

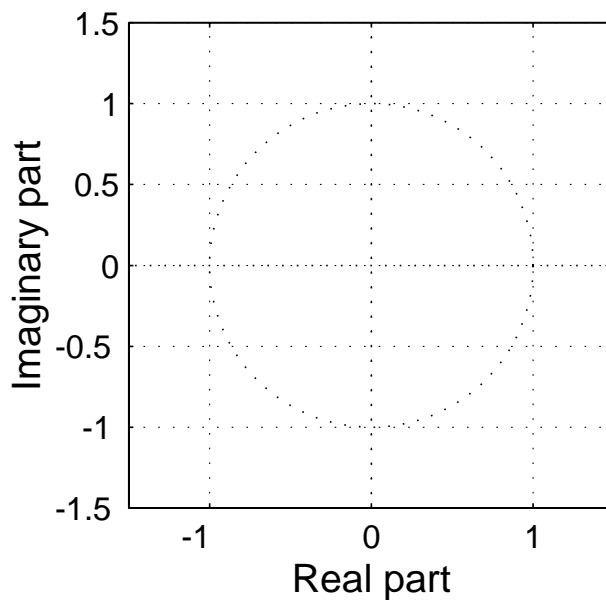
PROBLEM Fall-04-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z -transform system function:

$$H(z) = (1 + 0.8z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

- (a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

- (b) Determine *all* the zeros of $H(z)$ and plot them in the z -plane.



- (c) If the input is of the form $x[n] = s[n] + A \cos(\omega_0 n + \phi)$, where $s[n]$ is a speech signal, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN your answer.**

PROBLEM Fall-04-Q.3.2:

For each of the following expressions, select the correct match from the second list below.
(The operator $*$ denotes convolution.)

(a) $e^{-t}u(t) * u(t - 4)$

(b) $\delta(t - 1) * \delta(t - 3)$

(c) $e^{-t}u(t)\delta(t - 4)$

(d) $\frac{d}{dt} \{e^{-t}u(t - 4)\}$

(e) $u(4)$

(f) $u(t - 1) * u(t - 3)$

(g) $e^{-t}u(t) * \delta(t - 4)$

(h) $\int_{-\infty}^0 \delta(t - 4) dt$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t - 4)$

[2] $u(t - 4)$

[3] $-e^{-t}u(t - 4) + e^{-4}\delta(t - 4)$

[4] $(t - 4)u(t - 4)$

[5] $(1 - e^{-t+4})u(t - 4)$

[6] $e^{-(t-4)}u(t - 4)$

[7] $e^{-4}\delta(t - 4)$

[8] 0

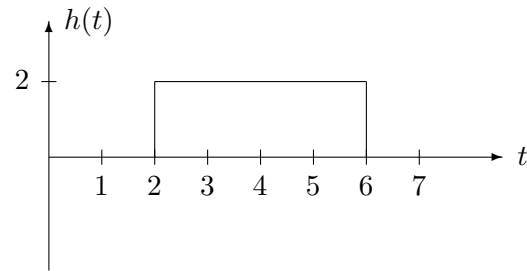
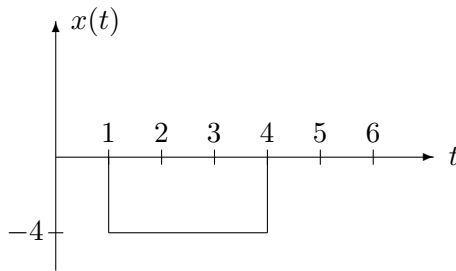
[9] $\delta(t - 4)$

[10] 1

[11] e^{-4}

PROBLEM Fall-04-Q.3.3:

The following figure shows the signal $x(t) = -4u(t-1)+4u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t-2)-2u(t-6)$.



(a) Sketch $h(8 - \tau)$ as a function of τ in the space below.

(b) Determine the value of the output of the LTI system, $y(t)$, at $t = 8$; that is, determine $y(8)$. It is not necessary to evaluate $y(t)$ for all t , only for $t = 8$. Note: This problem may be answered without performing any integration.

(c) $y(t)$ reaches its minimum value for $T_1 \leq t \leq T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2 .

$$y_{min} = \underline{\hspace{2cm}}$$

$$T_1 = \underline{\hspace{2cm}} \text{ sec}$$

$$T_2 = \underline{\hspace{2cm}} \text{ sec}$$

PROBLEM Fall-04-Q.3.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. *Write each answer in the box provided.* (The operator $*$ denotes convolution.)

(a) $x(t) = \delta(t - 3) \sin(\pi t)$

(b) $x(t) = -\frac{4}{3}e^{-2t/3}u(t) + 2\delta(t)$

(c) $x(t) = \frac{4}{3}e^{(-2+j3)t}u(t)$

(d) $x(t) = \Im\{\delta(t - 4) * e^{j\pi t}\}$

(e) $x(t) = u(t - 2) - u(t - 4)$

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \frac{4/3}{2 + j(\omega - 3)}$

[1] $X(j\omega) = \frac{1}{2\pi}e^{-j3\omega} * [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)]$

[2] $X(j\omega) = \frac{-9}{2 + j3\omega}$

[3] $X(j\omega) = j\pi\delta(\omega + \pi) - j\pi\delta(\omega - \pi)$

[4] $X(j\omega) = 2e^{-j3\omega} \frac{\sin(3\omega)}{\omega}$

[5] $X(j\omega) = 2e^{-j3\omega} \frac{\sin(\omega)}{\omega}$

[6] $X(j\omega) = \frac{j6\omega}{2 + j3\omega}$

[7] $X(j\omega) = 0$

[8] $X(j\omega) = j2e^{-j3\omega} \sin(3\omega)$

[9] $X(j\omega) = \frac{\sin(\omega)}{\omega/2}$

PROBLEM Fall-04-Q.3.1:

A discrete-time system (FIR filter) is defined by the following z -transform system function:

$$H(z) = (1 + 0.8z^{-1})(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})$$

- (a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system. Give the numerical values of all filter coefficients.

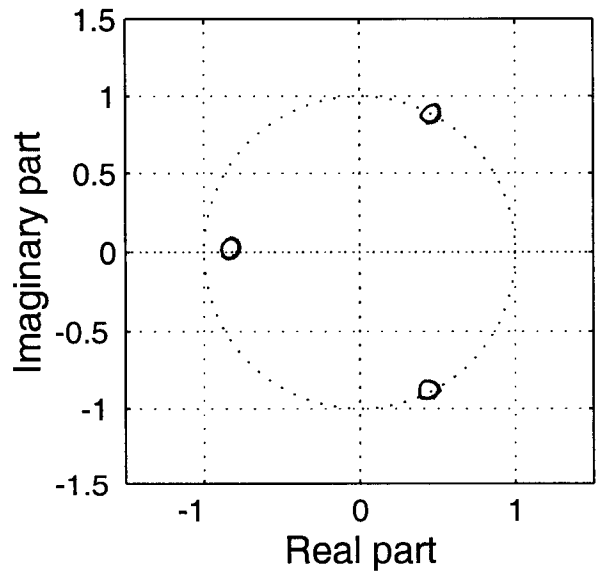
$$\begin{aligned} H(z) &= (1 + 0.8z^{-1})(1 - z^{-1} + z^{-2}) \\ &= 1 - 0.2z^{-1} + 0.2z^{-2} + 0.8z^{-3} \end{aligned}$$

$$y[n] = x[n] - 0.2x[n-1] + 0.2x[n-2] + 0.8x[n-3]$$

- (b) Determine *all* the zeros of $H(z)$ and plot them in the z -plane.

ZEROS AT:

$$\begin{aligned} z &= -0.8 \\ z &= e^{\pm j\frac{\pi}{3}} \end{aligned}$$



- (c) If the input is of the form $x[n] = s[n] + A \cos(\omega_0 n + \phi)$, where $s[n]$ is a speech signal, for what value of frequency ω_0 (in the range $0 < \omega_0 < \pi$) will the filter completely remove the sinusoidal component? **EXPLAIN** your answer.

$$\omega_0 = \frac{\pi}{3}$$

$$H(j\frac{\pi}{3}) = H(-j\frac{\pi}{3}) = 0$$

PROBLEM Fall-04-Q.3.2:

For each of the following expressions, select the correct match from the second list below.
(The operator $*$ denotes convolution.)

(a) $\boxed{[5]}$ $e^{-t}u(t) * u(t - 4)$

(b) $\boxed{[9]}$ $\delta(t - 1) * \delta(t - 3)$

(c) $\boxed{[7]}$ $e^{-t}u(t)\delta(t - 4)$

(d) $\boxed{[3]}$ $\frac{d}{dt} \{e^{-t}u(t - 4)\}$

(e) $\boxed{[10]}$ $u(4)$

(f) $\boxed{[4]}$ $u(t - 1) * u(t - 3)$

(g) $\boxed{[6]}$ $e^{-t}u(t) * \delta(t - 4)$

(h) $\boxed{[8]}$ $\int_{-\infty}^0 \delta(t - 4)dt$

Each of the expressions above is equivalent to one (and only one) of the expressions below:

[1] $-e^{-t}u(t - 4)$

[2] $u(t - 4)$

[3] $-e^{-t}u(t - 4) + e^{-4}\delta(t - 4)$

[4] $(t - 4)u(t - 4)$

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[7] $e^{-4}\delta(t - 4)$

[8] 0

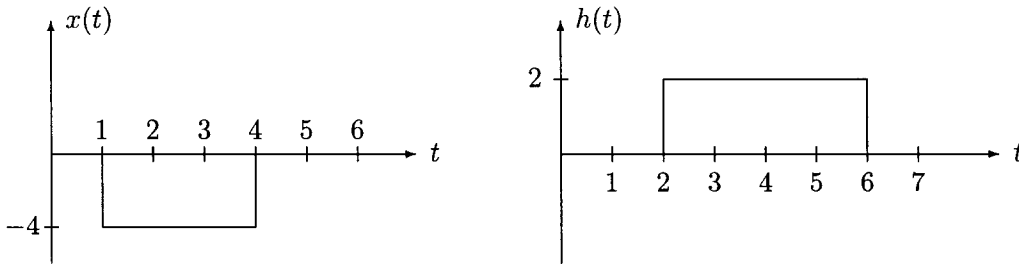
[9] $\delta(t - 4)$

[10] 1

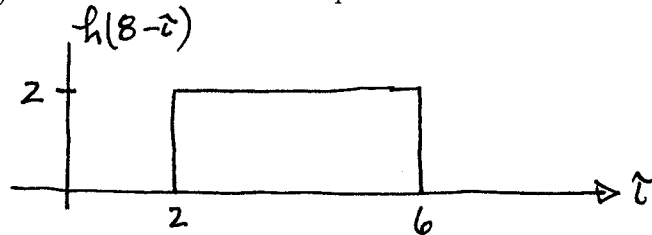
[11] e^{-4}

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The following figure shows the signal $x(t) = -4u(t-1) + 4u(t-4)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right) is $h(t) = 2u(t-2) - 2u(t-6)$.



- (a) Sketch $h(8 - \tau)$ as a function of τ in the space below.



- (b) Determine the value of the output of the LTI system, $y(t)$, at $t = 8$; that is, determine $y(8)$. It is not necessary to evaluate $y(t)$ for all t , only for $t = 8$. Note: This problem may be answered without performing any integration.

$$y(8) = \int_{-\infty}^{\infty} x(\tau) h(8 - \tau) d\tau = -16$$

- (c) $y(t)$ reaches its minimum value for $T_1 \leq t \leq T_2$. Find the minimum value, y_{min} and also the values for T_1 and T_2 .

$$y_{min} = \underline{-24}$$

$$T_1 = \underline{6} \text{ sec}$$

$$T_2 = \underline{7} \text{ sec}$$

PROBLEM Fall-04-Q.3.4:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. *Write each answer in the box provided.* (The operator * denotes convolution.)

(a) $x(t) = \delta(t - 3) \sin(\pi t)$

[7]

(b) $x(t) = -\frac{4}{3}e^{-2t/3}u(t) + 2\delta(t)$

[6]

(c) $x(t) = \frac{4}{3}e^{(-2+j3)t}u(t)$

[0]

(d) $x(t) = \Im\{\delta(t - 4) * e^{j\pi t}\}$

[3]

(e) $x(t) = u(t - 2) - u(t - 4)$

[5]

Each of the time signals above has a Fourier transform that can be found in the list below.

[0] $X(j\omega) = \frac{4/3}{2 + j(\omega - 3)}$

[1] $X(j\omega) = \frac{1}{2\pi}e^{-j3\omega} * [j\pi u(\omega + \pi) - j\pi u(\omega - \pi)]$

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