GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 7-Dec-04 COURSE: ECE-2025

NAME: ______________________________ GT # ____________________

LAST, FIRST (e.g. gtg123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning **CLEARLY** to receive partial credit.
  Explanations are also REQUIRED to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	

PROBLEM FALL-04-F.1:

In each of the following cases, ***simplify the expression as much as possible.***

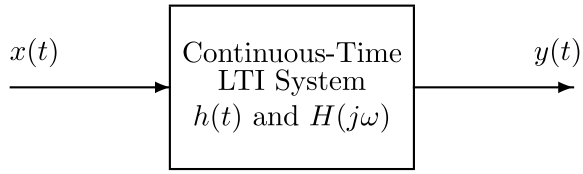
(a) $e^{-2t}\delta(t+3) =$ ____________________

(b) $u(t-3) * \delta(t-2) =$ ____________________

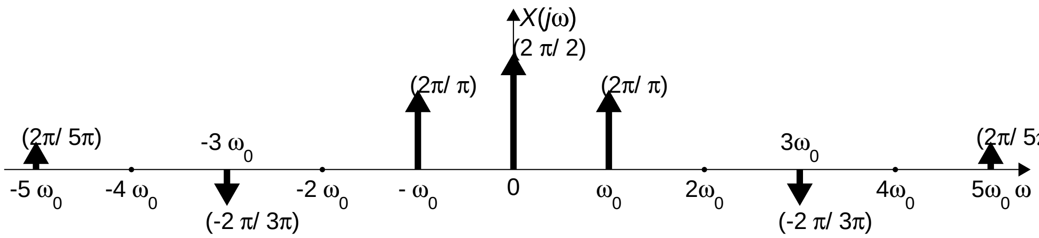
(c) $\displaystyle\int_{-10}^{5} \cos(10\tau)\delta(\tau-3)d\tau =$ ____________________

(d) $x[n] = 5\sqrt{2}\cos(0.3\pi n + \pi/4) + 5\cos(0.3\pi n + \pi) =$ ____________________

PROBLEM FALL-04-F.2:



The periodic input to the above LTI system has the Fourier transform $X(j\omega)$ drawn below:



where the dark arrows denote impulses. For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform, $X(j\omega)$. [*Circle the correct answer. There is only one correct answer in each case.*]

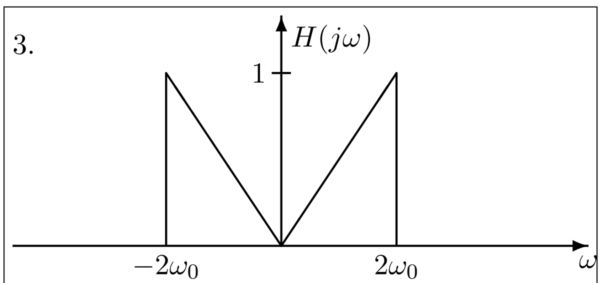
(a) $y(t) = 2(\omega_0/\pi)\cos(\omega_0 t + \pi/2)$	(1)	(2)	(3)	(4)	(5)	(6)
(b) $y(t) = x(t-1) - \frac{1}{2}$	(1)	(2)	(3)	(4)	(5)	(6)
(c) $y(t) = \dfrac{1}{\pi}\cos(\omega_0 t)$	(1)	(2)	(3)	(4)	(5)	(6)
(d) $y(t) = \frac{1}{2}$	(1)	(2)	(3)	(4)	(5)	(6)
(e) $y(t) = \dfrac{1}{2} + \dfrac{4}{3\pi}\cos(\omega_0 t)$	(1)	(2)	(3)	(4)	(5)	(6)

The possible filters are described by the following equations and graphs.[1]

1. $H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

2. $H(j\omega) = e^{-j\omega}$

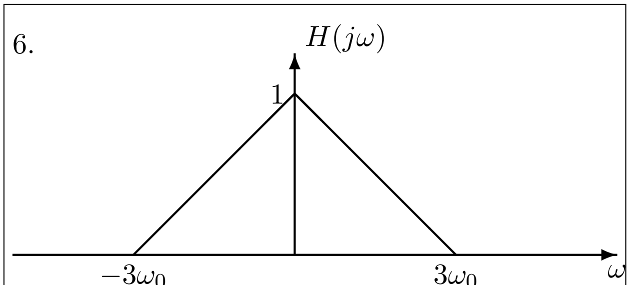
3.



4. $H(j\omega) = \begin{cases} j\omega & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

5. $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ e^{-j\omega} & |\omega| > \omega_0/2 \end{cases}$

6.

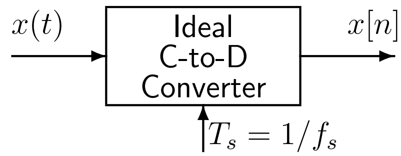


[1] *Some of these may not be needed as answers.*

PROBLEM FALL-04-F.3:

The individual parts of this problem are independent.

(a) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine two possible positive values of the input frequency of $x(t)$ that are less than 2000Hz.:



ANS 1: =

ANS 2: =

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;
xx = (3/pi) * cos(pi*1.25*nn + pi/3);
soundsc(xx,fsamp)
```

Although the sinusoid was not written to have a frequency of 2300 Hz, it is possible to play out the vector `xx` so that it sounds like a 2300 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 2300 Hz tone. Write your answer as an integer.

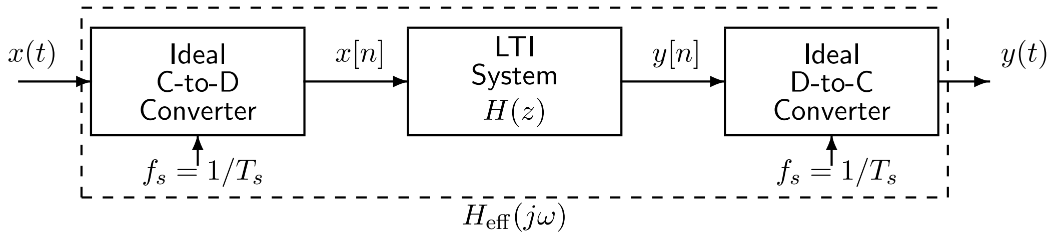
`fsamp =` Hz

(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by:
$x(t) = \Im m\{e^{-j1200\pi t} + e^{j2000\pi t}\}$.

ANS = samples/sec.

PROBLEM FALL-04-F.4:

Consider the following system for discrete-time filtering of a continuous-time signal:

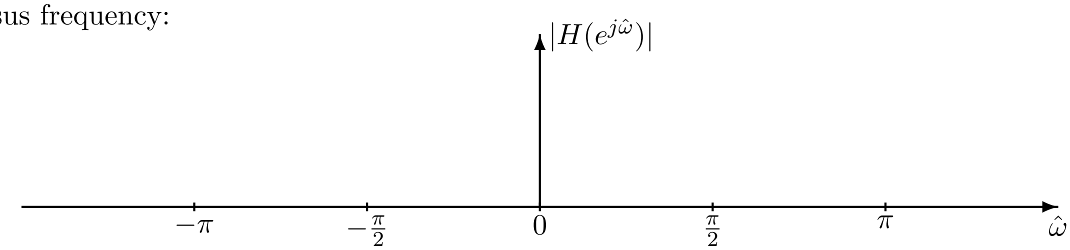


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,1,0,-1],xx)
```

where `xx` is an array of samples of $x[n]$ and `yy` holds samples of $y[n]$.

(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



(b) Assume that the input signal $x(t)$ is a sum of cosines:

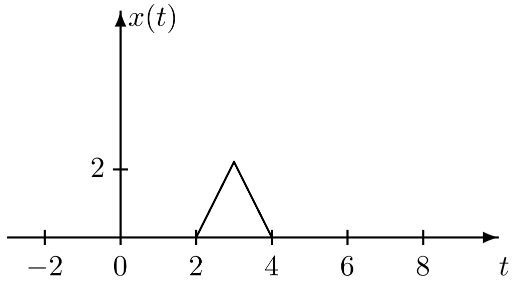
$$x(t) = 8\cos(960\pi t + \pi/3) + 5\cos(600\pi t - \pi/4)$$

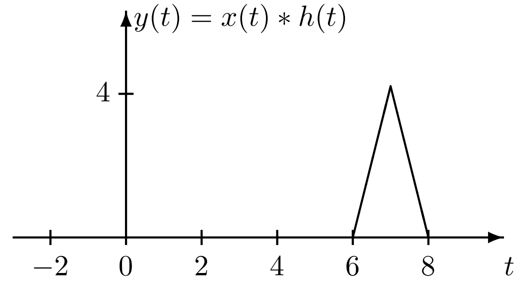
For this input signal, determine the output signal $y(t)$ when the sampling rate is $f_s = \mathbf{480}$ **samples/sec.** Your answer should be expressed as a sum of cosines.

PROBLEM FALL-04-F.5:

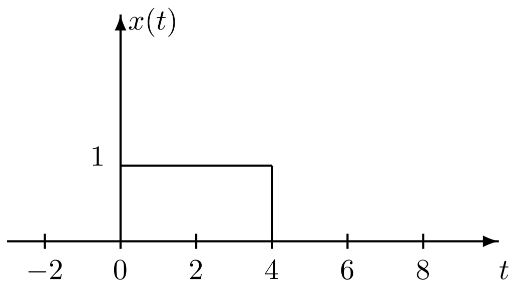
The parts of this problem are completely independent.

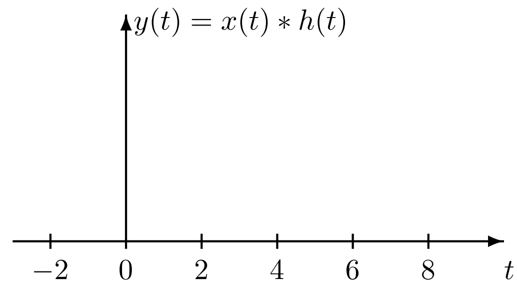
(a) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) =$



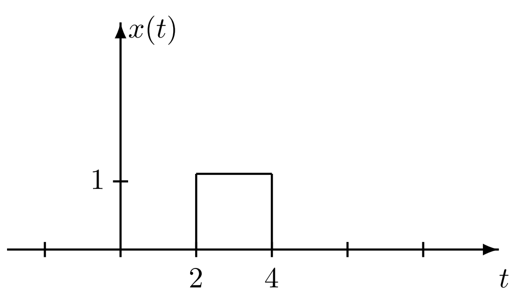


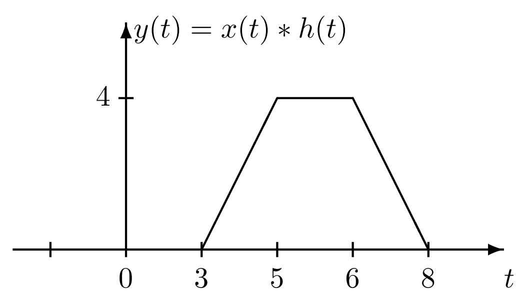
(b) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*





(c) If $h(t) = c[u(t-a) - u(t-b)]$ and $y(t) = x(t) * h(t)$, determine the values of a, b, and c, where $x(t)$ is given by the graph on the left and $y(t)$ is given by the graph on the right.





$a =$

$b =$

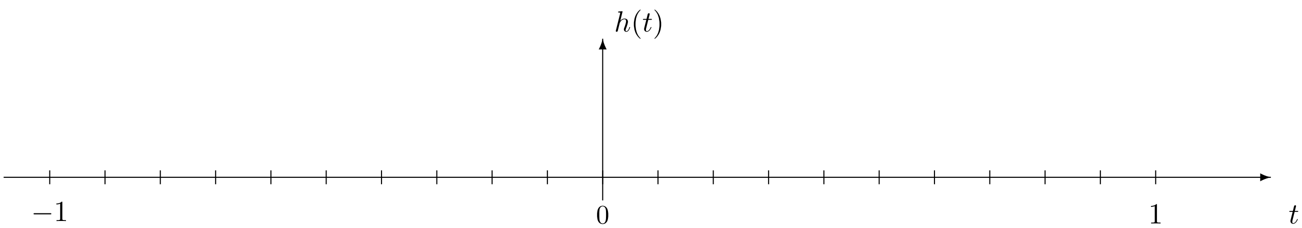
$c =$

PROBLEM FALL-04-F.6:

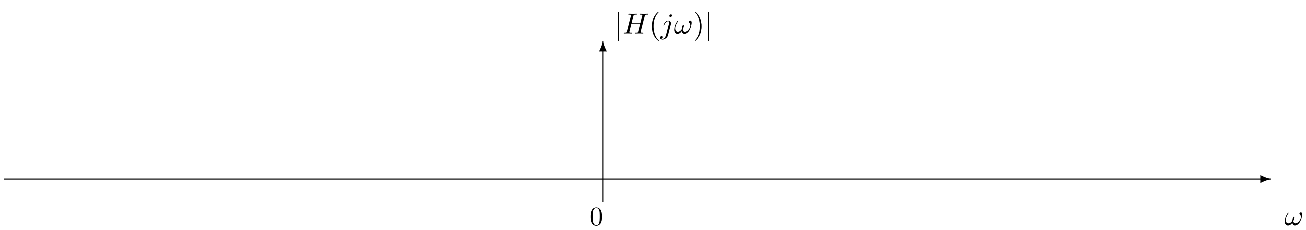
A linear time-invariant system has impulse response

$$h(t) = \frac{5\sin[(10/3)\pi t]}{\pi t}.$$

(a) Sketch $h(t)$ very carefully on the graph below for the indicated range of values of t. Label the maximum value and the points where $h(t) = 0$.



(b) Determine $H(j\omega)$, the Fourier transform of $h(t)$. Then, plot the frequency response magnitude, $|H(j\omega)|$, for this system on the graph below. Label your plot carefully to receive full credit.

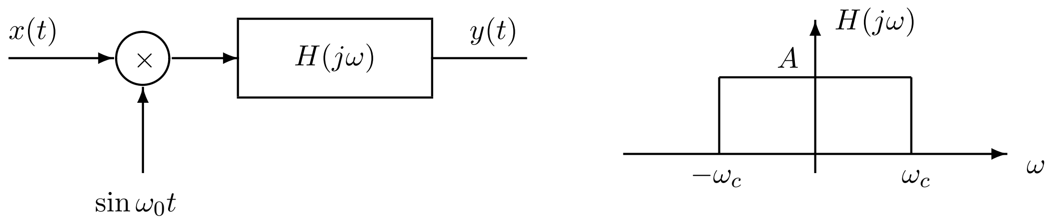


(c) Determine the output $y(t)$ of the LTI system when the input is

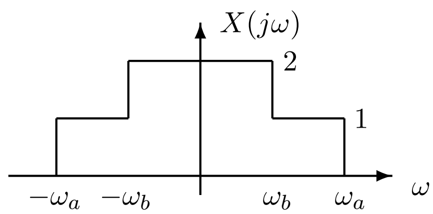
$$x(t) = \frac{2\sin(6\pi(t - 0.4))}{\pi(t - 0.4)}.$$

PROBLEM FALL-04-F.7:
The two parts of this problem are independent.

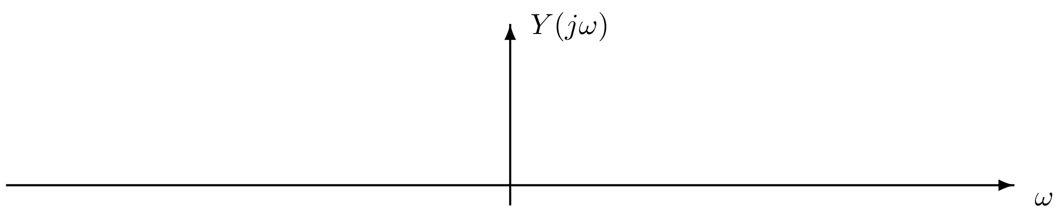
(a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine.



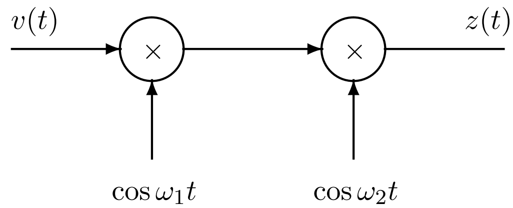
Assume that $x(t)$ has the spectrum, $X(j\omega)$ shown below



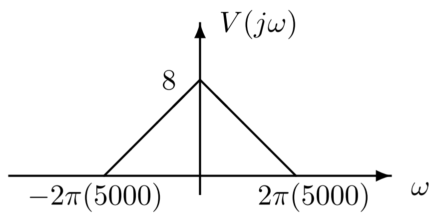
Sketch the spectrum, $Y(j\omega)$ of the output of the scrambler on the axes below if $\omega_a/(2\pi) =$ 5 kHz, $\omega_b/(2\pi) = 2$ kHz, $\omega_c/(2\pi) = 10$ kHz, and $\omega_0/(2\pi) = 10$ kHz. Be sure to LABEL YOUR PLOT.

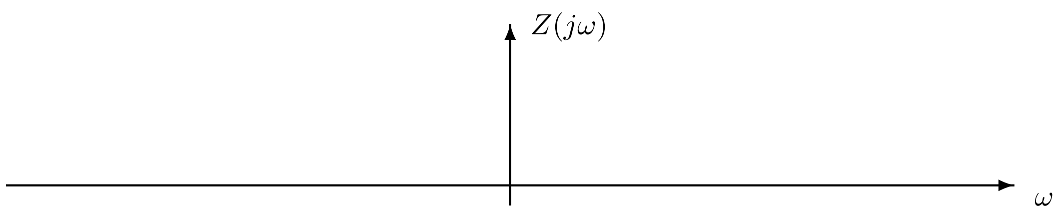


(b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.

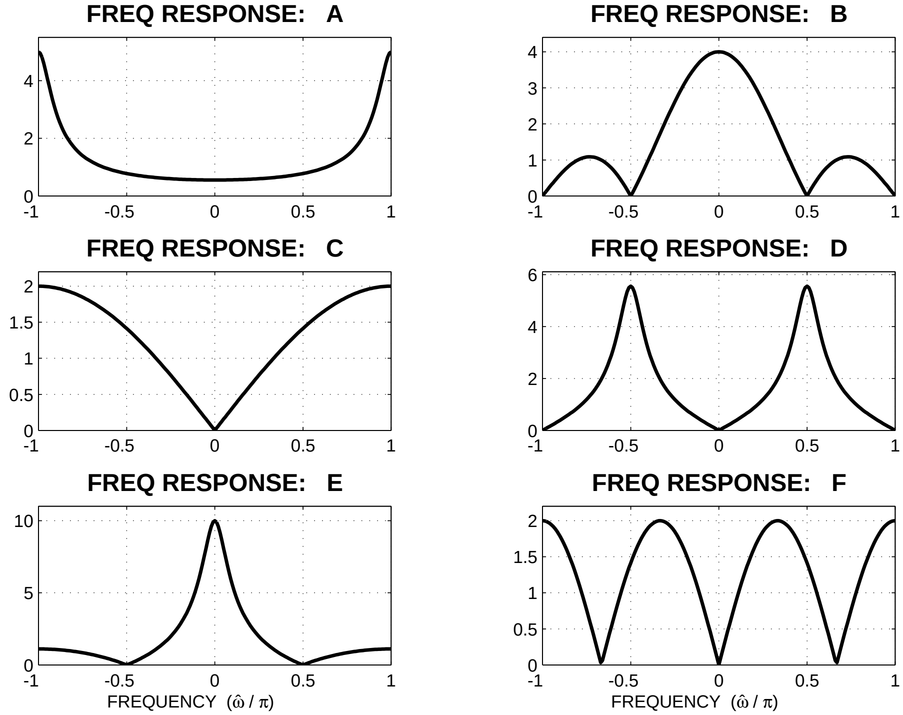


Let $f_1 = \omega_1/(2\pi) = 45$ kHz and $f_2 = \omega_2/(2\pi) = 10$ kHz. Sketch the spectrum $Z(j\omega)$ assuming that $V(j\omega)$ has the shape shown in the figure below.





PROBLEM FALL-04-F.8:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$, a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = -0.8y[n-1] + x[n]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-2}}{1+0.64z^{-2}}$

$\mathcal{S}_3$: $H(z) = \displaystyle\sum_{k=0}^{3} z^{-k}$

$\mathcal{S}_4$: $H(z) = \dfrac{1+z^{-2}}{1-0.8z^{-1}}$

$\mathcal{S}_5$: $H(z) = \dfrac{1}{1+0.8z^{-1}} + \dfrac{-1}{1-0.8z^{-1}}$

$\mathcal{S}_6$: $y[n] = -0.64y[n-2] + x[n] - x[n-2]$

$\mathcal{S}_7$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_8$: $H(z) = 1 - z^{-3}$

Mark your answer in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_{\#}$)
A		B	
C		D	
E		F	

PROBLEM FALL-04-F.1:

In each of the following cases, ***simplify the expression as much as possible.***

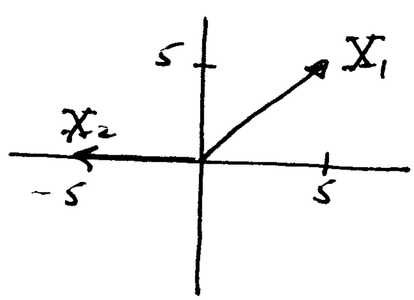
(a) $e^{-2t}\delta(t+3) =$ $e^{6}\delta(t+3)$

(b) $u(t-3) * \delta(t-2) =$ $u(t-5)$

(c) $\int_{-10}^{5} \cos(10\tau)\delta(\tau-3)d\tau =$ $\cos(30)$

(d) $x[n] = 5\sqrt{2}\cos(0.3\pi n + \pi/4) + 5\cos(0.3\pi n + \pi) =$ $5\cos(0.3\pi n + \pi/2)$

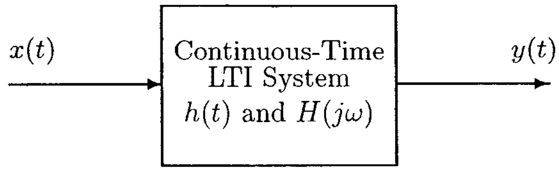
$X_1 = 5\sqrt{2}\, e^{j\frac{\pi}{4}}$ $\quad X_2 = 5e^{j\pi}$



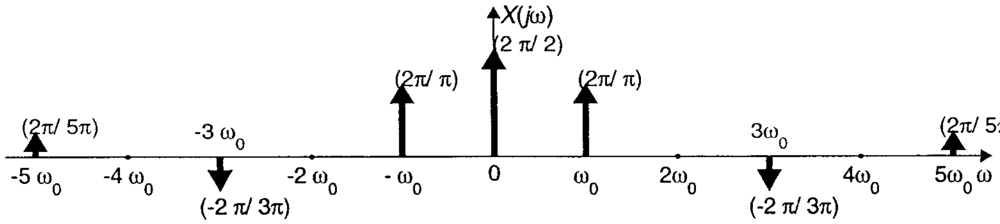
$X_1 + X_2 = 5e^{j\frac{\pi}{2}}$

$x[n] = 5\cos(0.3\pi n + \frac{\pi}{2})$

PROBLEM FALL-04-F.2:



The periodic input to the above LTI system has the Fourier transform $X(j\omega)$ drawn below:



where the dark arrows denote impulses. For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform, $X(j\omega)$. [*Circle the correct answer. There is only one correct answer in each case.*]

(a) $y(t) = 2(\omega_0/\pi)\cos(\omega_0 t + \pi/2)$ (1) (2) (3) (4) (5) (6)

(b) $y(t) = x(t-1) - \frac{1}{2}$ (1) (2) (3) (4) (5) (6)

(c) $y(t) = \frac{1}{\pi}\cos(\omega_0 t)$ (1) (2) (3) (4) (5) (6)

(d) $y(t) = \frac{1}{2}$ (1) (2) (3) (4) (5) (6)

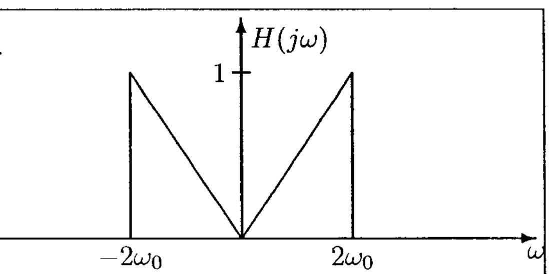
(e) $y(t) = \frac{1}{2} + \frac{4}{3\pi}\cos(\omega_0 t)$ (1) (2) (3) (4) (5) (6)

The possible filters are described by the following equations and graphs.[1]

1. $H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$

2. $H(j\omega) = e^{-j\omega}$

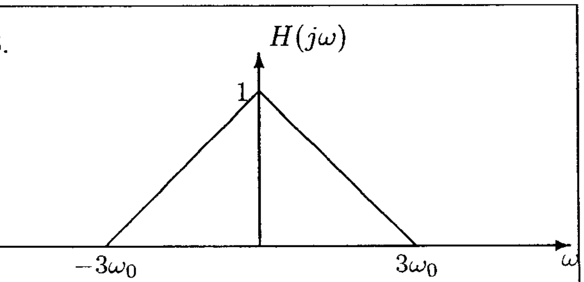
3.



4. $H(j\omega) = \begin{cases} j\omega & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$

5. $H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ e^{-j\omega} & |\omega| > \omega_0/2 \end{cases}$

6.

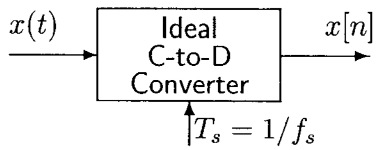


[1] *Some of these may not be needed as answers.*

PROBLEM FALL-04-F.3:

The individual parts of this problem are independent.

(a) If the output from an ideal C/D converter is $x[n] = 33\cos(0.5\pi n)$, and the sampling rate is 2000 samples/sec, then determine two possible positive values of the input frequency of $x(t)$ that are less than 2000Hz.:



ANS 1: = 1000π rad/sec.	ANS 2: = 3000π rad/sec.

or 500 Hz. or 1500 Hz.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;
xx = (3/pi) * cos(pi*1.25*nn + pi/3);
soundsc(xx,fsamp)
```

Although the sinusoid was not written to have a frequency of 2300 Hz, it is possible to play out the vector xx so that it sounds like a 2300 Hz tone. Determine the value of `fsamp` that should be used to play the vector xx as a 2300 Hz tone. Write your answer as an integer.

fsamp = 6,133 Hz

$$\hat{\omega} = \omega T_s \qquad \frac{5\pi}{4} \equiv \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$$

$$\frac{3\pi}{4} = 2\pi(2300)\,T_s$$

$$\frac{1}{T_s} = \frac{18400}{3} = 6,133$$

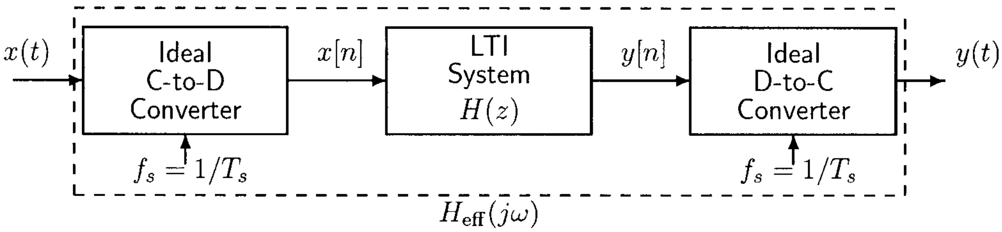
(c) Determine the Nyquist rate for sampling the signal $x(t)$ defined by:
$x(t) = \Im m\{e^{-j1200\pi t} + e^{j2000\pi t}\}$.

ANS = 2,000 samples/sec.

$$\omega_b = 2000\pi$$
$$f_b = 1000\ Hz.$$

PROBLEM FALL-04-F.4:

Consider the following system for discrete-time filtering of a continuous-time signal:

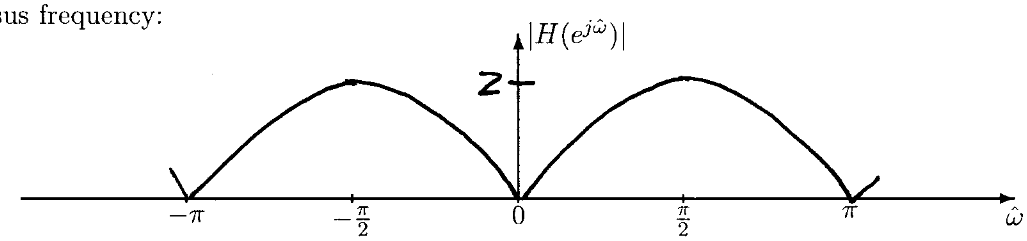


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,1,0,-1],xx)
```

where xx is an array of samples of $x[n]$ and yy holds samples of $y[n]$.

(a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - e^{-j3\hat{\omega}}$$
$$= e^{-j2\hat{\omega}}\left(e^{j\hat{\omega}} - e^{-j\hat{\omega}}\right) = j2\sin\hat{\omega}\, e^{-j2\hat{\omega}}$$

(b) Assume that the input signal $x(t)$ is a sum of cosines:

$$x(t) = 8\cos(960\pi t + \pi/3) + 5\cos(600\pi t - \pi/4)$$

For this input signal, determine the output signal $y(t)$ when the sampling rate is $f_s = 480$ **samples/sec.** Your answer should be expressed as a sum of cosines.

$$x[n] = 8\cos\left(\frac{960\pi n}{480} + \frac{\pi}{3}\right) + 5\cos\left(\frac{600\pi n}{480} - \frac{\pi}{4}\right)$$
$$= 8\cos\left(\frac{\pi}{3}\right) + 5\cos\left(\frac{5\pi}{4}n - \frac{\pi}{4}\right)$$
$$= 4 + 5\cos\left(\frac{3\pi}{4}n + \frac{\pi}{4}\right)$$

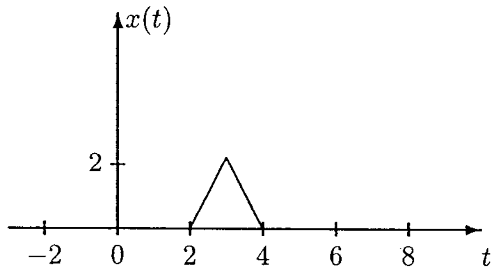
$$\Longrightarrow \quad y[n] = 0 + 5\sqrt{2}\cos\left(\frac{3\pi}{4}n - \pi + \frac{\pi}{4}\right)$$
$$= 5\sqrt{2}\cos\left(\frac{3\pi}{4}n - \frac{3\pi}{4}\right)$$

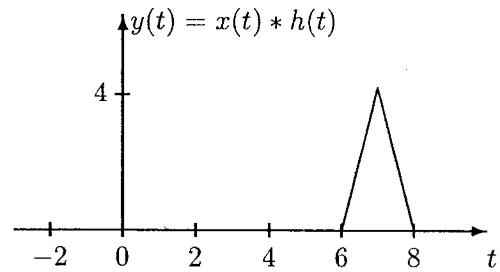
$$\boxed{y(t) = 5\sqrt{2}\cos\left(360\pi t - \frac{3\pi}{4}\right)}$$

PROBLEM FALL-04-F.5:

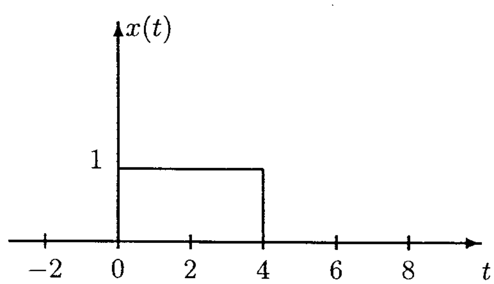
The parts of this problem are completely independent.

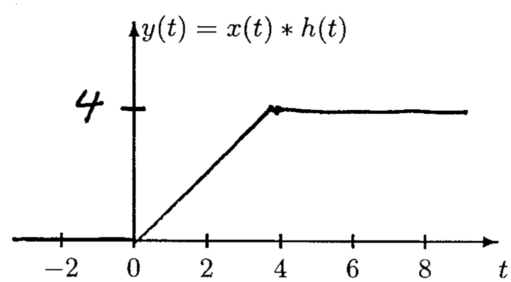
(a) Given that $y(t) = x(t) * h(t)$, find $h(t)$. $h(t) = 2\delta(t-4)$



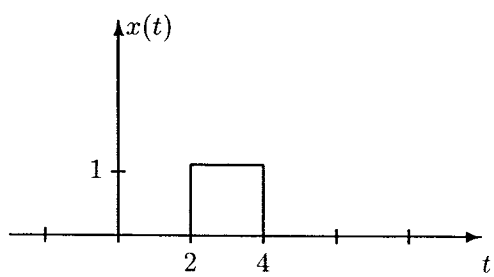


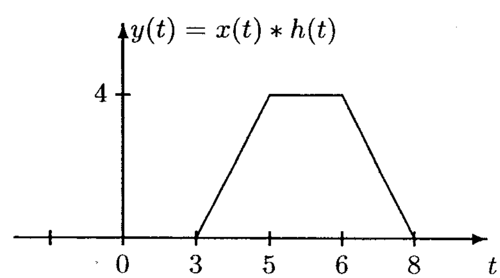
(b) If $h(t) = u(t)$, plot $y(t) = x(t) * h(t)$ on the graph on the right. *Be sure to label the $y(t)$ axis.*





(c) If $h(t) = c[u(t-a) - u(t-b)]$ and $y(t) = x(t) * h(t)$, determine the values of a, b, and c, where $x(t)$ is given by the graph on the left and $y(t)$ is given by the graph on the right.





$a =$ 1

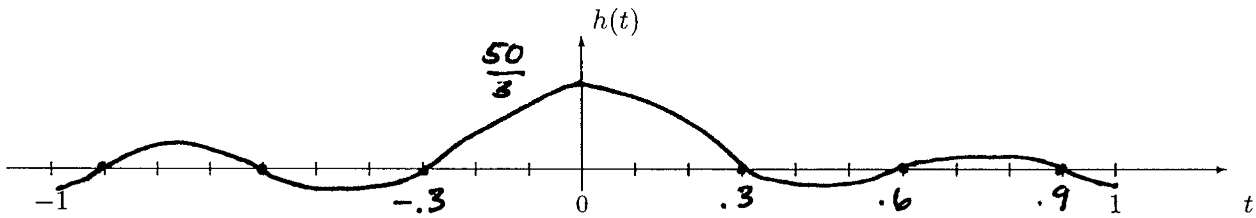
$b =$ 4

$c =$ 2

PROBLEM FALL-04-F.6:
A linear time-invariant system has impulse response

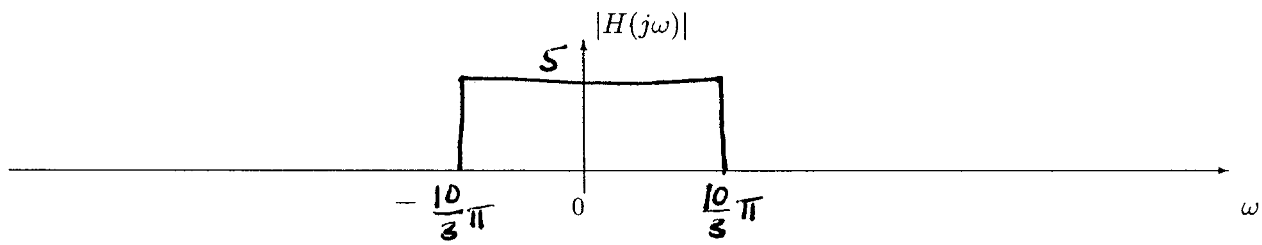
$$h(t) = \frac{5\sin[(10/3)\pi t]}{\pi t}.$$

(a) Sketch $h(t)$ very carefully on the graph below for the indicated range of values of t. Label the maximum value and the points where $h(t) = 0$.



(b) Determine $H(j\omega)$, the Fourier transform of $h(t)$. Then, plot the frequency response magnitude, $|H(j\omega)|$, for this system on the graph below. Label your plot carefully to receive full credit.

(SEE BELOW)



(c) Determine the output $y(t)$ of the LTI system when the input is

$$x(t) = \frac{2\sin(6\pi(t-0.4))}{\pi(t-0.4)}.$$

$$y(t) = \frac{10\sin\left(\frac{10}{3}\pi(t-0.4)\right)}{\pi(t-0.4)}$$

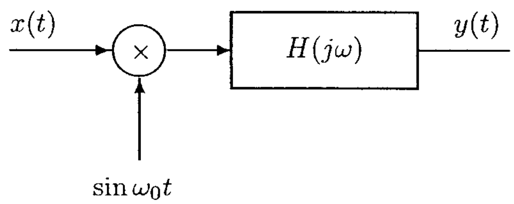
$$\frac{\sin\left[\frac{10}{3}\pi t\right]}{\pi t} \longleftrightarrow \begin{cases} 1, & |\omega| < \frac{10}{3}\pi \\ 0, & \text{OTHERWISE} \end{cases}$$

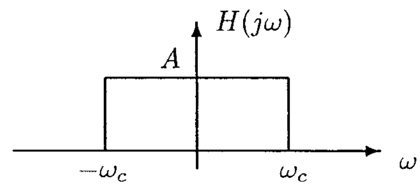
$$\frac{5\sin\frac{10}{3}\pi t}{\pi t} \longleftrightarrow \begin{cases} 5, & |\omega| < \frac{10}{3}\pi \\ 0, & \text{OTHERWISE} \end{cases}$$

PROBLEM FALL-04-F.7:

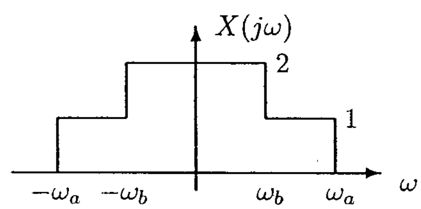
The two parts of this problem are independent.

(a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine..

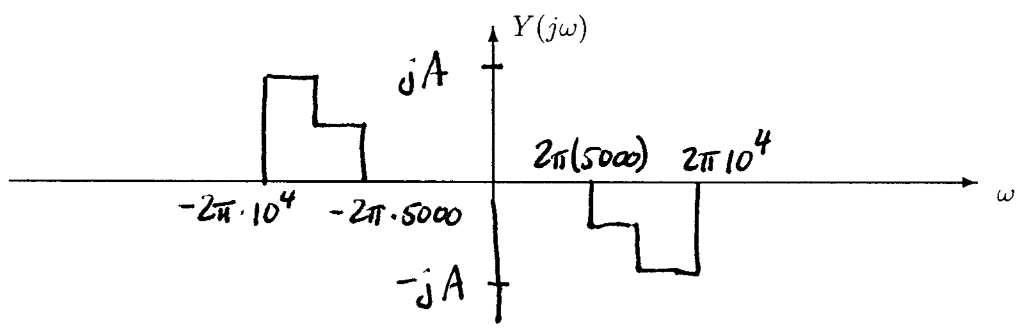




Assume that $x(t)$ has the spectrum, $X(j\omega)$ shown below

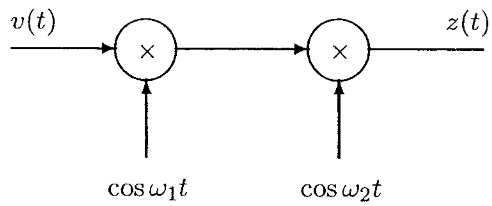


Sketch the spectrum, $Y(j\omega)$ of the output of the scrambler on the axes below if $\omega_a/(2\pi) =$ 5 kHz, $\omega_b/(2\pi) =$ 2 kHz, $\omega_c/(2\pi) =$ 10 kHz, and $\omega_0/(2\pi) =$ 10 kHz. Be sure to LABEL YOUR PLOT.

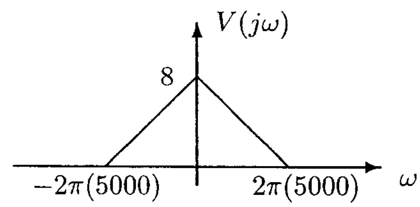


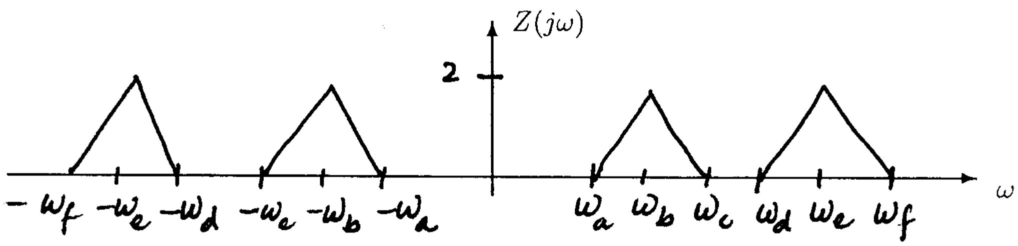
$$\sin \omega_0 t \longleftrightarrow -j\pi\delta(\omega-\omega_0)+j\pi\delta(\omega+\omega_0)$$

(b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.



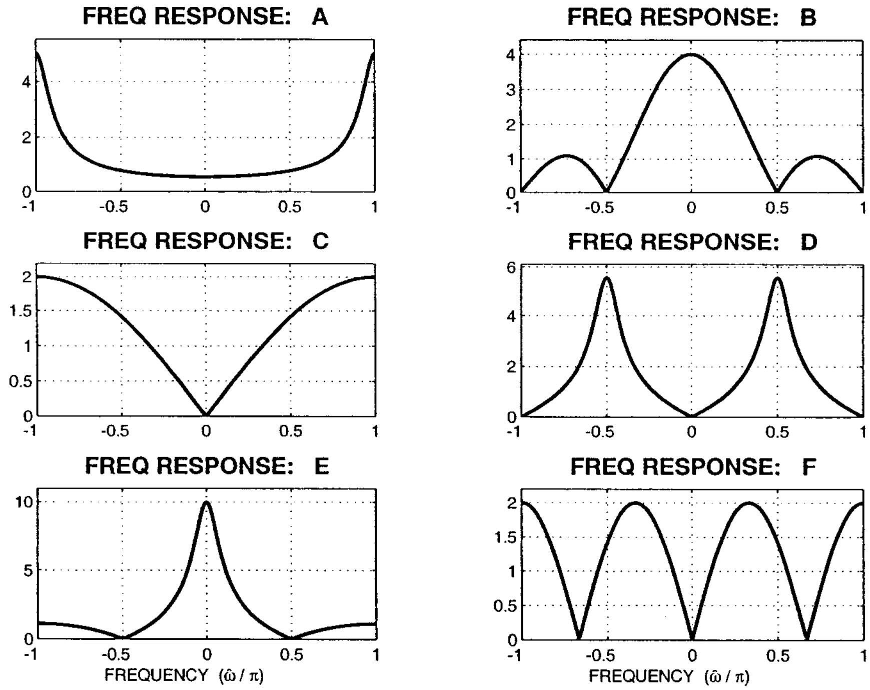
Let $f_1 = \omega_1/(2\pi) = 45$ kHz and $f_2 = \omega_2/(2\pi) = 10$ kHz. Sketch the spectrum $Z(j\omega)$ assuming that $V(j\omega)$ has the shape shown in the figure below.





$$\omega_b = \omega_1 - \omega_2 = 2\pi(35{,}000)$$
$$\omega_e = \omega_1 + \omega_2 = 2\pi(55{,}000)$$
$$\omega_a = \omega_b - 2\pi(5000) = 2\pi(30{,}000)$$
$$\omega_c = \omega_b + 2\pi(5000) = 2\pi(40{,}000)$$
$$\omega_d = \omega_e - 2\pi(5000) = 2\pi(50{,}000)$$
$$\omega_f = \omega_e + 2\pi(5000) = 2\pi(60{,}000)$$

PROBLEM FALL-04-F.8:



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an $H(z)$, a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is $\hat{\omega}/\pi$.

$\mathcal{S}_1$: $y[n] = -0.8y[n-1] + x[n]$

$\mathcal{S}_2$: $H(z) = \dfrac{1+z^{-2}}{1+0.64z^{-2}}$

$\mathcal{S}_3$: $H(z) = \displaystyle\sum_{k=0}^{3} z^{-k}$

$\mathcal{S}_4$: $H(z) = \dfrac{1+z^{-2}}{1-0.8z^{-1}}$

$\mathcal{S}_5$: $H(z) = \dfrac{1}{1+0.8z^{-1}} + \dfrac{-1}{1-0.8z^{-1}}$

$\mathcal{S}_6$: $y[n] = -0.64y[n-2] + x[n] - x[n-2]$

$\mathcal{S}_7$: $y[n] = x[n] - x[n-1]$

$\mathcal{S}_8$: $H(z) = 1 - z^{-3}$

Mark your answer in the following table:

FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)	FREQUENCY RESPONSE	SYSTEM ($\mathcal{S}_\#$)
A	1	B	3
C	7	D	6
E	4	F	8