ECE-2025 Spring-2005

Lecture 25
Review: Digital Filtering of Continuous-Time Signals
29-Apr-05

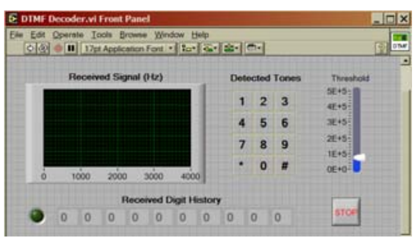
FINAL EXAM

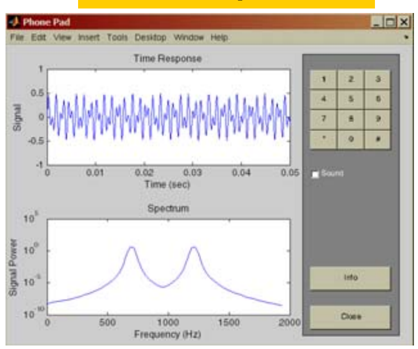
- **Final Exam: Monday @ 2:50pm, 2-May**
    - Review Session on Sunday, 1-May (**6:00pm**)
- FORMULA PAGE: **ONE** page **HAND-WRITTEN**
    - **Tables 11.2 and 11.3 will be supplied with the exam**
    - **Z-transform tables also**
- COVERAGE / EMPHASIS?
    - **Fourier Transform**
        - Sampling, Filtering & Spectrum
    - Digital Filters: IIR & FIR & H(z)
    - Sampling & Aliasing
    - Problems from Quizzes #3 and #4.
    - Concepts from Labs #11 and #12
    - **Homework** & Old Quizzes

DTMF Hardware Demo

National Instruments LabView + Speedy33 DTMF decoder

MATLAB: phone.m





Senior Design Course(s)

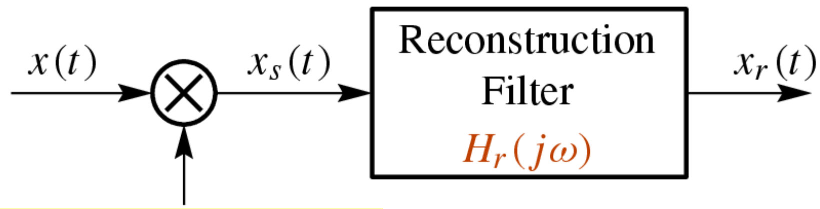
- Graduation requires
    - ECE-4000 *Project Engineering*
    - ECE-4006 *Design Project*
        - Can specialize in different areas, e.g., DSP
        - Real-Time DSP Projects
- DSP concentration
    - ECE-3075 *Random Signals*
    - ECE-4270 *DSP*
    - ECE-4271 *Applications of DSP*
    - ECE-4273 *ASICs for DSP*

Lecture

LECTURE OBJECTIVES

- Sampling Theorem Revisited
  - GENERAL: in the FREQUENCY DOMAIN
  - Fourier transform of sampled signal
  - Reconstruction from samples
- **Effective Frequency Response**
- Important FT properties
  - Convolution ←→ multiplication
  - Frequency shifting

Sampling: Freq. Domain



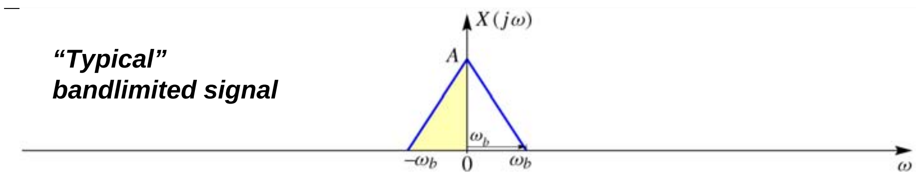
$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

EXPECT FREQUENCY SHIFTING !!!

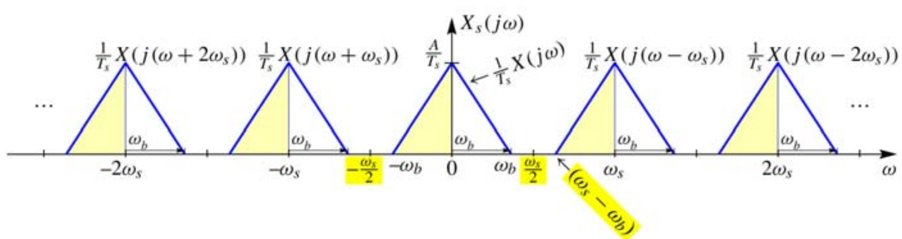
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

Frequency-Domain Representation of Sampling

"Typical" bandlimited signal

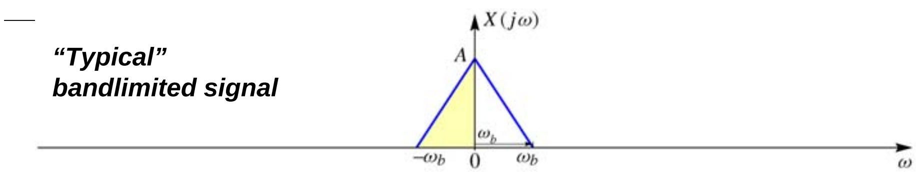


$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

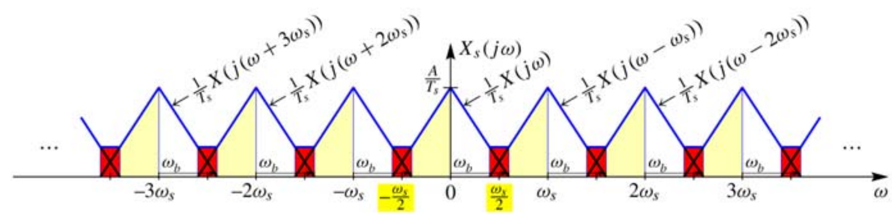


Aliasing Distortion

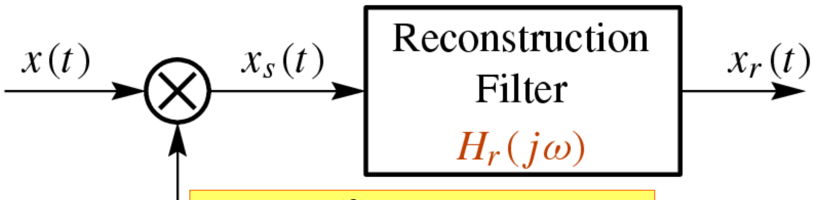
"Typical" bandlimited signal



- If $\omega_s < 2\omega_b$, the copies of $X(j\omega)$ overlap, and we have aliasing distortion.



Reconstruction of $x(t)$

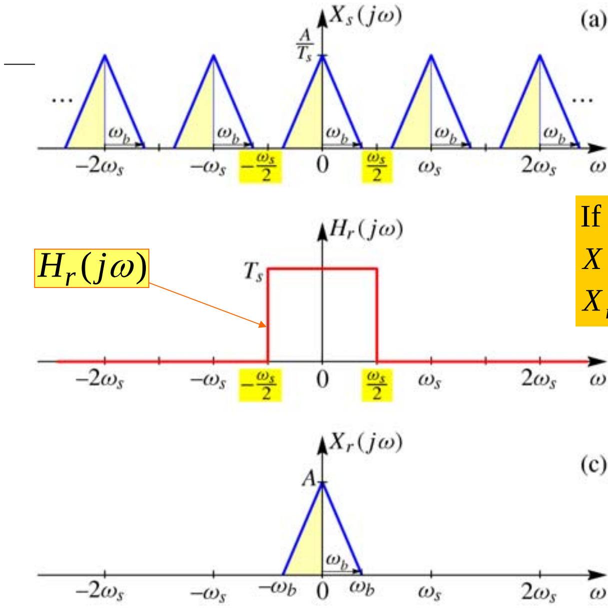


$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega)X_s(j\omega)$$

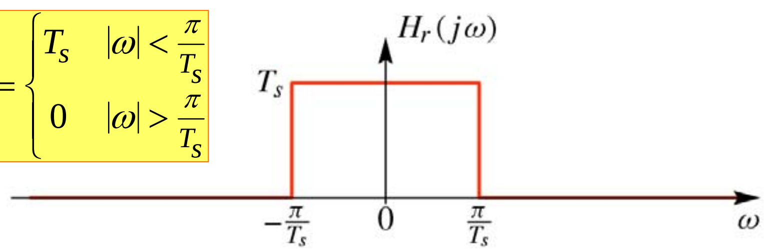
Reconstruction: Frequency-Domain



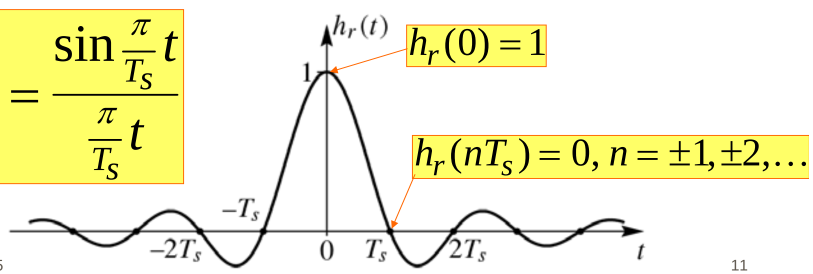
If $\omega_s > 2\omega_b$, the copies of $X(j\omega)$ do not overlap, so $X_r(j\omega) = H_r(j\omega)X_s(j\omega)$

Ideal Reconstruction Filter

$$H_r(j\omega) = \begin{cases} T_s & |\omega| < \frac{\pi}{T_s} \\ 0 & |\omega| > \frac{\pi}{T_s} \end{cases}$$



$$h_r(t) = \frac{\sin\frac{\pi}{T_s}t}{\frac{\pi}{T_s}t}$$



Signal Reconstruction

$$x_r(t) = h_r(t) * x_s(t) = h_r(t) * \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h_r(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

Ideal bandlimited interpolation formula

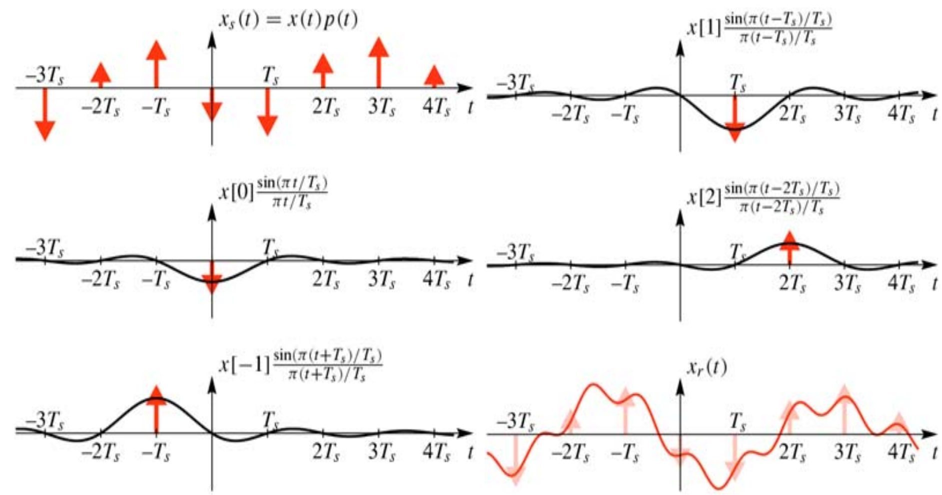
Shannon Sampling Theorem

- **"SINC" Interpolation** is the ideal
  - PERFECT RECONSTRUCTION
  - of BANDLIMITED SIGNALS

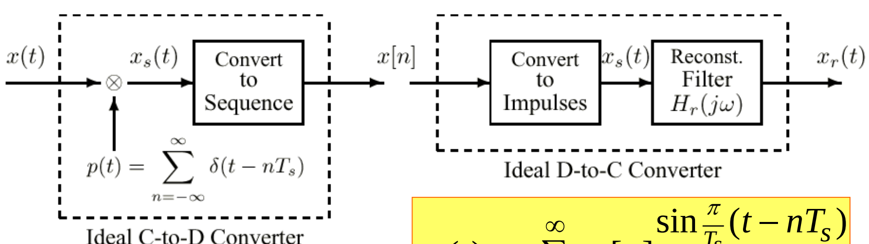
A signal $x(t)$ with bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ can be reconstructed exactly from samples taken with sampling rate $\omega_s = 2\pi/T_s \geq 2\omega_b$ using the following bandlimited interpolation formula:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\left[\frac{\pi}{T_s}(t - nT_s)\right]}{\frac{\pi}{T_s}(t - nT_s)}.$$

Reconstruction in Time-Domain



Ideal C-to-D and D-to-C



$x[n] = x(nT_s)$

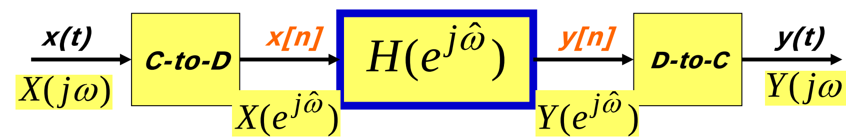
$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

Ideal Sampler ***Ideal bandlimited interpolator***

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega) X_s(j\omega)$$

DT Filtering of CT Signals

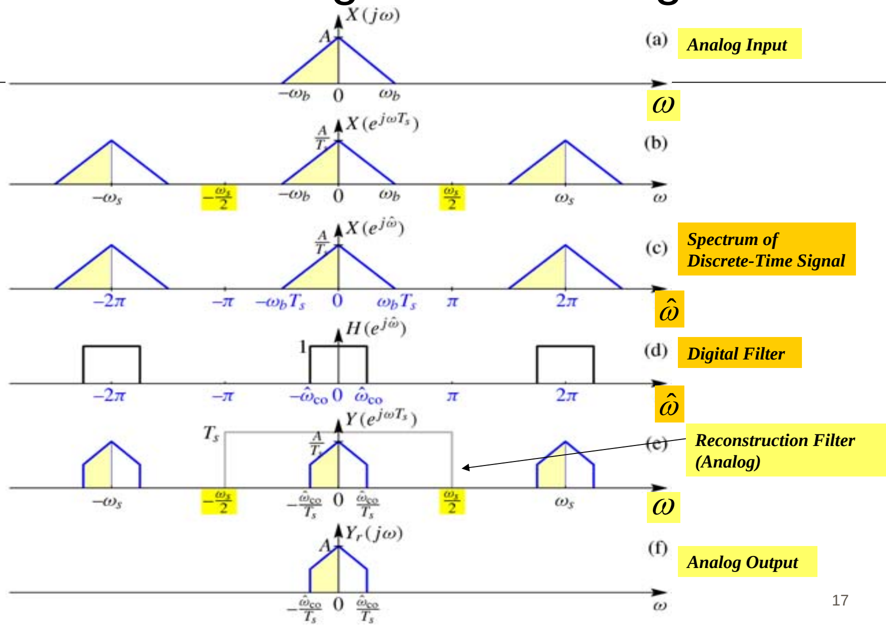


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega) X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ \textbf{UNDEFINED NOT LTI} & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

- So, we can plot:
- Scaled Freq. Axis

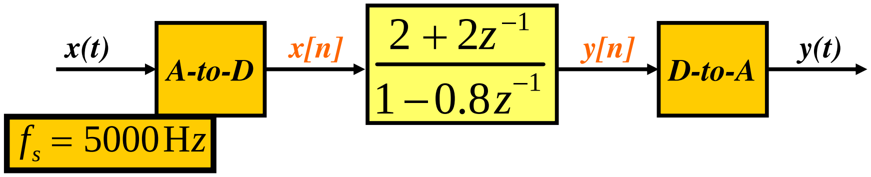
DIGITAL FILTER

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

ANALOG FREQUENCY

Equivalent Systems

- Given:



$$\frac{2+2z^{-1}}{1-0.8z^{-1}}$$

$$f_s = 5000\,\text{Hz}$$

- "Effective Analog System" for $\omega<(2\pi f_s)/2$

x(t) → $H(j\omega) = \dfrac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$ → y(t)

Effective Frequency Response

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$

NO Aliasing, Because 2(1000) < 5000

- The discrete-time frequency response is

$$H(e^{j\hat{\omega}}) = \frac{2+2e^{-j\hat{\omega}}}{1-0.8e^{-j\hat{\omega}}}$$

- Then the Effective Frequency Response is

$$H(j\omega) = \frac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$$

Mathematical Elegance

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Fourier ***(Inverse***

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Fourier Analysis (Forward Transform)

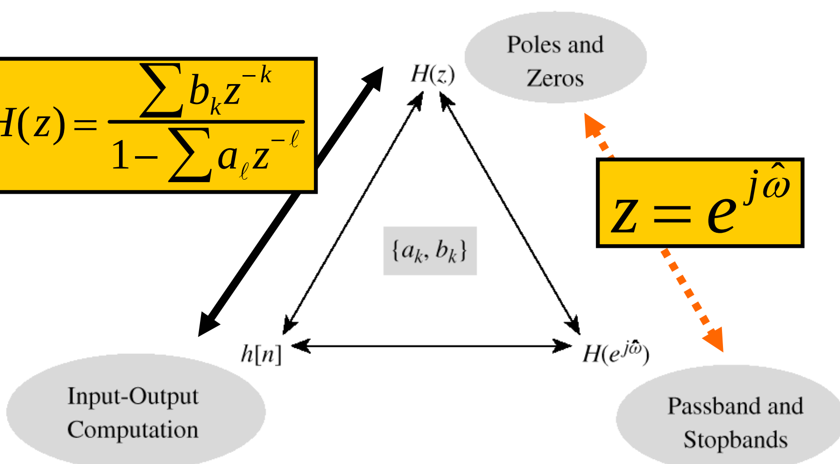
$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$

IMPORTANT CONCEPTS

- ALL Signals have Frequency Content
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- FILTERS alter the Frequency Content
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- 3 Domains for Analysis

THREE DOMAINS

$$H(z) = \frac{\sum b_k z^{-k}}{1 - \sum a_\ell z^{-\ell}}$$



$$z = e^{j\hat{\omega}}$$

Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k,\ b_k\}$ play a central role.

THE FUTURE

- Circuits & Laplace Transforms