

ECE-2025

Spring-2005

Lecture 4
Spectrum Representation
24-Jan-05

Web-CT Info

- Bulletin Board has all **OFFICIAL** msgs
- Lectures are being posted
 - PDF format (4 per page)
- Upcoming Events:
 - Quiz #1 on on 4-Feb (Friday)
 - Review Session likely on Thursday, 3-Feb

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Lab Info

- Lab #1 Report
 - Turn in at beginning of your lab time
 - **Ask your grading TA about his/her format**
- Lab #2 is posted
 - Monday sections start Lab #1 on 24-Jan
 - Lab #2 on 31-Jan
- Finish Instructor Verification in Lab
 - Come to lab PREPARED
- Computer Problems?
help@ece.gatech.edu

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Homework Info

- Written HW #2 due THIS WEEK
 - In Recitation, at the beginning
 - **Monday Recitations turn in HW #1 on Monday, 24-Jan**
 - Format your HW (see guidelines)
- HW #1 Solutions have been posted
- HW #3 is out; will be covered on Quiz #1
- HW, Lab and MATLAB Help:
 - Mon, Tues, Wed: 6pm Burger-Henry-216
 - Every week during the semester

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PRINTING QUOTA

- ECE Labs have printers, but...
- Limit your printing to essentials
 - Your account has a quota
- 10 pages/week
 - 2000 students
 - 3 courses/student
 - 15 weeks/semester
 - 900,000 pages !

James THURBER

- It is better to know some of the questions than all of the answers

HISTORY

- Which company's first successful product was a sine-wave generator?
 - Variable frequency
 - Lab Instrument



Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 3, Section 3-1
- Other Reading:
 - Appendix A: Complex Numbers
- Next Lecture: Ch 3, Sects 3-2, 3-3, 3-7 & 3-8

LECTURE OBJECTIVES

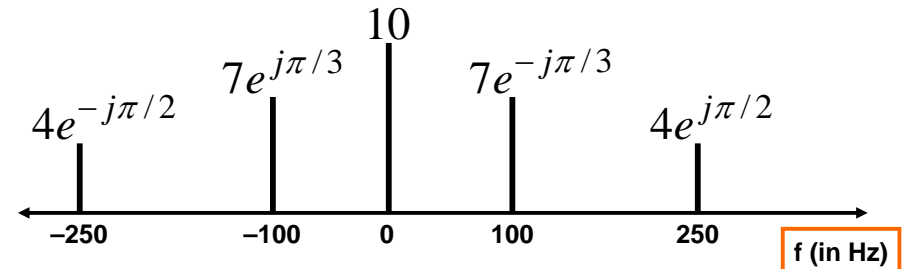
- Sinusoids with **DIFFERENT** Frequencies
 - SYNTHESIZE by Adding Sinusoids

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

- **SPECTRUM** Representation
 - Graphical Form shows **DIFFERENT** Freqs

FREQUENCY DIAGRAM

- Plot Complex Amplitude vs. Freq



Another FREQ. Diagram

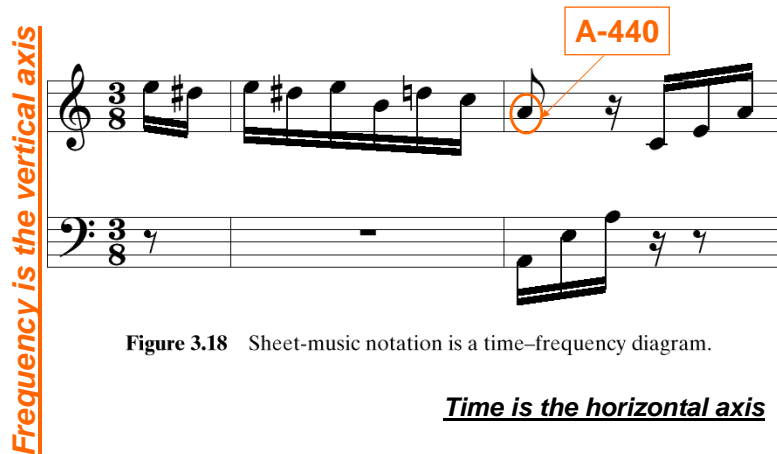


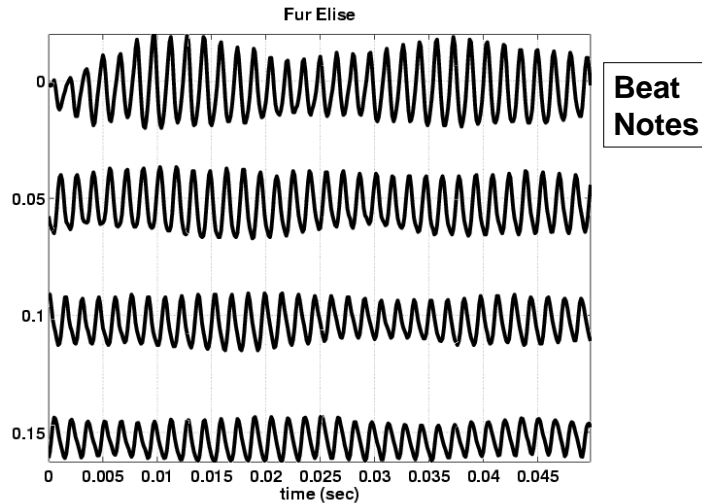


Figure 3.18 Sheet-music notation is a time–frequency diagram.

MOTIVATION

- Synthesize **Complicated** Signals
 - Musical Notes 
 - Piano uses 3 strings for many notes
 - Chords: play several notes simultaneously
 - Human Speech 
 - Vowels have dominant frequencies
 - Application: computer generated speech
 - Can **all** signals be generated this way?
 - Sum of sinusoids?

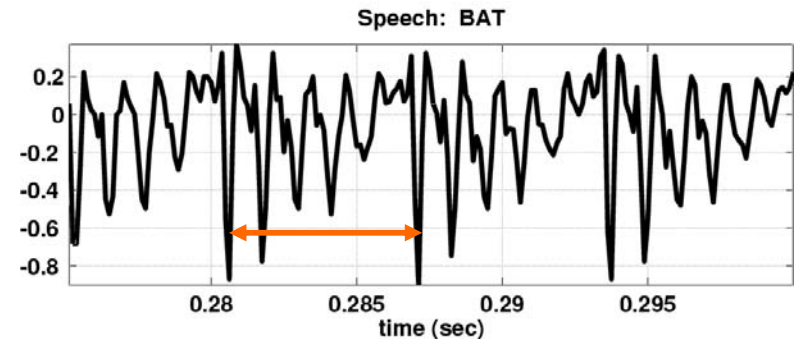
Fur Elise WAVEFORM



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Speech Signal: BAT

- Nearly **Periodic** in Vowel Region
 - Period is (Approximately) $T = 0.0065$ sec



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Euler's Formula Reversed

- Solve for **cosine** (or sine)

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$e^{-j\omega t} = \cos(-\omega t) + j \sin(-\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

$$e^{j\omega t} + e^{-j\omega t} = 2 \cos(\omega t)$$

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

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INVERSE Euler's Formula

- Solve for **cosine** (or sine)

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j}(e^{j\omega t} - e^{-j\omega t})$$

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SPECTRUM Interpretation

- Cosine = sum of 2 complex exponentials:

$$A \cos(7t) = \frac{A}{2} e^{j7t} + \frac{A}{2} e^{-j7t}$$

One has a positive frequency
The other has **negative** freq.
Amplitude of each is half as big

NEGATIVE FREQUENCY

- Is negative frequency real?
- Doppler Radar provides an example
 - Police radar measures speed by using the Doppler shift principle
 - Let's assume 400Hz \leftrightarrow 60 mph
 - **+400Hz** means towards the radar
 - **-400Hz** means away (opposite **direction**)
 - Think of a train whistle

SPECTRUM of SINE

- Sine = sum of 2 complex exponentials:

$$\begin{aligned} A \sin(7t) &= \frac{A}{2j} e^{j7t} - \frac{A}{2j} e^{-j7t} \\ &= \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t} \end{aligned}$$

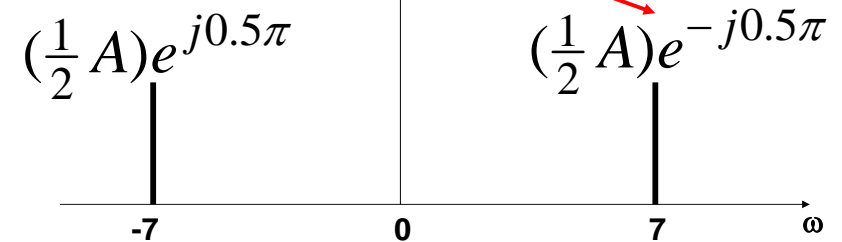
$$\frac{-1}{j} = j = e^{j0.5\pi}$$

- Positive freq. has phase = -0.5π
- Negative freq. has phase = **$+0.5\pi$**

GRAPHICAL SPECTRUM

EXAMPLE of SINE

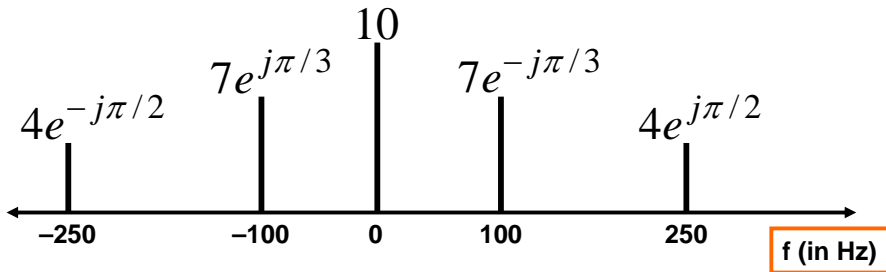
$$A \sin(7t) = \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$



AMPLITUDE, PHASE & FREQUENCY are shown

SPECTRUM ---> SINUSOID

- Add the spectrum components:



What is the formula for the signal $x(t)$?

Gather (A, ω , ϕ) information

- | | |
|----------------|---------------------|
| ▪ Frequencies: | ▪ Amplitude & Phase |
| ▪ -250 Hz | ▪ 4 $-\pi/2$ |
| ▪ -100 Hz | ▪ 7 $+\pi/3$ |
| ▪ 0 Hz | ▪ 10 0 |
| ▪ 100 Hz | ▪ 7 $-\pi/3$ |
| ▪ 250 Hz | ▪ 4 $+\pi/2$ |

Note the **conjugate phase**

DC is another name for zero-freq component

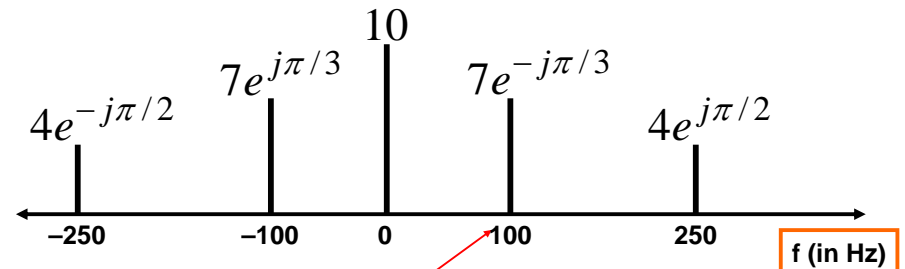
DC component always has $\phi=0$ or π (for real $x(t)$)

Add Spectrum Components-1

- | | |
|----------------|---------------------|
| ▪ Frequencies: | ▪ Amplitude & Phase |
| ▪ -250 Hz | ▪ 4 $-\pi/2$ |
| ▪ -100 Hz | ▪ 7 $+\pi/3$ |
| ▪ 0 Hz | ▪ 10 0 |
| ▪ 100 Hz | ▪ 7 $-\pi/3$ |
| ▪ 250 Hz | ▪ 4 $+\pi/2$ |

$$x(t) = 10 + 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Add Spectrum Components-2



$$x(t) = 10 + 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Simplify Components

$$x(t) = 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi(100)t} + 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi(250)t}$$

Use Euler's Formula to get **REAL** sinusoids:

$$A\cos(\omega t + \varphi) = \frac{1}{2}Ae^{j\varphi}e^{j\omega t} + \frac{1}{2}Ae^{-j\varphi}e^{-j\omega t}$$

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FINAL ANSWER

$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

So, we get the general form:

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$


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Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

$$x(t) = X_0 + \sum_{k=1}^N \Re\{X_k e^{j2\pi f_k t}\}$$

$$\Re\{z\} = \frac{1}{2}z + \frac{1}{2}z^*$$

$$X_k = A_k e^{j\varphi_k}$$

Frequency = f_k

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{1}{2}X_k e^{j2\pi f_k t} + \frac{1}{2}X_k^* e^{-j2\pi f_k t} \right\}$$

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Example: Synthetic Vowel

- Sum of 5 Frequency Components

f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	-0.185
1600	$(1657 - j13520)$	13,621	-1.449
1700	$4723 + j0$	4723	0

Table 3.1: Complex amplitudes for harmonic signal that approximates the vowel sound "ah".

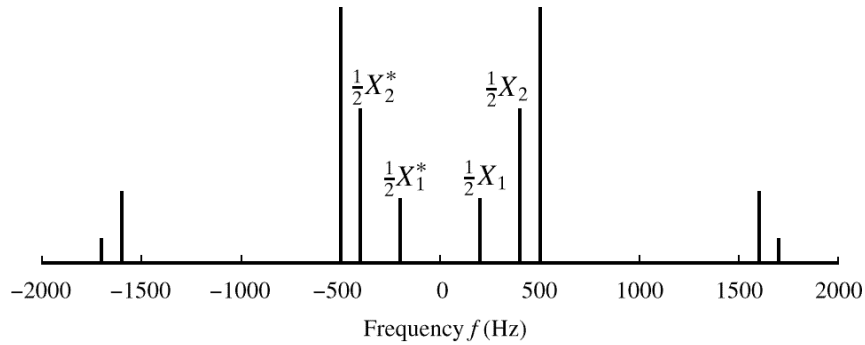
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SPECTRUM of VOWEL

- Note: Spectrum has $0.5X_k$ (except X_{DC})
- Conjugates in negative frequency

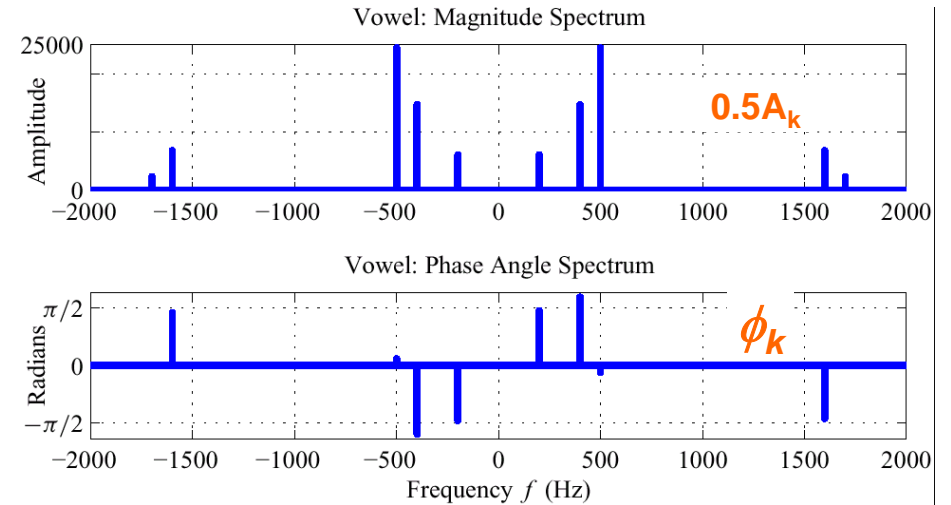


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SPECTRUM of VOWEL (Polar Format)



Vowel Waveform (sum of all 5 components)

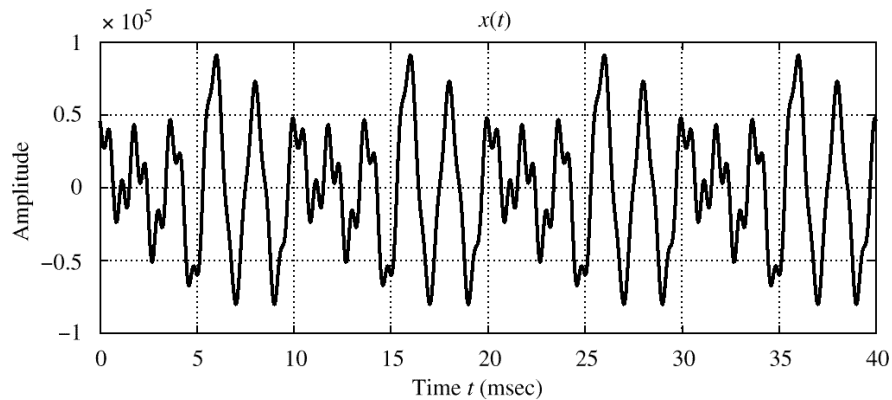


Figure 3.11 Sum of all of the terms in (3.3.4). Note that the period is 10 msec, which equals $1/f_0$.

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