find the input/output equation in terms of an input $x[n]$ and an output $y[n]$ for the system implementing this $H(z)$.

(b) Suppose we want to implement this system in MATLAB using the command:

```
yy = filter(bb,aa,xx)
```

What should the variables `bb` and `aa` contain?

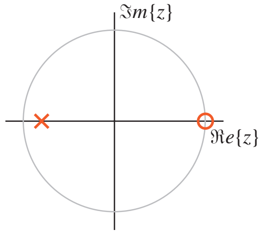
PROBLEM 13.4*:
Determine the inverse z-transforms of the following:

(a) $H_a(z) = \dfrac{1 + z^{-1}}{1 - 0.5z^{-1}}$.

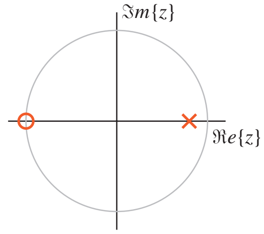
(b) $H_b(z) = \dfrac{0.5}{1 + 0.75e^{j0.3\pi}z^{-1}} + \dfrac{0.5}{1 + 0.75e^{-j0.3\pi}z^{-1}}$.

(c) $H_c(z) = \dfrac{0.6 + z^{-1}}{1 + 0.6z^{-1}}$.

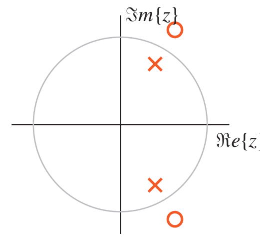
PROBLEM 13.5:



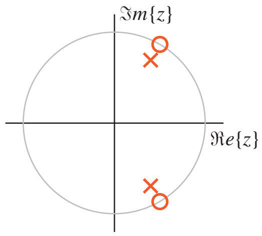
Pole–Zero Plot #1



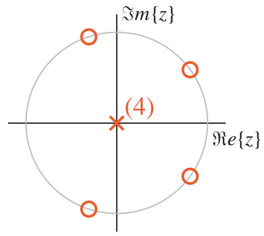
Pole–Zero Plot #2



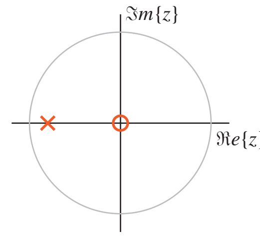
Pole–Zero Plot #3



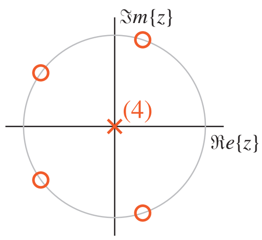
Pole–Zero Plot #4



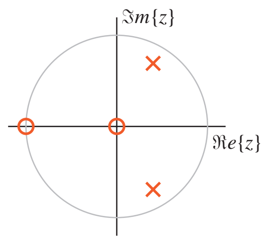
Pole–Zero Plot #5



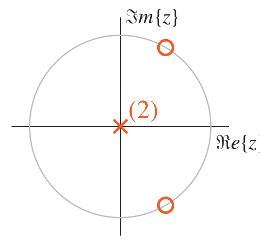
Pole–Zero Plot #6



Pole–Zero Plot #7



Pole–Zero Plot #8



Pole–Zero Plot #9

For each of systems below[2] determine which of the pole-zero diagrams, (#1, #2, #3, #4, #5, #6, #7, #8, #9), is a match. *Note:* the unit circle is shown for reference.

$$\mathcal{S}_1: \qquad H(z) = \frac{1+z^{-1}}{1-0.8z^{-1}}$$

$$\mathcal{S}_2: \qquad y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$$

$$\mathcal{S}_3: \qquad H(z) = \frac{8-8z^{-1}+8z^{-2}}{1-0.8z^{-1}+0.64z^{-2}}$$

$$\mathcal{S}_4: \qquad y[n] = -0.8y[n-1] + 2x[n]$$

$$\mathcal{S}_5: \qquad H(z) = \frac{1.8(1+z^{-1})}{1-0.8z^{-1}+0.64z^{-2}}$$

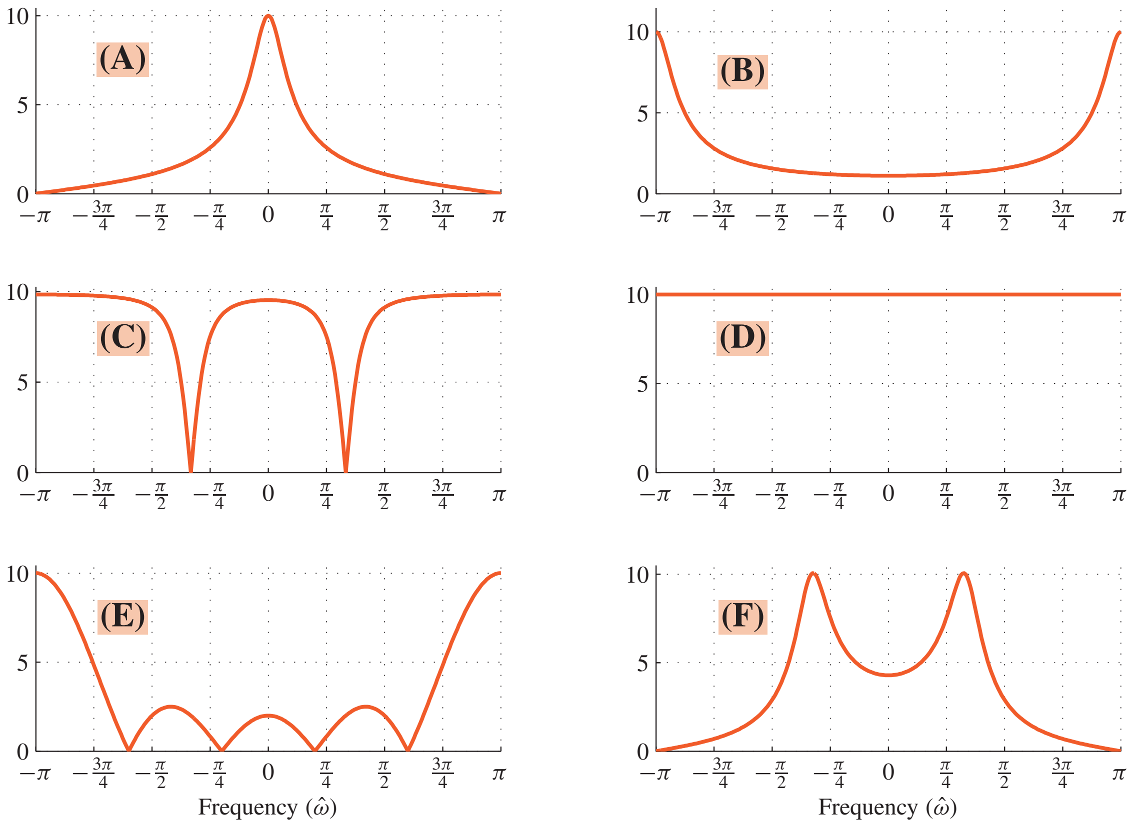
$$\mathcal{S}_6: \qquad y[n] = \frac{10}{3}(x[n] - x[n-1] + x[n-2])$$

$$\mathcal{S}_7: \qquad H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$\mathcal{S}_8: \qquad y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$$

[2]These same systems are also used in the next problem.

PROBLEM 13.6*:



For each of the discrete-time systems below, determine which of the frequency response (magnitude) plots, (A, B, C, D, E, F, or None), is a match. *Note:* the frequency axis is $\hat{\omega}$. ***Hint:*** You might find it helpful to work the previous problem before this one even though it does not have a star.

$$\mathcal{S}_1: \quad H(z) = \frac{1 + z^{-1}}{1 - 0.8z^{-1}}$$

$$\mathcal{S}_2: \quad y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4]$$

$$\mathcal{S}_3: \quad H(z) = \frac{8 - 8z^{-1} + 8z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_4: \quad y[n] = -0.8y[n-1] + 2x[n]$$

$$\mathcal{S}_5: \quad H(z) = \frac{1.8(1 + z^{-1})}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$\mathcal{S}_6: \quad y[n] = \frac{10}{3}(x[n] - x[n-1] + x[n-2])$$

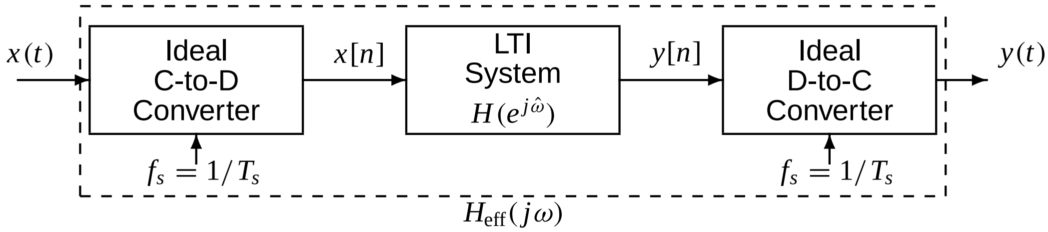
$$\mathcal{S}_7: \quad H(z) = 2(1 - z^{-1} + z^{-2} - z^{-3} + z^{-4})$$

$$\mathcal{S}_8: \quad y[n] = 0.8y[n-1] - 0.64y[n-2] + 6.4x[n] - 8x[n-1] + 10x[n-2]$$

PROBLEM 13.7*:

This type of problem has often appeared on the Final Exam.

Consider the following system for discrete-time filtering of a continuous-time signal:

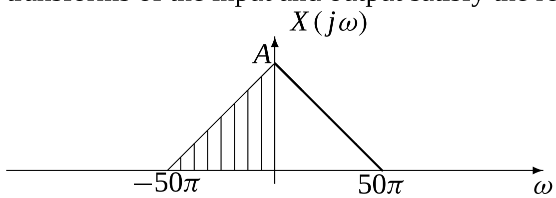


(a) Suppose that the discrete-time system is defined by the difference equation

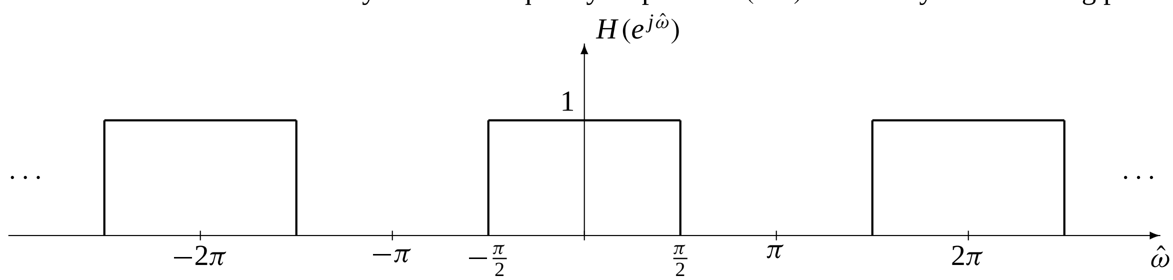
$$y[n] = 0.8y[n-1] + x[n] + x[n-2],$$

and the sampling rate of the input is $f_s = 200$ samples/second. Determine an expression for $H_{\text{eff}}(j\omega)$, the overall effective frequency response of the above system. Use this result to find the output $y(t)$ when the input to the overall system is $x(t) = 2\cos(100\pi t)$.

(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that the Fourier transforms of the input and output satisfy the relation $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$?



(c) Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:



Now, if $\boxed{f_s = 200 \text{ samples/sec}}$, make a carefully labeled plot of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system.

(d) For the input in part (b) and the system in part (c), what is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered; i.e., what is the minimum f_s such that $Y(j\omega) = X(j\omega)$?