

ECE-2025

Spring-2005

Lecture 11

Linearity, Time-Invariance
and Convolution

21-Feb-05

Info: Web-CT, Lab, HW

- Listen to speech synthesis for Lab #5
- Read the Pre-Labs
 - Run MATLAB **Debugger** for Lab #6
- Lab #6: Image Processing
 - Sampling & Zooming
 - Interpolation
 - Needed in Digital Cameras
- Quiz #1 grading issues?
 - Resolve them no later than today.

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DEBUGGING

- “Any Fool” can write code
- Debugging is the interesting part
 - It takes brain power !!!
- HOWEVER,
 - Assume the **stupid** mistake is the problem

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Lecture

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 5, Sections 5-5 and 5-6
 - Section 5-4 will be covered, but not “in depth”
- Other Reading:
 - Recitation: Ch. 5, Sects 5-6, 5-7 & 5-8
 - **CONVOLUTION**
 - Next Lecture: start Chapter 6

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LECTURE OBJECTIVES

- **GENERAL** PROPERTIES of FILTERS
 - LINEARITY
 - TIME-INVARIANCE
 - ==> **CONVOLUTION**
- BLOCK DIAGRAM REPRESENTATION
 - Components for **Hardware**
 - **Connect** Simple Filters Together to Build More Complicated Systems

LTI SYSTEMS

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OVERVIEW

- IMPULSE RESPONSE, $h[n]$
 - FIR case: same as $\{b_k\}$
- CONVOLUTION
 - GENERAL: $y[n] = h[n] * x[n]$
 - GENERAL CLASS of SYSTEMS
 - LINEAR and TIME-INVARIANT
- **ALL LTI** systems have $h[n]$ & use convolution

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DIGITAL FILTERING



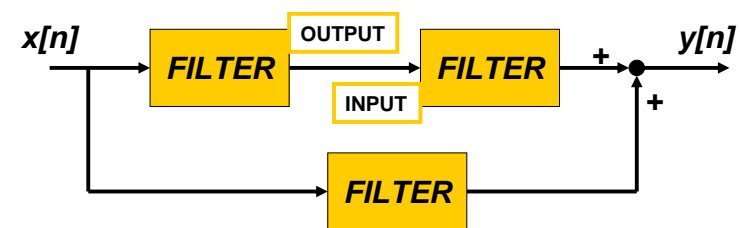
- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIME SIGNALS
 - FUNCTIONS of n , the "time index"
 - INPUT $x[n]$
 - OUTPUT $y[n]$

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BUILDING BLOCKS



- BUILD UP COMPLICATED FILTERS
 - FROM SIMPLE **MODULES**
 - Ex: FILTER **MODULE** MIGHT BE 3-pt FIR

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GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

- DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- For example, $b_k = \{3, -1, 2, 1\}$

$$\begin{aligned} y[n] &= \sum_{k=0}^3 b_k x[n-k] \\ &= 3x[n] - x[n-1] + 2x[n-2] + x[n-3] \end{aligned}$$

MATLAB for FIR FILTER

- $yy = \text{conv}(bb, xx)$

- VECTOR **bb** contains Filter Coefficients

- DSP-First: $yy = \text{firfilt}(bb, xx)$

- FILTER COEFFICIENTS $\{b_k\}$

conv2()
for images

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

SPECIAL INPUT SIGNALS

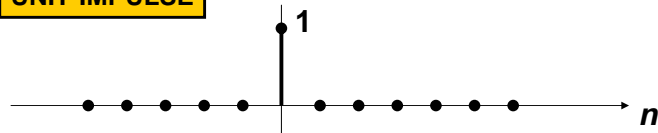
- $x[n] = \text{SINUSOID}$

FREQUENCY RESPONSE

- $x[n]$ has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



FIR IMPULSE RESPONSE

- Convolution = Filter Definition

- Filter Coeffs = Impulse Response

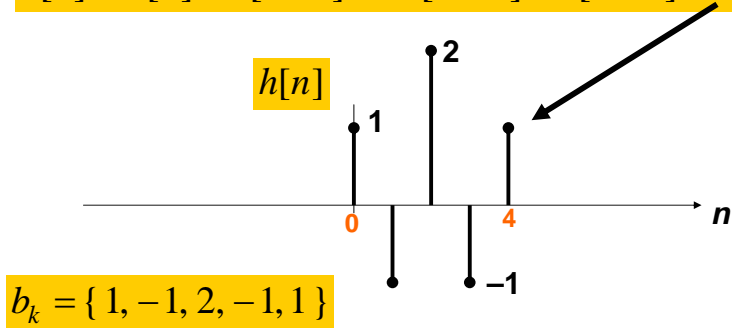
n	$n < 0$	0	1	2	3	...	M	$M+1$	$n > M+1$
$x[n] = \delta[n]$	0	1	0	0	0	0	0	0	0
$y[n] = h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$h[n] = \sum_{k=0}^M b_k \delta[n-k]$$

MATH FORMULA for $h[n]$

- Use **SHIFTED** IMPULSES to write $h[n]$

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



$$b_k = \{1, -1, 2, -1, 1\}$$

LTI: Convolution Sum

- Output = Convolution of $x[n]$ & $h[n]$**

- NOTATION: $y[n] = h[n] * x[n]$

- Here is the FIR case:

$$y[n] = \sum_{k=0}^M h[k]x[n-k]$$

FINITE LIMITS

Same as b_k

FINITE LIMITS

CONVOLUTION Example

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$

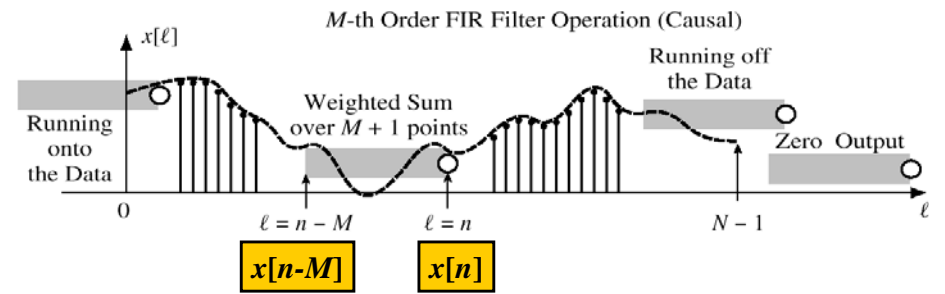
$$x[n] = u[n]$$

n	-1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	-1	2	-1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	-1	-1	-1	-1	-1	-1	-1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	-1	-1	-1	-1	-1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

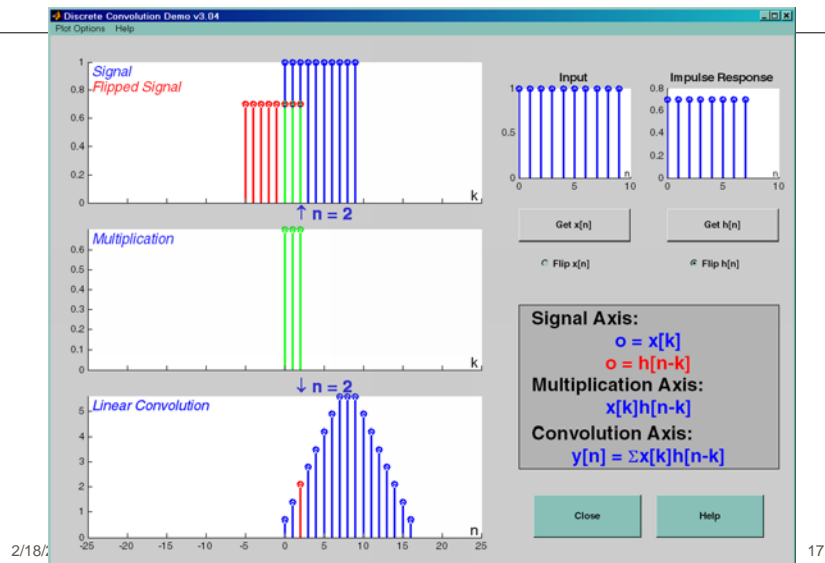
GENERAL FIR FILTER

- SLIDE** a Length- L WINDOW over $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$



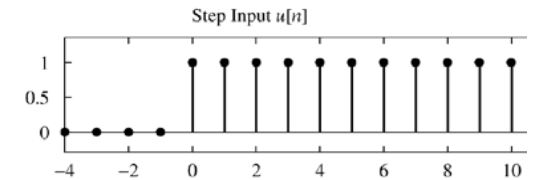
DCONVDEMO: MATLAB GUI



POP QUIZ

- FIR Filter is “FIRST DIFFERENCE”
 - $y[n] = x[n] - x[n-1]$
- INPUT is “UNIT STEP”

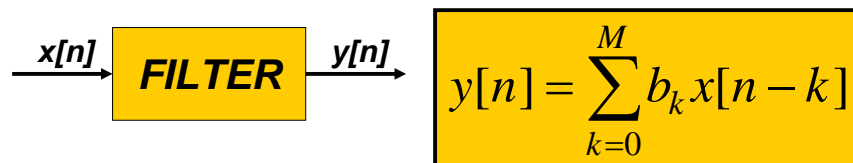
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



- Find $y[n]$

$$y[n] = u[n] - u[n-1] = \delta[n]$$

HARDWARE STRUCTURES

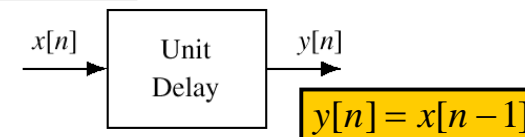
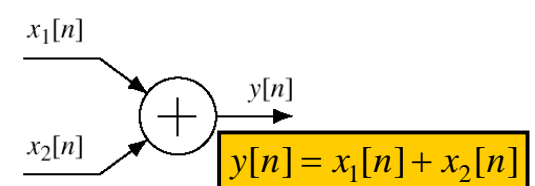
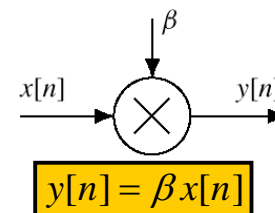


- INTERNAL STRUCTURE of “FILTER”
 - WHAT COMPONENTS ARE NEEDED?
 - HOW DO WE “HOOK” THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

HARDWARE ATOMS

- Add, Multiply & Store

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$



FIR STRUCTURE

- Direct Form

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

SIGNAL FLOW GRAPH

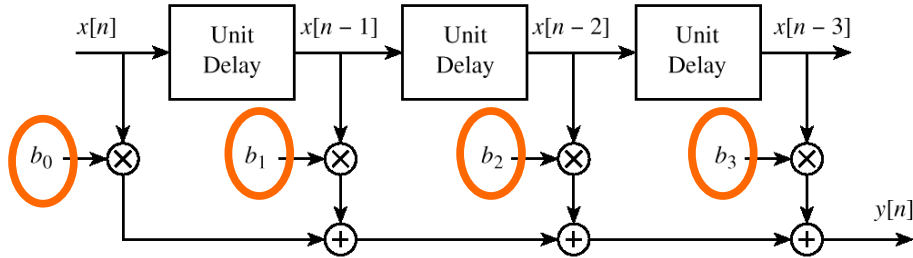
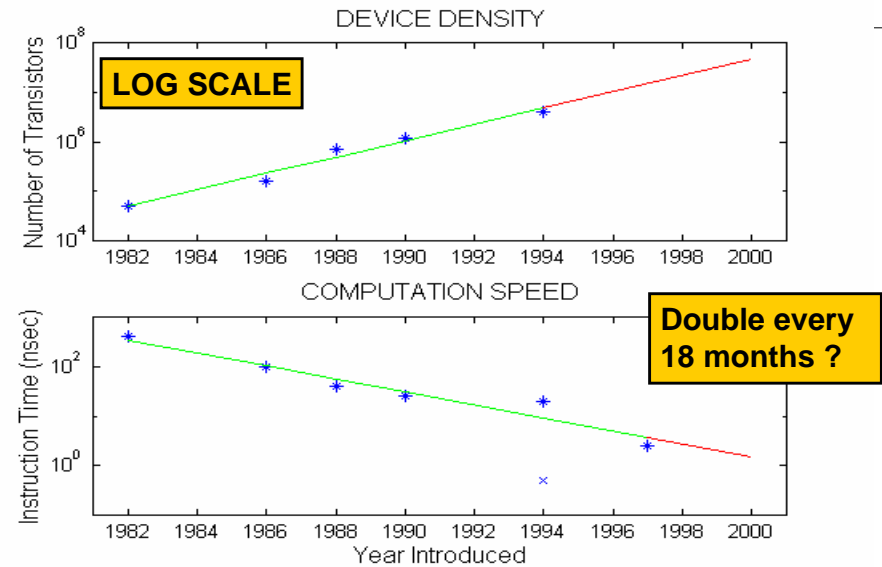


Figure 5.13 Block-diagram structure for the M th order FIR filter.

Moore's Law for TI DSPs



SYSTEM PROPERTIES



- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
 - “No output prior to input”

TIME-INVARIANCE

- IDEA:
 - “Time-Shifting the input will cause the **same** time-shift in the output”
- EQUIVALENTLY,
 - We can prove that
 - The time origin ($n=0$) is picked arbitrary

TESTING Time-Invariance

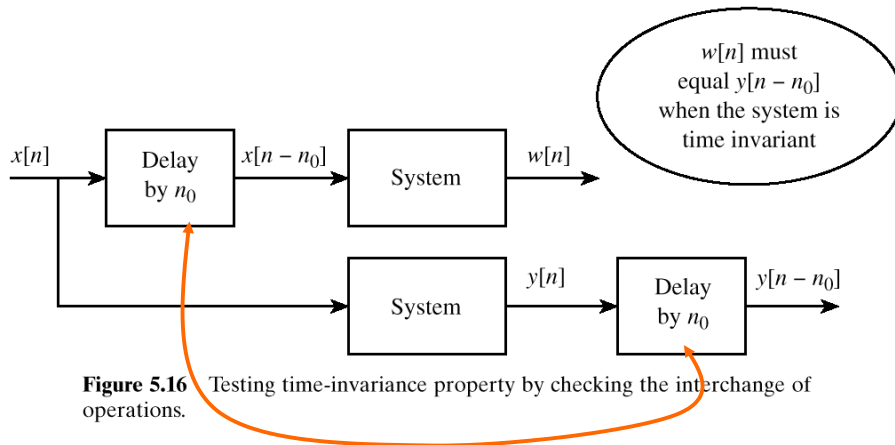


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
 - “Doubling $x[n]$ will double $y[n]$ ”
- SUPERPOSITION:
 - “Adding two inputs gives an output that is the sum of the individual outputs”

TESTING LINEARITY

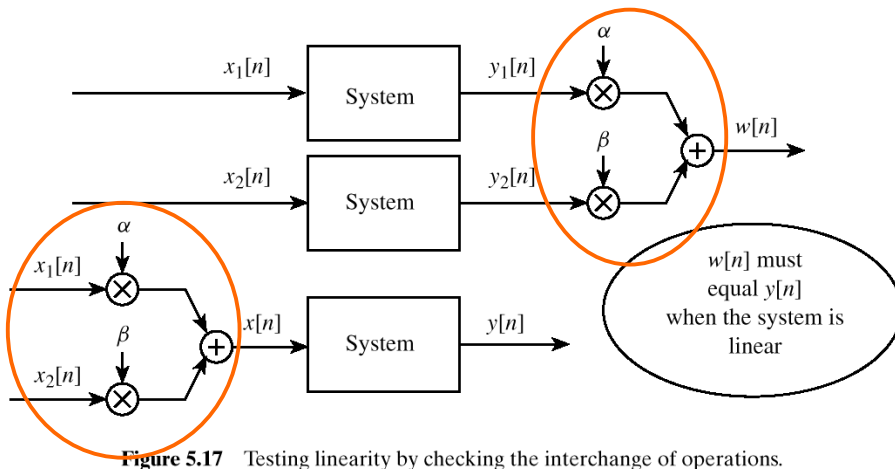


Figure 5.17 Testing linearity by checking the interchange of operations.

LTI SYSTEMS

- LTI: **L**inear & **T**ime-**I**nvariant
- COMPLETELY CHARACTERIZED by:
 - **IMPULSE RESPONSE** $h[n]$
 - **CONVOLUTION**: $y[n] = x[n] * h[n]$
 - The “rule” defining the system can ALWAYS be re-written as convolution
- FIR Example: $h[n]$ is same as b_k

POP QUIZ

- FIR Filter is “FIRST DIFFERENCE”

- $y[n] = x[n] - x[n - 1]$

- Write output as a convolution

- Need impulse response

$$h[n] = \delta[n] - \delta[n - 1]$$

- Then, another way to compute the output:

$$y[n] = (\delta[n] - \delta[n - 1]) * x[n]$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?

- NO, LTI SYSTEMS can be rearranged !!!

- WHAT ARE THE FILTER COEFFS? $\{b_k\}$

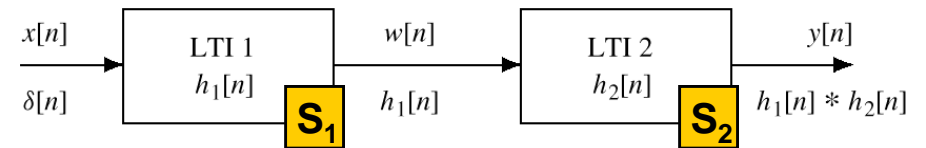


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- Find “overall” $h[n]$ for a cascade ?

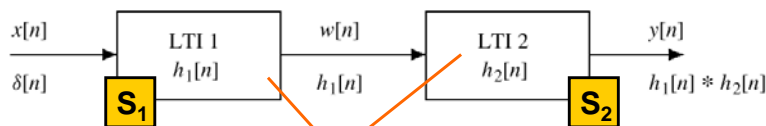


Figure 5.19 A Cascade of Two LTI Systems.

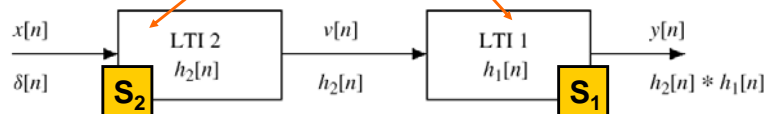
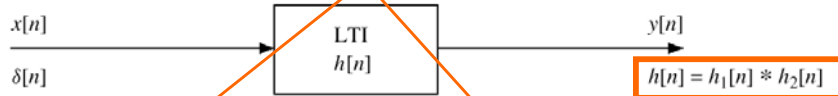


Figure 5.20 Switching the order of cascaded LTI systems.