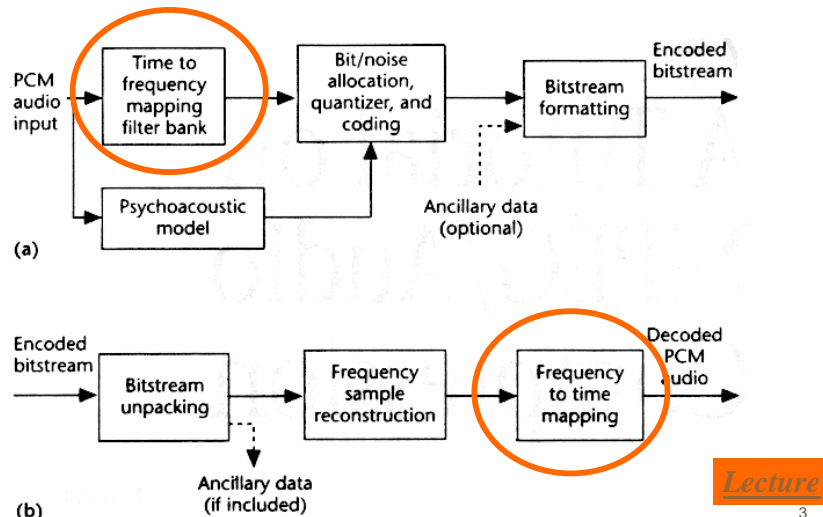


Lecture 8
Sampling & Aliasing
11-Feb-05

Info

- Lab #4
 - **Formal** Lab Report will be worth 150 pts
- **NO Lab on Tuesday, 15-Feb**
 - **Instead, Open lab, 12:30-3:30pm, for help on Lab #4**
 - Lab #5 starts on Wed
- Honor Code !!!
 - **Don't exchange anything written or any electronic files**
- Quiz #1: Resolve grading issues
 - NO LATER than Monday, Feb 21st
 - After that, no scores will be changed
 - Check WebCT for graders of individual problems
 - #1 and #2: Casinovi
 - #3: Juang
 - #4: McClellan

MP-3 Block Diagram



READING ASSIGNMENTS

- This Lecture:
 - **Chap 4, Sections 4-1 and 4-2**
 - Replaces Ch 4 in DSP First, pp. 83-94
- Other Reading:
 - Recitation: Strobe Demo (Sect 4-3)
 - Next Lecture: **Chap. 4 Sects. 4-4 and 4-5**

LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
 - **Sampling Theorem**
 - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, $x[n]$
 - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

↑
ALIASING

SYSTEMS Process Signals



- PROCESSING GOALS:
 - Change $x(t)$ into $y(t)$
 - For example, more BASS
 - Improve $x(t)$, e.g., image deblurring
 - Extract Information from $x(t)$

System IMPLEMENTATION

- ANALOG/ELECTRONIC:
 - Circuits: resistors, capacitors, op-amps

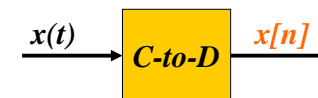


- DIGITAL/MICROPROCESSOR
 - Convert $x(t)$ to **numbers** stored in memory



SAMPLING $x(t)$

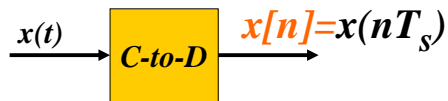
- SAMPLING PROCESS
 - Convert $x(t)$ to **numbers** $x[n]$
 - “n” is an integer; $x[n]$ is a sequence of values
 - Think of “n” as the storage address in memory
- UNIFORM SAMPLING at $t = nT_s$
 - IDEAL: $x[n] = x(nT_s)$



SAMPLING RATE, f_s

SAMPLING RATE (f_s)

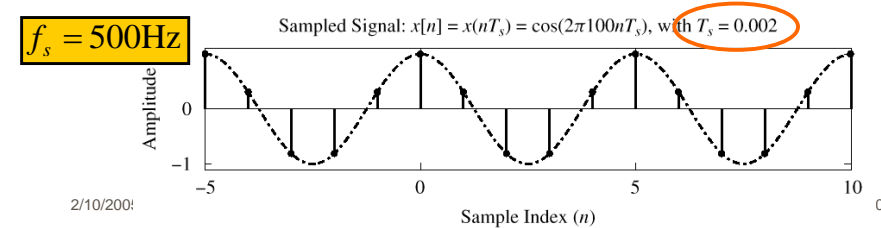
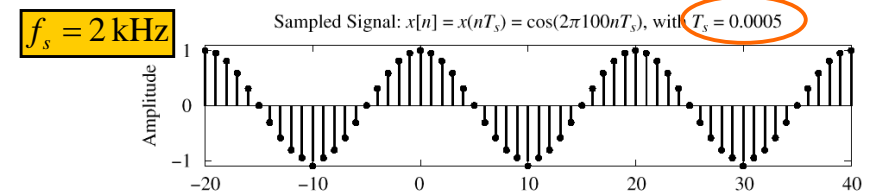
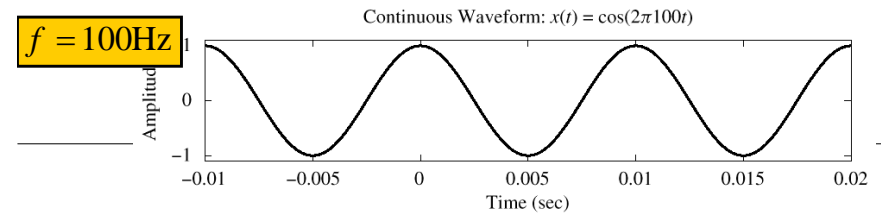
- $f_s = 1/T_s$
 - NUMBER of SAMPLES PER SECOND
- $T_s = 125 \text{ microsec} \rightarrow f_s = 8000 \text{ samples/sec}$
 - UNITS ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING at $t = nT_s = n/f_s$
 - IDEAL: $x[n] = x(nT_s) = x(n/f_s)$



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SAMPLING THEOREM

HOW OFTEN ?

- DEPENDS on FREQUENCY of SINUSOID
- ANSWERED by SHANNON/NYQUIST Theorem
- ALSO DEPENDS on **RECONSTRUCTION**

Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{\max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

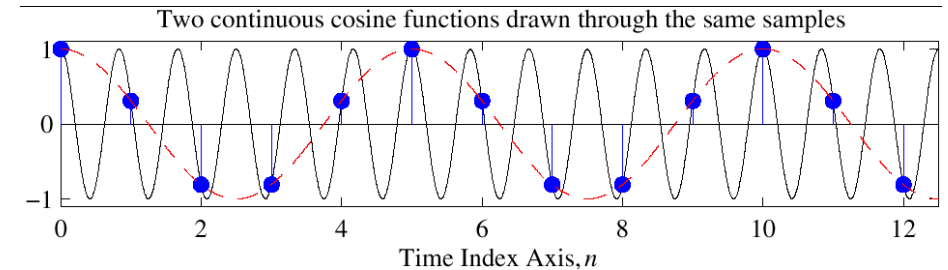
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Reconstruction? Which One?

Given the samples, draw a sinusoid through the values



$$x[n] = \cos(0.4\pi n)$$

When n is an integer
 $\cos(0.4\pi n) = \cos(2.4\pi n)$

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STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
 - A list of numbers stored in memory
- EXAMPLE: audio CD
- CD rate is 44,100 samples per second
 - 16-bit samples
 - Stereo uses 2 channels
- Number of bytes for 1 minute is
 - $2 \times (16/8) \times 60 \times 44100 = 10.584$ Mbytes

DISCRETE-TIME SINUSOID

- Change $x(t)$ into $x[n]$ **DERIVATION**

$$x(t) = A \cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A \cos(\omega nT_s + \varphi)$$

$$x[n] = A \cos((\omega T_s)n + \varphi)$$

$$x[n] = A \cos(\hat{\omega}n + \varphi)$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

DEFINE DIGITAL FREQUENCY

DIGITAL FREQUENCY $\hat{\omega}$

- $\hat{\omega}$ VARIES from **0** to **2π** , as f varies from 0 to the sampling frequency
- UNITS are radians, **not** rad/sec
 - DIGITAL FREQUENCY is NORMALIZED

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

SPECTRUM (DIGITAL)

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f_s = 1 \text{ kHz}$$

$$\frac{1}{2} X^*$$

-0.2π

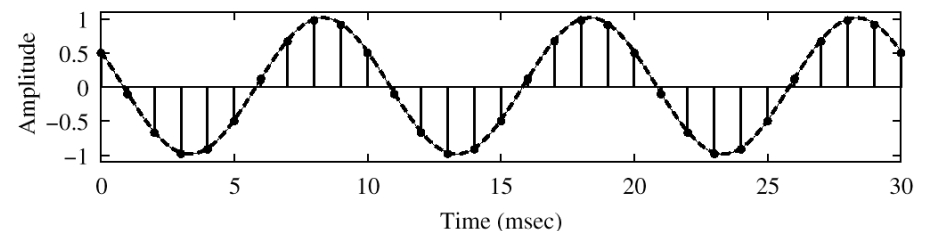
$$\frac{1}{2} X$$

$2\pi(0.1)$

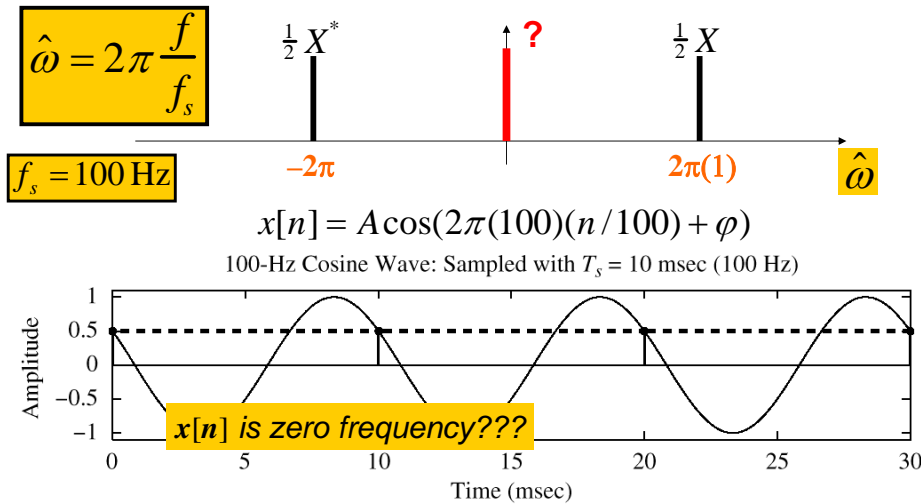
$$\hat{\omega}$$

$$x[n] = A \cos(2\pi(100)(n/1000) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 1$ msec (1000 Hz)



SPECTRUM (DIGITAL) ???



The REST of the STORY

- Spectrum of $x[n]$ has more than one line for each complex exponential
 - Called **ALIASING**
 - **MANY SPECTRAL LINES**
- SPECTRUM is PERIODIC with period = 2π
 - Because

$A \cos(\hat{\omega}n + \varphi) = A \cos((\hat{\omega} + 2\pi)n + \varphi)$

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ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$
 - $x_1(t) = \cos(400\pi t)$ sampled at $f_s = 1000$ Hz
 - $x_1[n] = \cos(400\pi \frac{n}{1000}) = \cos(0.4\pi n)$
 - $x_2(t) = \cos(2400\pi t)$ sampled at $f_s = 1000$ Hz
 - $x_2[n] = \cos(2400\pi \frac{n}{1000}) = \cos(2.4\pi n)$
 - $x_2[n] = \cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n) = \cos(0.4\pi n)$
 - $\Rightarrow x_2[n] = x_1[n]$
- $2400\pi - 400\pi = 2\pi(1000)$

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ALIASING DERIVATION-2

- Other Frequencies give the same $\hat{\omega}$
- If $x(t) = A \cos(2\pi(f + lf_s)t + \varphi)$
- $t \leftarrow \frac{n}{f_s}$
- and we want: $x[n] = A \cos(\hat{\omega}n + \varphi)$

then: $\hat{\omega} = \frac{2\pi(f + lf_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi lf_s}{f_s}$

$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$

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ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ or $-f_s$ to the FREQ of $x(t)$ gives exactly the same $x[n]$
 - The samples, $x[n] = x(n/f_s)$ are EXACTLY THE SAME VALUES
- GIVEN $x[n]$, WE CAN'T DISTINGUISH f_0 FROM $(f_0 + f_s)$ or $(f_0 + 2f_s)$

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NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

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SPECTRUM for $x[n]$

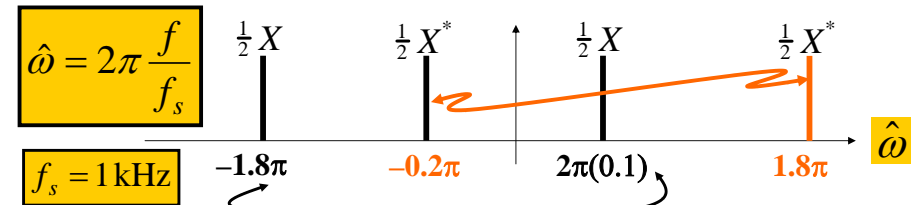
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE **ALL** SPECTRUM LINES
 - ALIASES
 - ADD MULTIPLES of 2π
 - SUBTRACT MULTIPLES of 2π
 - FOLDED ALIASES
 - (to be discussed later)
 - ALIASES of NEGATIVE FREQS

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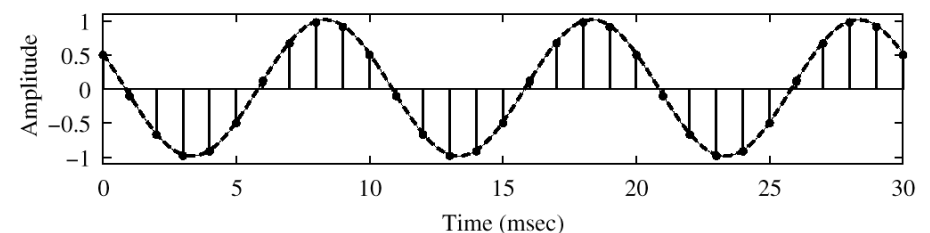
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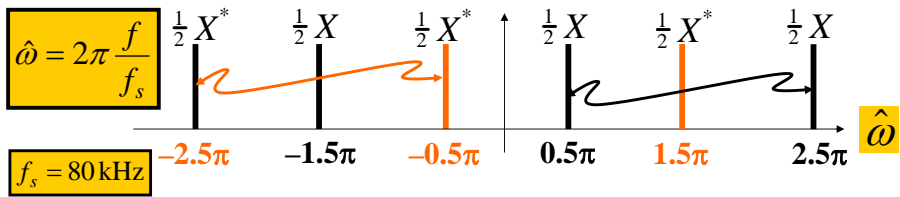
SPECTRUM (MORE LINES)



$x[n] = A \cos(2\pi(100)(n/1000) + \varphi)$
 100-Hz Cosine Wave: Sampled with $T_s = 1$ msec (1000 Hz)

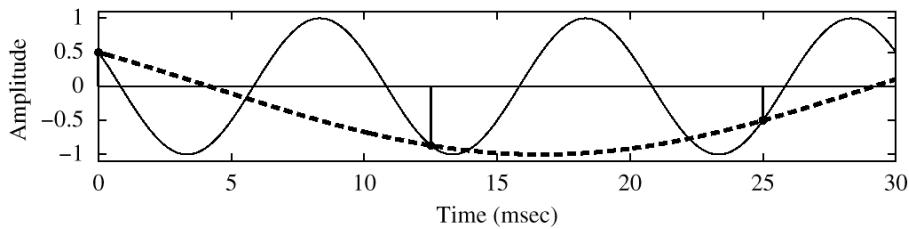


SPECTRUM (ALIASING CASE)

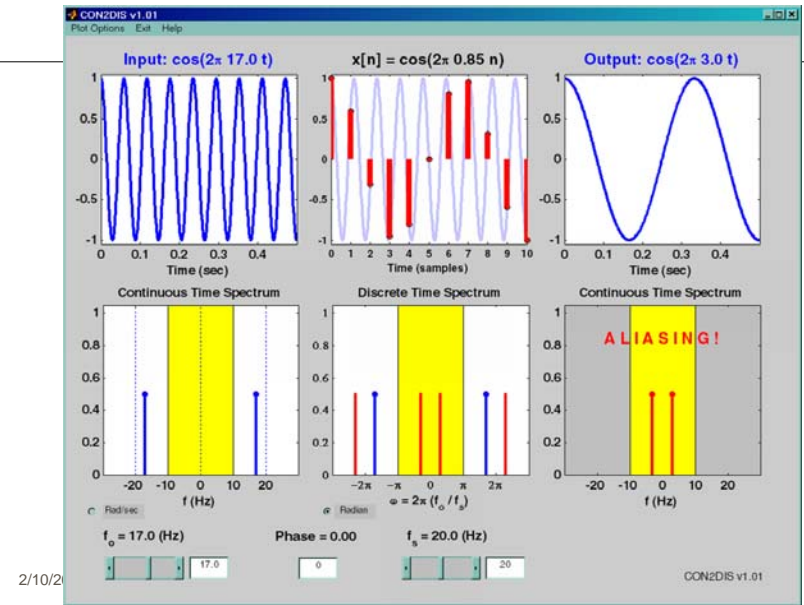


$$x[n] = A \cos(2\pi(100)(n/80) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 12.5 \text{ msec}$ (80 Hz)



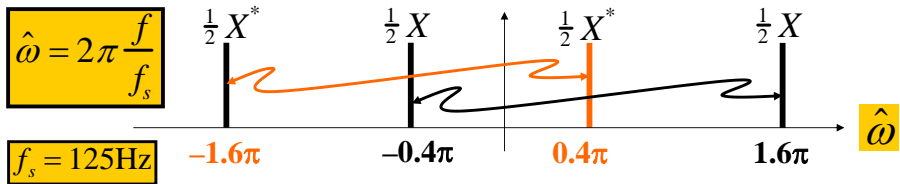
SAMPLING GUI (con2dis)



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SPECTRUM (FOLDING CASE)



$$x[n] = A \cos(2\pi(100)(n/125) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 8 \text{ msec}$ (125 Hz)

