

**GEORGIA INSTITUTE OF TECHNOLOGY**  
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING  
**FINAL EXAM**

DATE: 7-Dec-04

COURSE: ECE-2025

NAME: \_\_\_\_\_  
                    LAST,                    FIRST

GT # \_\_\_\_\_  
(e.g. gtg123a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

L02:Thurs-9:30 (Anderson)	L03:Tues-Noon (Williams)	L05:Tues-1:30 (Williams)
L06:Thurs-1:30 (Anderson)	L07:Tues-3:00 (Durey)	L08:Thurs-3:00 (Smith)
L09:Tues-4:30 (Durey)	L10:Thurs-4:30 (Smith)	L13:Mon-3:00 (McClellan)
L14:Wed-3:00 (Taylor)	L15:Mon-4:30 (Hayes)	L16:Wed-4:30 (Taylor)

- Write your name on the front page **ONLY**. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ( $8\frac{1}{2}'' \times 11''$ ) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning **CLEARLY** to receive partial credit.  
Explanations are also **REQUIRED** to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.  
Only these answers will be graded. Circle your answers, or write them in the boxes provided.  
If space is needed for scratch work, use the backs of previous pages.

<i>Problem</i>	<i>Value</i>	<i>Score</i>
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	

**PROBLEM Fall-04-F.1:**

In each of the following cases, *simplify the expression as much as possible.*

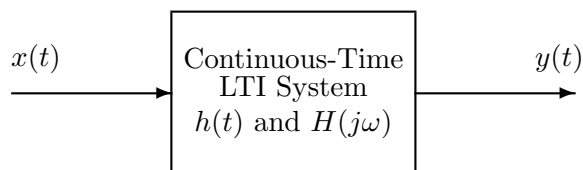
(a)  $(t^2 - 5t + 1)\delta(t - 4) =$

(b)  $u(t + 3) * \delta(t - 4) =$

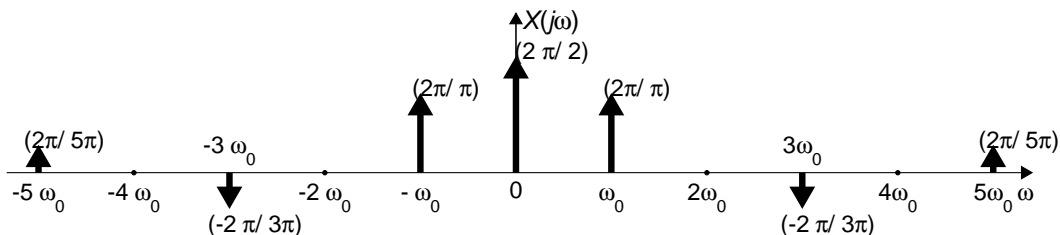
(c)  $\int_{-5}^5 \sin(\pi\tau/4)\delta(\tau + 3)d\tau =$

(d)  $x[n] = 5 \cos(0.3\pi n - \pi/4) + \frac{5}{\sqrt{2}} \cos(0.3\pi n + \pi/2) =$

**PROBLEM Fall-04-F.2:**



The periodic input to the above LTI system has the Fourier transform  $X(j\omega)$  drawn below:



where the dark arrows denote impulses. For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform,  $X(j\omega)$ . [Circle the correct answer. There is only one correct answer in each case.]

- |  |     |     |     |     |     |     |
|--|-----|-----|-----|-----|-----|-----|
| (a) $y(t) = \frac{1}{\pi} \cos(\omega_0 t)$                | (1) | (2) | (3) | (4) | (5) | (6) |
| (b) $y(t) = x(t - 1)$                                      | (1) | (2) | (3) | (4) | (5) | (6) |
| (c) $y(t) = 2(\omega_0/\pi) \cos(\omega_0 t + \pi/2)$      | (1) | (2) | (3) | (4) | (5) | (6) |
| (d) $y(t) = \frac{1}{2} + \frac{4}{3\pi} \cos(\omega_0 t)$ | (1) | (2) | (3) | (4) | (5) | (6) |
| (e) $y(t) = x(t - 1) - \frac{1}{2}$                        | (1) | (2) | (3) | (4) | (5) | (6) |

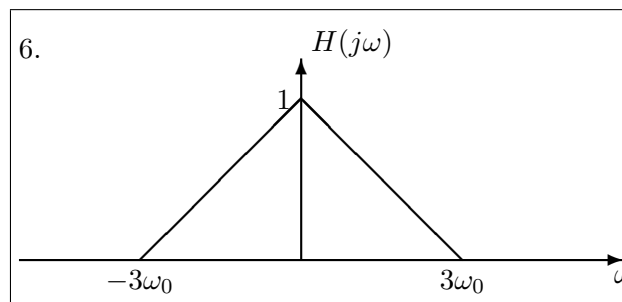
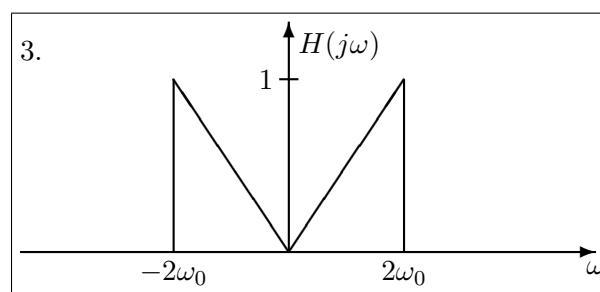
The possible filters are described by the following equations and graphs.<sup>1</sup>

$$1. H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$$

$$4. H(j\omega) = \begin{cases} j\omega & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$$

$$2. H(j\omega) = e^{-j\omega}$$

$$5. H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ e^{-j\omega} & |\omega| > \omega_0/2 \end{cases}$$

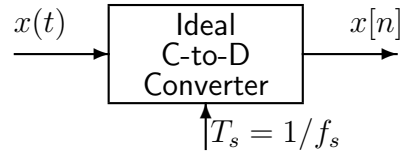


<sup>1</sup>Some of these may not be needed as answers.

### PROBLEM Fall-04-F.3:

The individual parts of this problem are independent.

- (a) If the output from an ideal C/D converter is  $x[n] = A \cos(0.25\pi n)$ , and the sampling rate is 10000 samples/sec, then determine two possible positive values of the input frequency of  $x(t)$  that are less than 10000Hz.:



ANS 1: =

ANS 2: =

- (b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;  
xx = (3/pi) * cos(pi*1.25*nn + pi/3);  
soundsc(xx,fsamp)
```

Although the sinusoid was not written to have a frequency of 1800 Hz, it is possible to play out the vector **xx** so that it sounds like a 1800 Hz tone. Determine the value of **fsamp** that should be used to play the vector **xx** as a 1800 Hz tone. Write your answer as an integer.

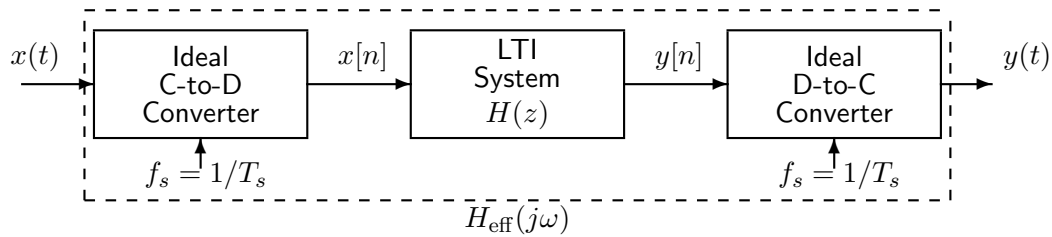
fsamp =  Hz

- (c) Determine the Nyquist rate for sampling the signal  $x(t)$  defined by:  
 $x(t) = \Im\{e^{-j4000\pi t} + e^{j3000\pi t}\}.$

ANS =  samples/sec.

**PROBLEM Fall-04-F.4:**

Consider the following system for discrete-time filtering of a continuous-time signal:

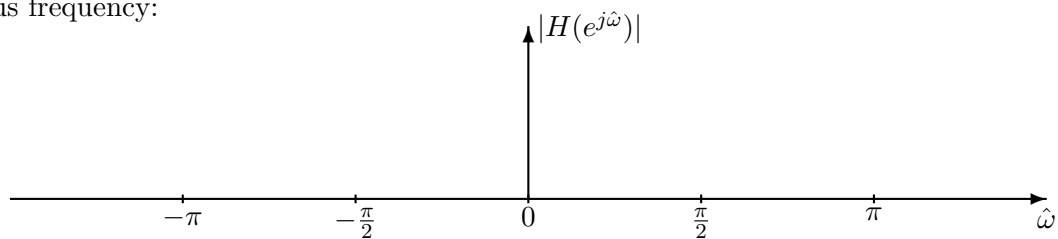


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,0,1,0,-1],xx)
```

where **xx** is an array of samples of  $x[n]$  and **yy** holds samples of  $y[n]$ .

- (a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



- (b) Assume that the input signal  $x(t)$  is a sum of cosines:

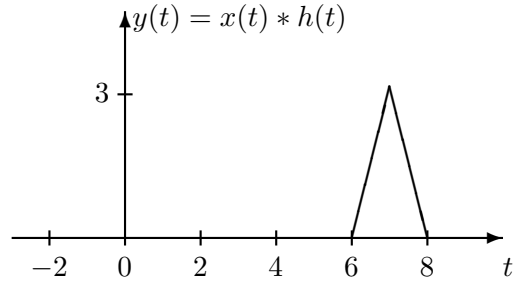
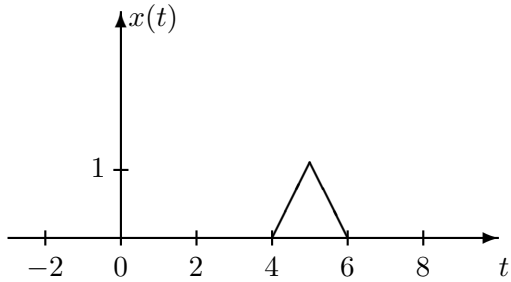
$$x(t) = 9 \cos(600\pi t - \pi/3) + 3 \cos(800\pi t + \pi/4)$$

For this input signal, determine the output signal  $y(t)$  when the sampling rate is  $f_s = 600$  **samples/sec**. Your answer should be expressed as a sum of cosines.

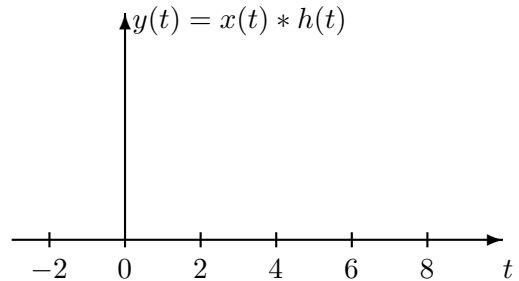
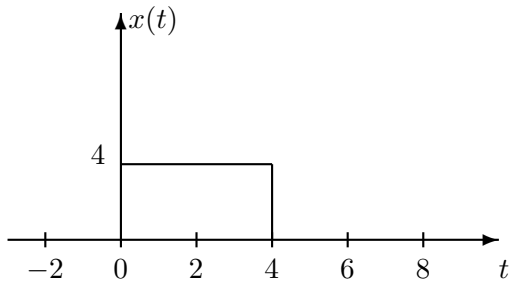
**PROBLEM Fall-04-F.5:**

The parts of this problem are completely independent.

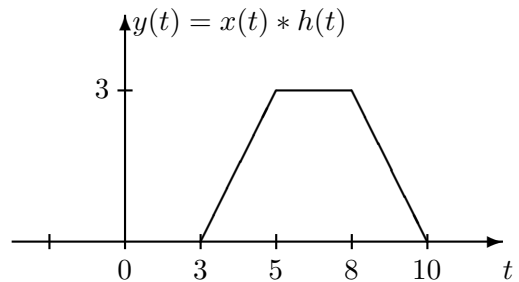
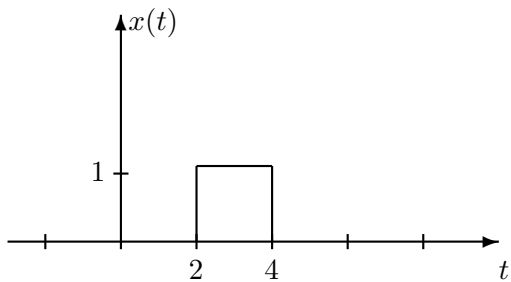
(a) Given that  $y(t) = x(t) * h(t)$ , find  $h(t)$ .  $h(t) =$



(b) If  $h(t) = u(t)$ , plot  $y(t) = x(t) * h(t)$  on the graph on the right. *Be sure to label the y(t) axis.*



(c) If  $h(t) = c[u(t-a) - u(t-b)]$  and  $y(t) = x(t) * h(t)$ , determine the values of  $a$ ,  $b$ , and  $c$ , where  $x(t)$  is given by the graph on the left and  $y(t)$  is given by the graph on the right.



$a =$

$b =$

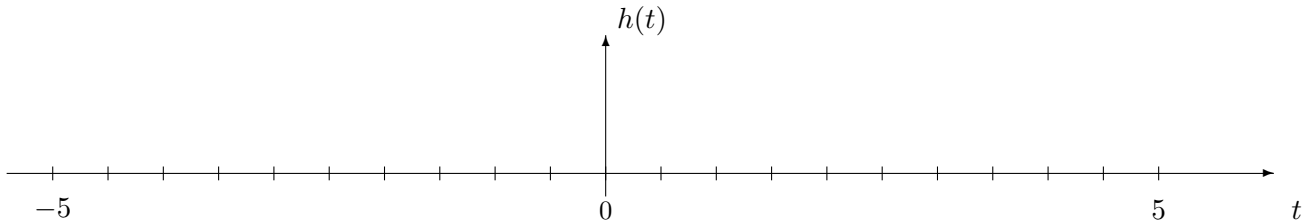
$c =$

**PROBLEM Fall-04-F.6:**

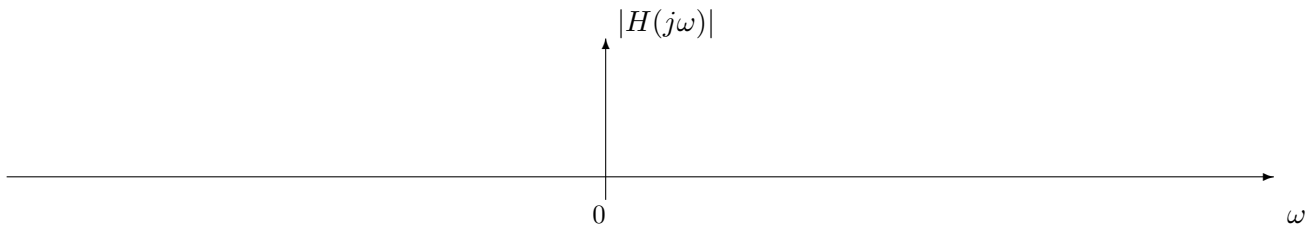
A linear time-invariant system has impulse response

$$h(t) = \frac{100 \sin(0.5\pi t)}{\pi t}.$$

- (a) Sketch  $h(t)$  very carefully on the graph below for the indicated range of values of  $t$ . Label the maximum value and the points where  $h(t) = 0$ .



- (b) Determine  $H(j\omega)$ , the Fourier transform of  $h(t)$ . Then, plot the frequency response magnitude,  $|H(j\omega)|$ , for this system on the graph below. Label your plot carefully to receive full credit.



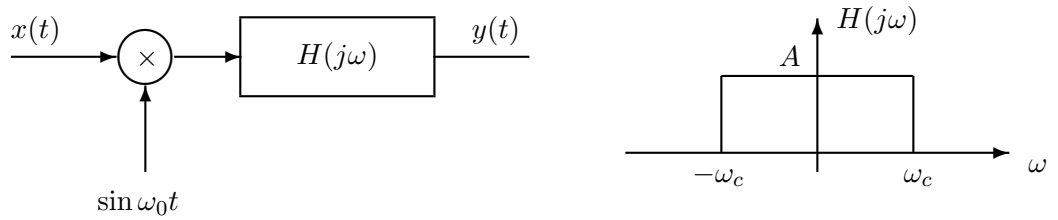
- (c) Determine the output  $y(t)$  of the LTI system when the input is

$$x(t) = \frac{5 \sin(2\pi(t - 3))}{\pi(t - 3)}.$$

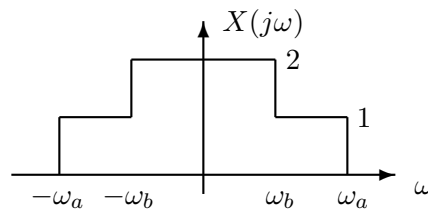
**PROBLEM Fall-04-F.7:**

The two parts of this problem are independent.

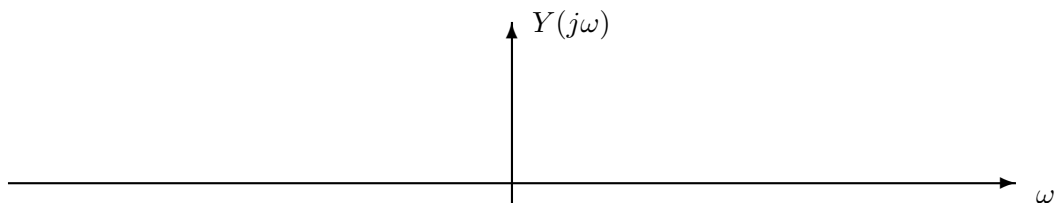
- (a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine.



Assume that  $x(t)$  has the spectrum,  $X(j\omega)$  shown below

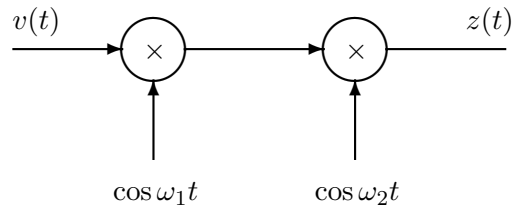


Sketch the spectrum,  $Y(j\omega)$  of the output of the scrambler on the axes below if  $\omega_a/(2\pi) = 10$  kHz,  $\omega_b/(2\pi) = 4$  kHz,  $\omega_c/(2\pi) = 15$  kHz, and  $\omega_0/(2\pi) = 15$  kHz. Be sure to LABEL YOUR PLOT.

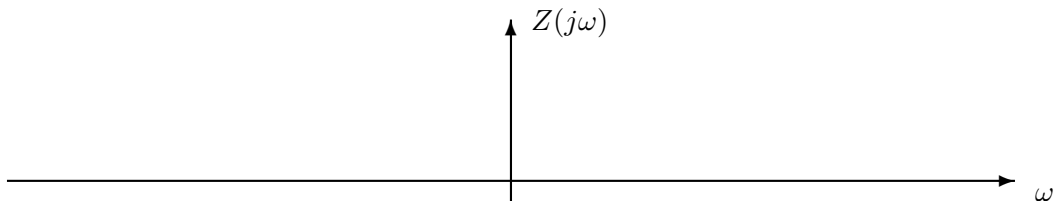
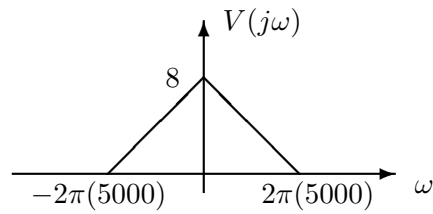




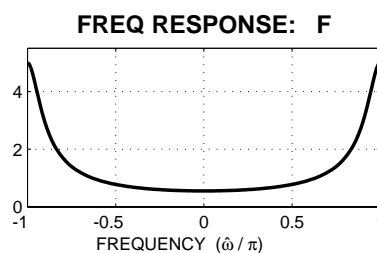
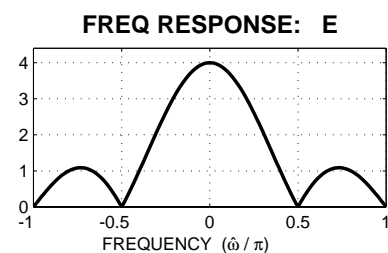
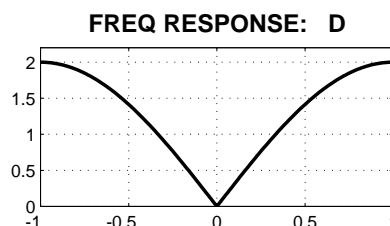
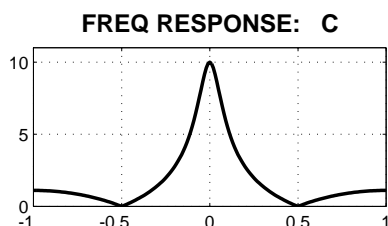
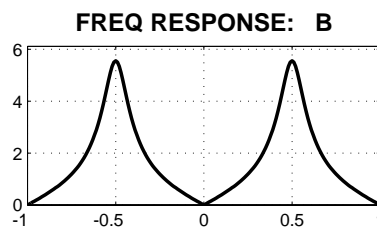
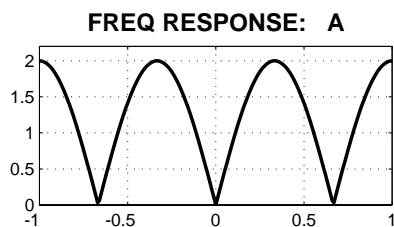
- (b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.



Let  $f_1 = \omega_1/(2\pi) = 50$  kHz and  $f_2 = \omega_2/(2\pi) = 20$  kHz. Sketch the spectrum  $Z(j\omega)$  assuming that  $V(j\omega)$  has the shape shown in the figure below.



**PROBLEM Fall-04-F.8:**



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an  $H(z)$ , a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is  $\hat{\omega}/\pi$ .

$\mathcal{S}_1 : y[n] = -0.8y[n-1] + x[n]$

$\mathcal{S}_5 : H(z) = \frac{1}{1 + 0.8z^{-1}} + \frac{-1}{1 - 0.8z^{-1}}$

$\mathcal{S}_2 : H(z) = \frac{1 + z^{-2}}{1 + 0.64z^{-2}}$

$\mathcal{S}_6 : y[n] = -0.64y[n-2] + x[n] - x[n-2]$

$\mathcal{S}_3 : H(z) = \sum_{k=0}^3 z^{-k}$

$\mathcal{S}_7 : y[n] = x[n] - x[n-1]$

$\mathcal{S}_4 : H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}$

$\mathcal{S}_8 : H(z) = 1 - z^{-3}$

Mark your answer in the following table:

FREQUENCY RESPONSE	SYSTEM ( $\mathcal{S}_\#$ )	FREQUENCY RESPONSE	SYSTEM ( $\mathcal{S}_\#$ )
A		B	
C		D	
E		F	

**PROBLEM Fall-04-F.1:**

In each of the following cases, *simplify the expression as much as possible.*

$$(a) (t^2 - 5t + 1)\delta(t - 4) = \boxed{-3\delta(t-4)}$$

$$(b) u(t+3) * \delta(t-4) = \boxed{u(t-1)}$$

$$(c) \int_{-5}^5 \sin(\pi\tau/4)\delta(\tau+3)d\tau = \boxed{\sin(-3\pi/4) = -1/\sqrt{2}}$$

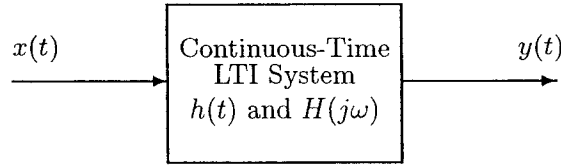
$$(d) x[n] = 5 \cos(0.3\pi n - \pi/4) + \frac{5}{\sqrt{2}} \cos(0.3\pi n + \pi/2) = \boxed{\frac{5}{\sqrt{2}} \cos(0.3\pi n)}$$

$$x[n] = \operatorname{Re} \left[ \left( 5e^{-j\pi/4} + \frac{5}{\sqrt{2}} e^{j\pi/2} \right) e^{j0.3\pi n} \right]$$

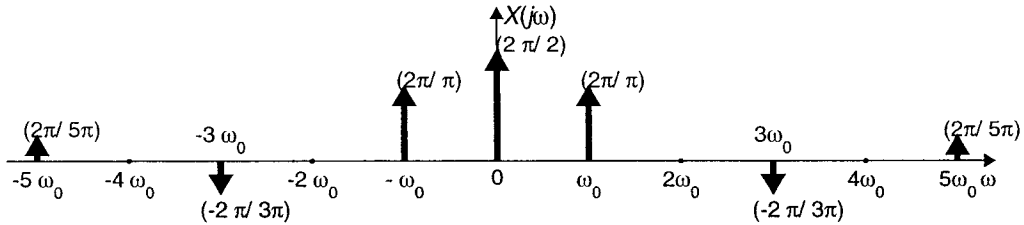
$$5e^{-j\pi/4} + \frac{5}{\sqrt{2}} e^{j\pi/2} = 5 \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \right) + \frac{5}{\sqrt{2}} j = \frac{5}{\sqrt{2}}$$

$$\Rightarrow x[n] = \frac{5}{\sqrt{2}} \cos(0.3\pi n)$$

**PROBLEM Fall-04-F.2:**



The periodic input to the above LTI system has the Fourier transform  $X(j\omega)$  drawn below:



where the dark arrows denote impulses. For the following outputs of the system, determine from the list below the frequency response of the system that could have produced that output when the input is the signal with the given Fourier transform,  $X(j\omega)$ . [Circle the correct answer. There is only one correct answer in each case.]

- |  |     |            |            |            |            |            |
|--|-----|------------|------------|------------|------------|------------|
| (a) $y(t) = \frac{1}{\pi} \cos(\omega_0 t)$                | (1) | (2)        | <b>(3)</b> | (4)        | (5)        | (6)        |
| (b) $y(t) = x(t - 1)$                                      | (1) | <b>(2)</b> | (3)        | (4)        | (5)        | (6)        |
| (c) $y(t) = 2(\omega_0/\pi) \cos(\omega_0 t + \pi/2)$      | (1) | (2)        | (3)        | <b>(4)</b> | (5)        | (6)        |
| (d) $y(t) = \frac{1}{2} + \frac{4}{3\pi} \cos(\omega_0 t)$ | (1) | (2)        | (3)        | (4)        | (5)        | <b>(6)</b> |
| (e) $y(t) = x(t - 1) - \frac{1}{2}$                        | (1) | (2)        | (3)        | (4)        | <b>(5)</b> | (6)        |

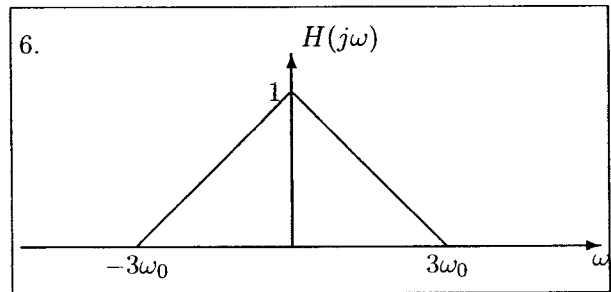
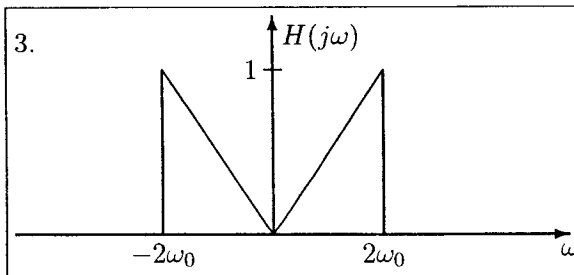
The possible filters are described by the following equations and graphs.<sup>1</sup>

$$1. H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < \omega_0/2 \\ 0 & |\omega| > \omega_0/2 \end{cases}$$

$$4. H(j\omega) = \begin{cases} j\omega & |\omega| < 3\omega_0/2 \\ 0 & |\omega| > 3\omega_0/2 \end{cases}$$

$$2. H(j\omega) = e^{-j\omega}$$

$$5. H(j\omega) = \begin{cases} 0 & |\omega| < \omega_0/2 \\ e^{-j\omega} & |\omega| > \omega_0/2 \end{cases}$$

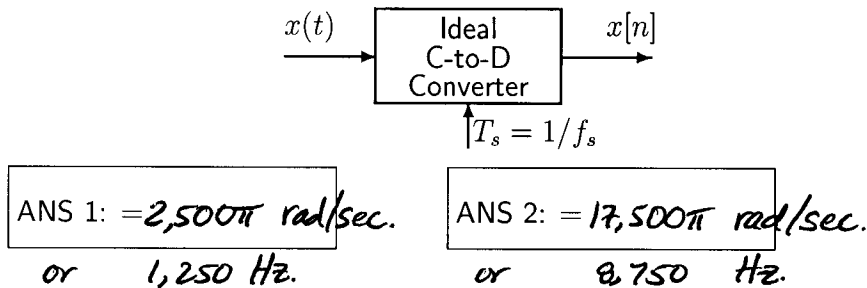


<sup>1</sup>Some of these may not be needed as answers.

**PROBLEM Fall-04-F.3:**

The individual parts of this problem are independent.

- (a) If the output from an ideal C/D converter is  $x[n] = A \cos(0.25\pi n)$ , and the sampling rate is 10000 samples/sec, then determine two possible positive values of the input frequency of  $x(t)$  that are less than 10000Hz.:



- (b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
nn = 0:44100;
xx = (3/pi) * cos(pi*1.25*nn + pi/3);
soundsc(xx, fsamp)
```

Although the sinusoid was not written to have a frequency of 1800 Hz, it is possible to play out the vector `xx` so that it sounds like a 1800 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 1800 Hz tone. Write your answer as an integer.

`fsamp = 4,800` Hz

$$\hat{\omega} = \omega T_s \quad \frac{5\pi}{4} \equiv \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$$

$$\frac{3\pi}{4} = 2\pi(1800)T_s$$

$$\frac{1}{T_s} = \frac{14,400}{3} = 4,800$$

- (c) Determine the Nyquist rate for sampling the signal  $x(t)$  defined by:  
 $x(t) = \Im\{e^{-j4000\pi t} + e^{j3000\pi t}\}$ .

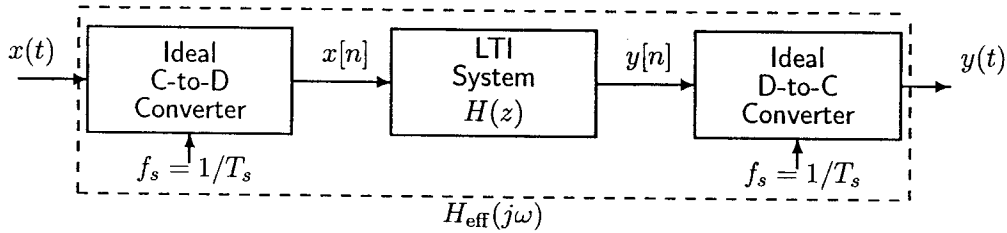
ANS = 4,000 samples/sec.

$$\omega_b = 4000\pi$$

$$f_b = 2000 \text{ Hz.}$$

**PROBLEM Fall-04-F.4:**

Consider the following system for discrete-time filtering of a continuous-time signal:

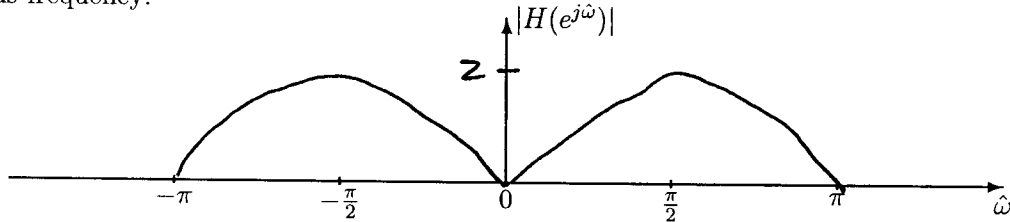


Assume that the discrete-time system is implemented using the MATLAB command:

```
yy=firfilt([0,0,1,0,-1],xx)
```

where  $xx$  is an array of samples of  $x[n]$  and  $yy$  holds samples of  $y[n]$ .

- (a) Determine the frequency response of the discrete-time filter and plot its magnitude response versus frequency:



$$\begin{aligned}
 H(e^{j\hat{\omega}}) &= e^{-j2\hat{\omega}} - e^{-j4\hat{\omega}} = e^{-j3\hat{\omega}} [e^{j\hat{\omega}} - e^{-j\hat{\omega}}] \\
 &= j2 \sin(\hat{\omega}) e^{-j3\hat{\omega}}
 \end{aligned}$$

- (b) Assume that the input signal  $x(t)$  is a sum of cosines:

$$x(t) = 9 \cos(600\pi t - \pi/3) + 3 \cos(800\pi t + \pi/4)$$

For this input signal, determine the ~~Fourier transform~~ of the output signal  $y(t)$  when the sampling rate is  $f_s = 600$  samples/sec. ~~Determine the output signal  $y[n]$ .~~ Your answer should be expressed as a sum of cosines.

$$\begin{aligned}
 x[n] &= 9 \cos\left(\frac{600\pi}{600} n - \frac{\pi}{3}\right) + 3 \cos\left(\frac{800\pi}{600} n + \frac{\pi}{4}\right) \\
 &= 9 \cos\left(n\pi - \frac{\pi}{3}\right) + 3 \cos\left(\frac{4\pi}{3} n + \frac{\pi}{4}\right) \\
 &= 9 \cos\left(n\pi - \frac{\pi}{3}\right) + 3 \cos\left(\frac{2\pi}{3} n - \frac{\pi}{4}\right)
 \end{aligned}$$

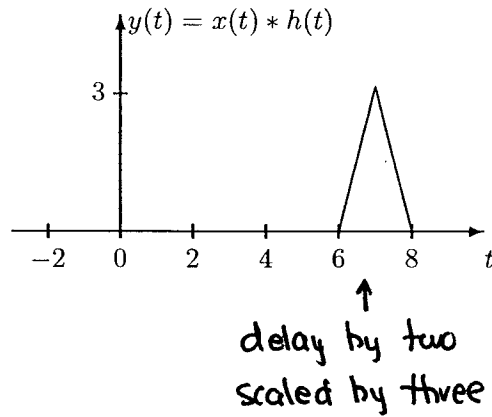
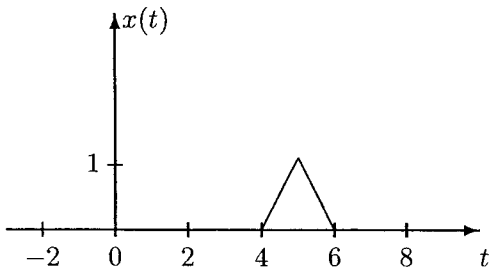
$$\Rightarrow y[n] = 0 + 3(2 \sin(\frac{2\pi}{3})) \cos\left(\frac{2\pi}{3} n - \frac{\pi}{4} + \frac{\pi}{2}\right)$$

$y[n] = 3\sqrt{3} \cos\left(\frac{2\pi}{3} n + \frac{\pi}{4}\right)$
$y(t) = 3\sqrt{3} \cos(400\pi t + \frac{\pi}{4})$

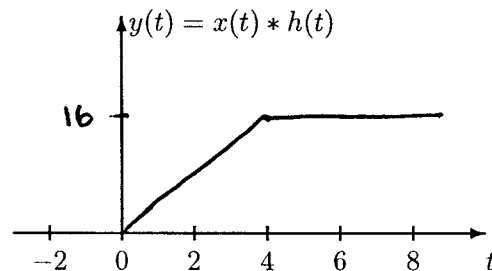
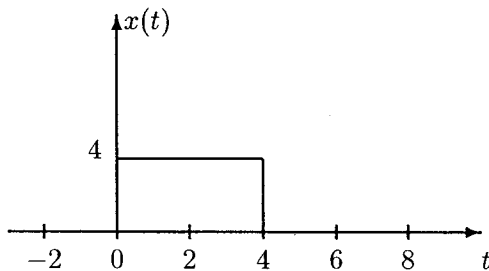
**PROBLEM Fall-04-F.5:**

The parts of this problem are completely independent.

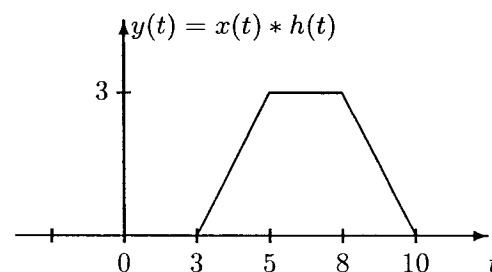
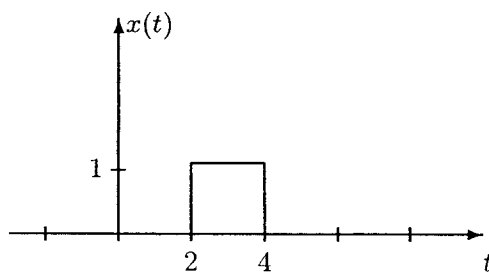
(a) Given that  $y(t) = x(t) * h(t)$ , find  $h(t)$ .  $h(t) = 3\delta(t-2)$



(b) If  $h(t) = u(t)$ , plot  $y(t) = x(t) * h(t)$  on the graph on the right. Be sure to label the  $y(t)$  axis.



(c) If  $h(t) = c[u(t-a) - u(t-b)]$  and  $y(t) = x(t) * h(t)$ , determine the values of  $a$ ,  $b$ , and  $c$ , where  $x(t)$  is given by the graph on the left and  $y(t)$  is given by the graph on the right.



$a = 1$

$b = 6$

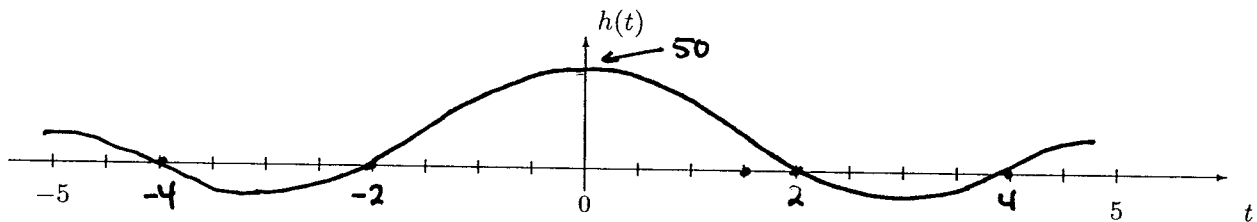
$c = 3/2$

**PROBLEM Fall-04-F.6:**

A linear time-invariant system has impulse response

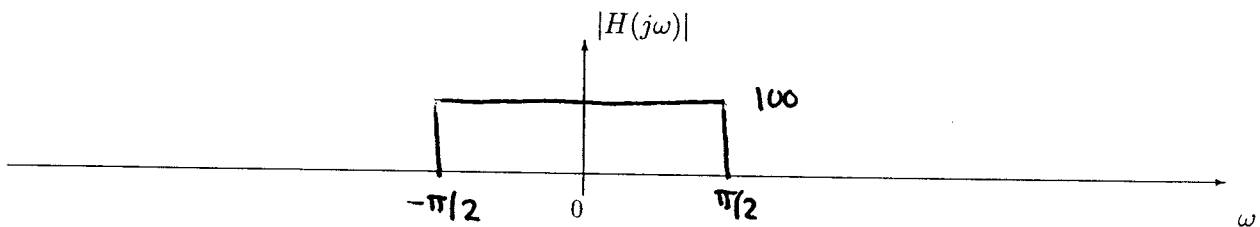
$$h(t) = \frac{100 \sin(0.5\pi t)}{\pi t}$$

- (a) Sketch  $h(t)$  very carefully on the graph below for the indicated range of values of  $t$ . Label the maximum value and the points where  $h(t) = 0$ .



- (b) Determine  $H(j\omega)$ , the Fourier transform of  $h(t)$ . Then, plot the frequency response magnitude,  $|H(j\omega)|$ , for this system on the graph below. Label your plot carefully to receive full credit.

$$H(j\omega) = 100 [u(\omega + \pi/2) - u(\omega - \pi/2)]$$



- (c) Determine the output  $y(t)$  of the LTI system when the input is

$$x(t) = \frac{5 \sin(2\pi(t-3))}{\pi(t-3)}$$

$$X(j\omega) = 5 e^{-j3\omega} [u(\omega + 2\pi) - u(\omega - 2\pi)]$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = 500 e^{-j3\omega} [u(\omega + \pi/2) - u(\omega - \pi/2)]$$

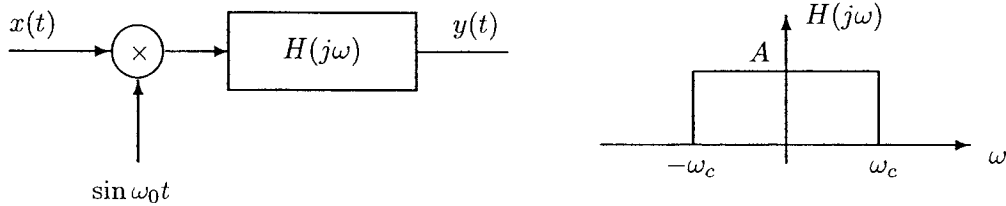
$$\Rightarrow y(t) = \frac{500 \sin(0.5\pi(t-3))}{\pi(t-3)}$$



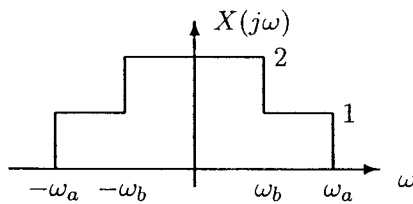
**PROBLEM Fall-04-F.7:**

The two parts of this problem are independent.

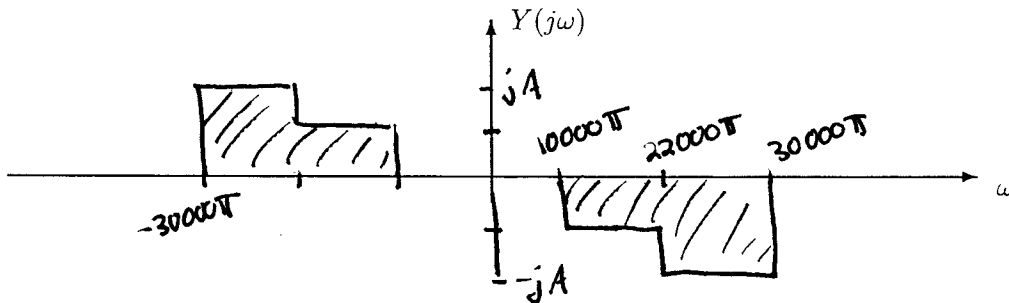
- (a) The system below is proposed as an alternative speech scrambler to the one in lab. Notice that the carrier signal is a sine instead of a cosine..



Assume that  $x(t)$  has the spectrum,  $X(j\omega)$  shown below

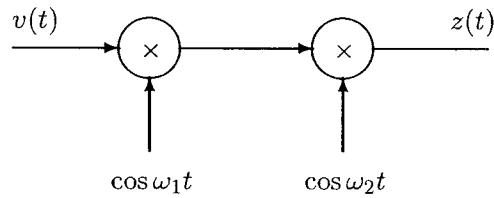


Sketch the spectrum,  $Y(j\omega)$  of the output of the scrambler on the axes below if  $\omega_a/(2\pi) = 10$  kHz,  $\omega_b/(2\pi) = 4$  kHz,  $\omega_c/(2\pi) = 15$  kHz, and  $\omega_0/(2\pi) = 15$  kHz. Be sure to LABEL YOUR PLOT.

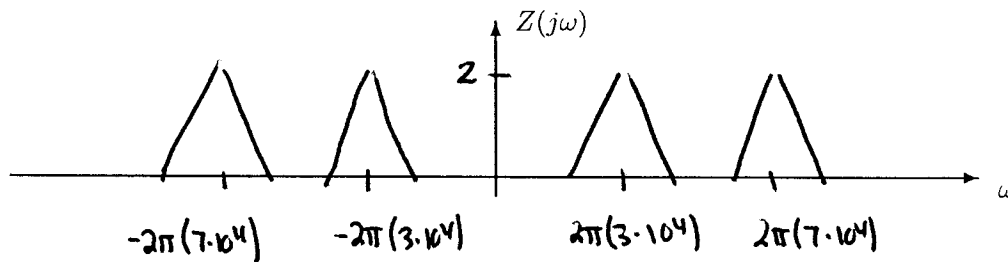
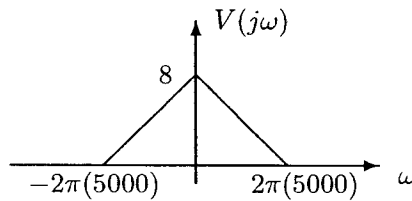


$$\sin(\omega_0 t) \longrightarrow -j\pi \delta(\omega - \omega_0) + j\pi \delta(\omega + \omega_0) :$$

- (b) Signals are often repeatedly moved from one portion of the spectrum to another by repeated mixing. This process is called **heterodyning**. A simple example is the cascade of two mixers shown below.

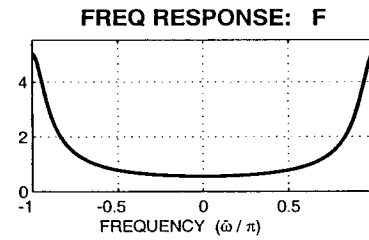
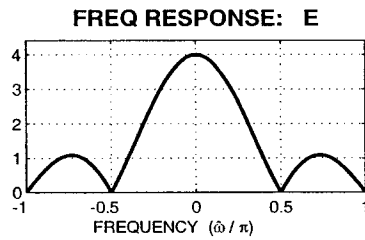
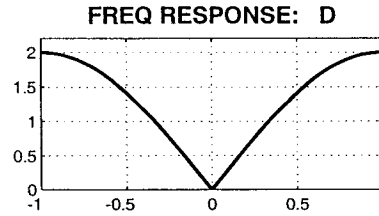
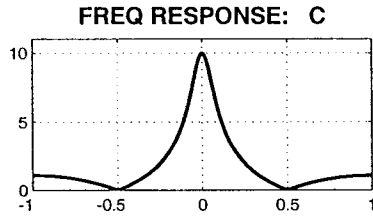
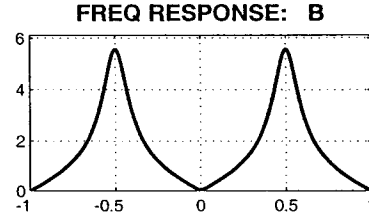
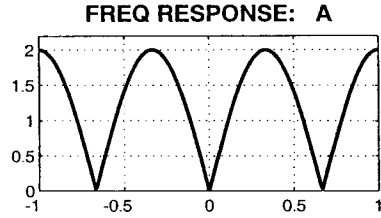


Let  $f_1 = \omega_1/(2\pi) = 50$  kHz and  $f_2 = \omega_2/(2\pi) = 20$  kHz. Sketch the spectrum  $Z(j\omega)$  assuming that  $V(j\omega)$  has the shape shown in the figure below.



$$\begin{aligned}
 z(t) &= v(t) \cos(\omega_1 t) \cos(\omega_2 t) \\
 &= \frac{1}{2} v(t) \left[ \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t \right] \\
 &= \frac{1}{2} v(t) \left[ \cos(2\pi \cdot 7 \cdot 10^4 t) + \cos(2\pi \cdot 3 \cdot 10^4 t) \right]
 \end{aligned}$$

**PROBLEM Fall-04-F.8:**



For each of the frequency response plots (A, B, C, D, E, F), determine which one of the following systems (specified by either an  $H(z)$ , a difference equation, or a MATLAB statement) matches the frequency response (magnitude only). NOTE: frequency axis is **normalized**; it is  $\hat{\omega}/\pi$ .

$S_1 : y[n] = -0.8y[n-1] + x[n]$

$S_5 : H(z) = \frac{1}{1 + 0.8z^{-1}} + \frac{-1}{1 - 0.8z^{-1}}$

$S_2 : H(z) = \frac{1 + z^{-2}}{1 + 0.64z^{-2}}$

$S_6 : y[n] = -0.64y[n-2] + x[n] - x[n-2]$

$S_3 : H(z) = \sum_{k=0}^3 z^{-k}$

$S_7 : y[n] = x[n] - x[n-1]$

$S_4 : H(z) = \frac{1 + z^{-2}}{1 - 0.8z^{-1}}$

$S_8 : H(z) = 1 - z^{-3}$

Mark your answer in the following table:

FREQUENCY RESPONSE	SYSTEM ( $S_{\#}$ )	FREQUENCY RESPONSE	SYSTEM ( $S_{\#}$ )
A	8	B	6
C	4	D	7
E	3	F	1