

Lecture 12

Frequency Response of FIR
25-Feb-05

Info: Web-CT, Lab, HW

- Lab #6: Image Processing
- Prob Set #7 due **next week**
- **Quiz #2 on 4-March (Friday)**
 - Coverage: HW #4, #5, #6, and #7
 - Chapters 3(FourierSeries), 4(Sampling), and 5(FIR Filters)

Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 6, Sections 6-1, 6-2, 6-3, 6-4, & 6-5
- Other Reading:
 - Recitation: Chapter 6
 - FREQUENCY RESPONSE EXAMPLES
 - Next Lecture: Chap. 6, Sects. 6-6, 6-7 & 6-8

LECTURE OBJECTIVES

- **SINUSOIDAL INPUT SIGNAL**
 - DETERMINE the FIR FILTER OUTPUT

- **FREQUENCY RESPONSE** of FIR

- PLOTTING vs. Frequency
- MAGNITUDE vs. Freq
- PHASE vs. Freq

$$H(e^{j\hat{\omega}}) = \left| H(e^{j\hat{\omega}}) \right| e^{j\angle H(e^{j\hat{\omega}})}$$

DOMAINS: Time & Frequency

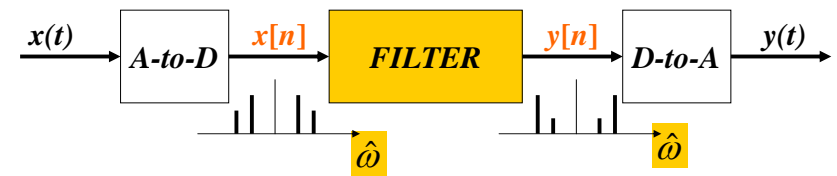
- **Time-Domain: “n” = time**
 - $x[n]$ discrete-time signal
 - $x(t)$ continuous-time signal
- **Frequency Domain (sum of sinusoids)**
 - Spectrum vs. f (Hz)
 - ANALOG vs. DIGITAL
 - Spectrum vs. ω -hat
- Move back and forth **QUICKLY**

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DIGITAL “FILTERING”



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
 - INPUT $x[n]$ = SUM of SINUSOIDS
 - Then, OUTPUT $y[n]$ = SUM of SINUSOIDS

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FILTERING EXAMPLE

7-point AVERAGER

- Removes cosine
 - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^6 \left(\frac{1}{7}\right)x[n-k]$$

3-point AVERAGER

- Changes A slightly

$$y_3[n] = \sum_{k=0}^2 \left(\frac{1}{3}\right)x[n-k]$$

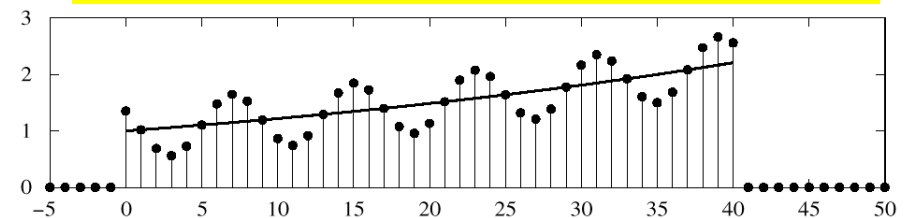
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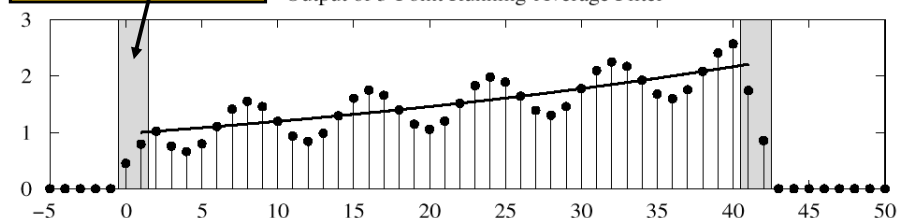
3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



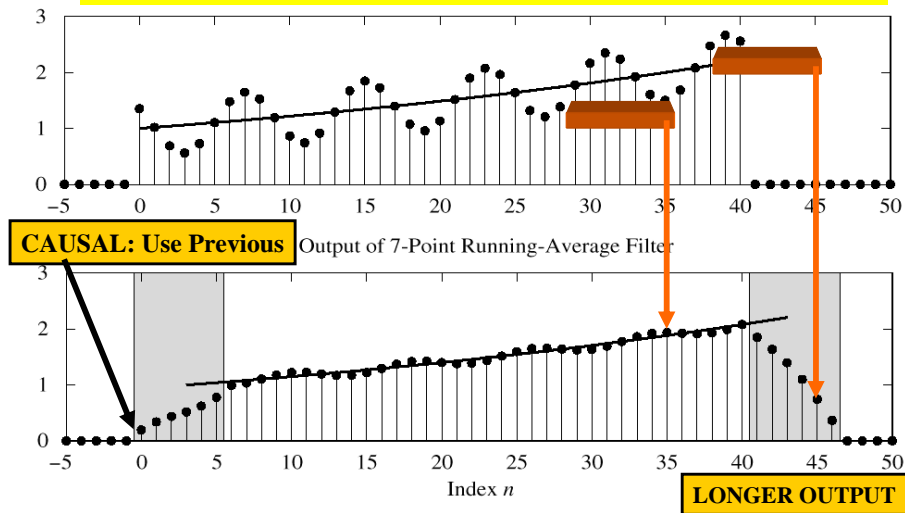
USE PAST VALUES

Output of 3-Point Running-Average Filter



7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



SINUSOIDAL RESPONSE

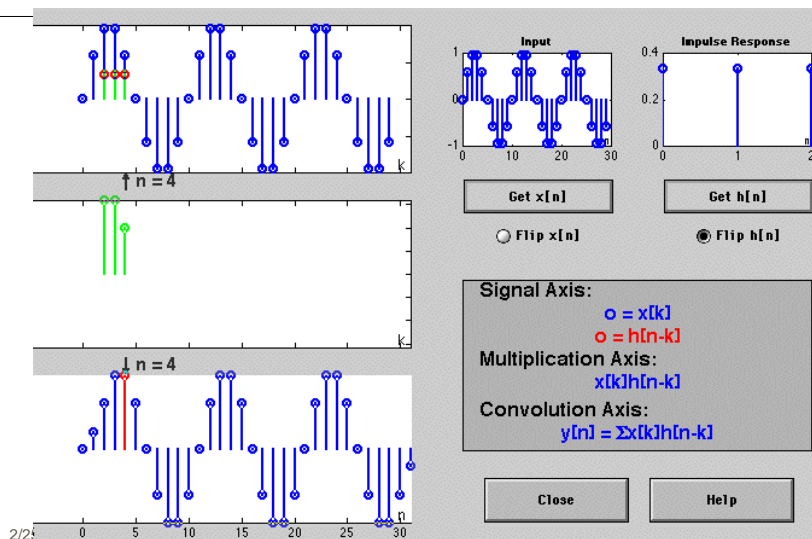
- INPUT: $x[n] = \text{SINUSOID}$
- OUTPUT: $y[n]$ will also be a **SINUSOID**
 - Different Amplitude and Phase
 - SAME** Frequency
- AMPLITUDE & PHASE CHANGE
 - Called the **FREQUENCY RESPONSE**

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DCONVDEMO: MATLAB GUI



COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi} e^{j\hat{\omega}n} \quad -\infty < n < \infty$$

$x[n]$ is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

FIR DIFFERENCE EQUATION

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COMPLEX EXP OUTPUT

- Use the FIR “Difference Equation”

$$\begin{aligned}
 y[n] &= \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M b_k A e^{j\phi} e^{j\hat{\omega}(n-k)} \\
 &= \left(\sum_{k=0}^M b_k e^{j\hat{\omega}(-k)} \right) A e^{j\phi} e^{j\hat{\omega}n} \\
 &= H(\hat{\omega}) A e^{j\phi} e^{j\hat{\omega}n}
 \end{aligned}$$

FREQUENCY RESPONSE

- At each frequency, we can **DEFINE**

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} \quad \leftarrow \text{FREQUENCY RESPONSE}$$

- Complex-valued formula
 - Has **MAGNITUDE** vs. frequency
 - And **PHASE** vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$

EXAMPLE 6.1

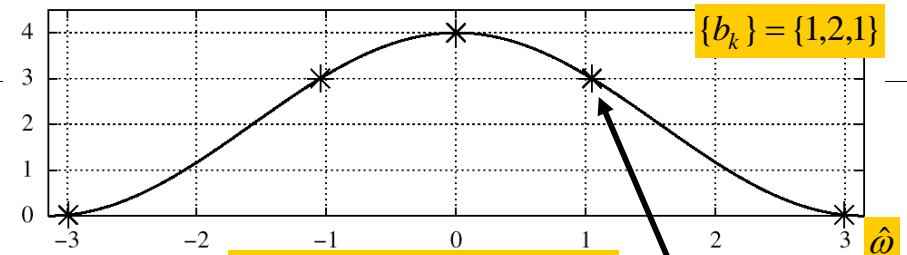
$$\{b_k\} = \{1, 2, 1\}$$

$$\begin{aligned}
 H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\
 &= e^{-j\hat{\omega}} (e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\
 &= e^{-j\hat{\omega}} (2 + 2\cos\hat{\omega})
 \end{aligned}$$

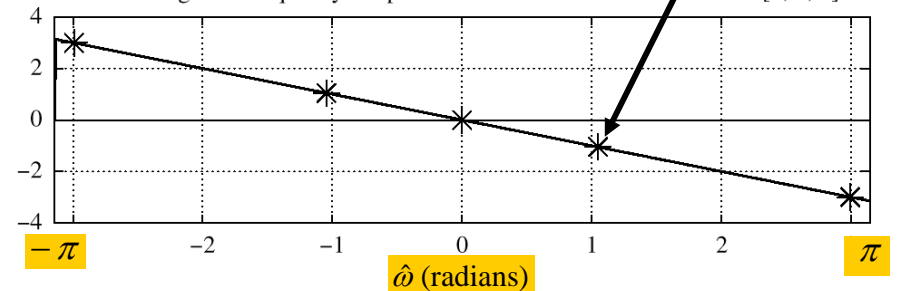
EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$
 Magnitude is $|H(e^{j\hat{\omega}})| = (2 + 2\cos\hat{\omega})$
 and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

Magnitude of Frequency Response of FIR Filter with Coefficients [1, 2, 1]

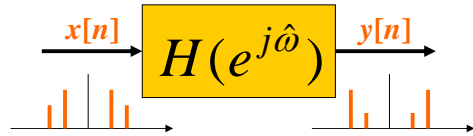


Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known
and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos \hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

$$y[n] = (3e^{-j\pi/3}) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

EX: COSINE INPUT

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\frac{\pi}{3}n + \frac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use
Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

MATLAB: FREQUENCY RESPONSE

- **HH = freqz(bb,1,ww)**
 - VECTOR **bb** contains Filter Coefficients
 - DSP-First: **HH = freekz(bb,1,ww)**
- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
 - **FREQUENCY RESPONSE**, or
 - IMPULSE RESPONSE $h[n]$
- **Sinusoid IN -----> Sinusoid OUT**
 - **At the SAME Frequency**

Time & Frequency Relation

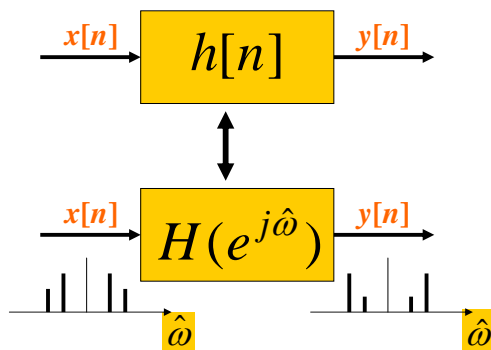
- Get Frequency Response from $h[n]$
 - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

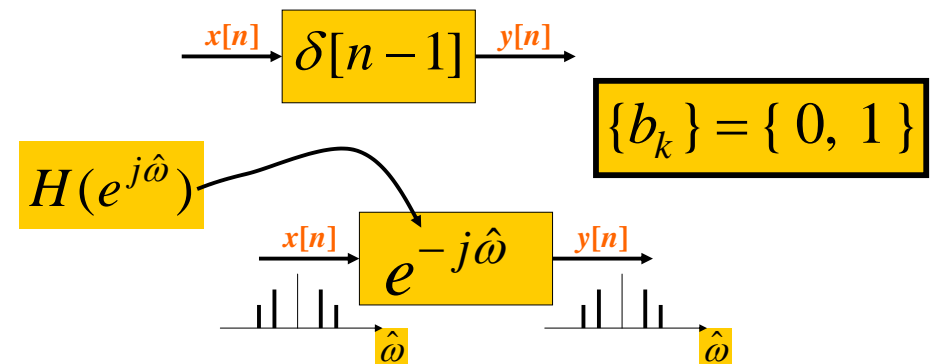
BLOCK DIAGRAMS

- Equivalent Representations



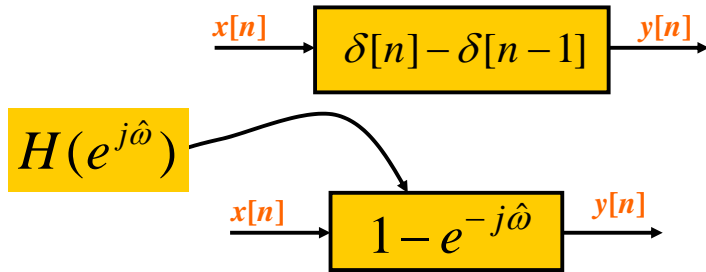
UNIT-DELAY SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n - 1]$

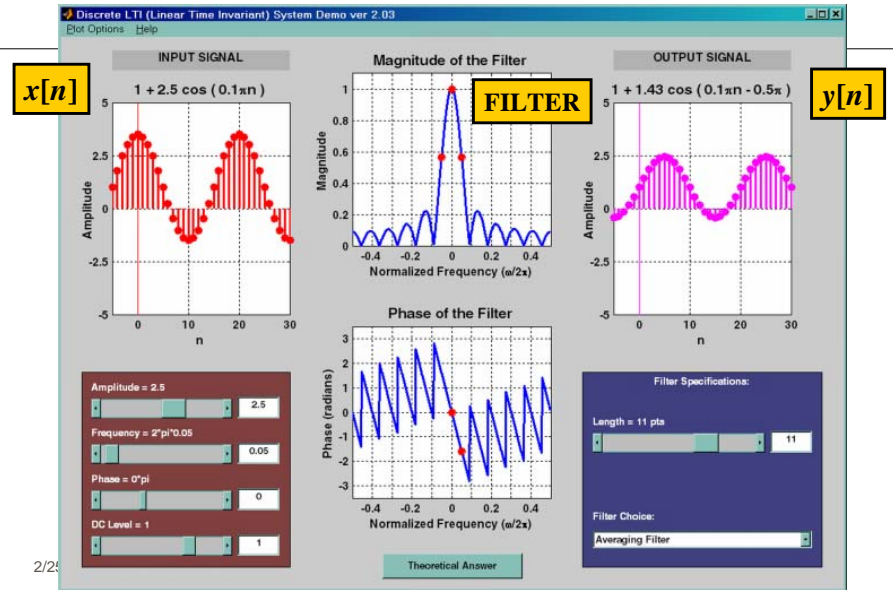


FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n - 1]$



DLTI Demo with Sinusoids



CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
 - NO, LTI SYSTEMS can be rearranged !!!
 - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
 - WHAT is the overall FREQUENCY RESPONSE ?

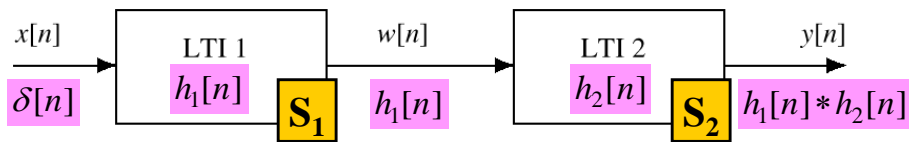


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- MULTIPLY** the Frequency Responses

