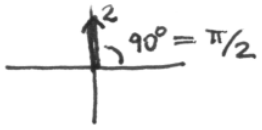
ECE-2025
Spring-2005
Solutions HW #1

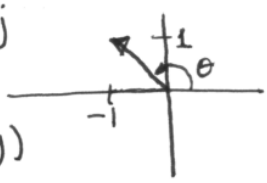
PROBLEM 1.1*:

(a) $z = j2$



$z = 2e^{j\pi/2} = 2\angle 90°$

(b) $z = -1 + j$



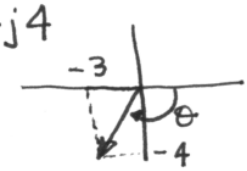
$(-1, 1) = (x, y)$

$\theta = 135° = 3\pi/4$ radians

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$z = \sqrt{2}\, e^{j3\pi/4} = \sqrt{2}\, e^{j2.356}$

(c) $z = -3 - j4$



$r = \sqrt{3^2+4^2} = 5$ $\quad 5\angle -126.87°$

$\theta = \text{Tan}^{-1}\left(\frac{-4}{-3}\right) = -126.87°$

convert to radians: $-126.87\left(\frac{\pi}{180}\right) = -2.21 = -0.705\pi$

$z = 5e^{-j0.705\pi} = 5e^{-j2.21}$

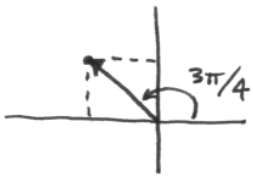
(d) $z = (0, -1)$

$\theta = -90° = -\pi/2$ rads.

$z = 1e^{-j\pi/2}$

PROBLEM 1.2*:

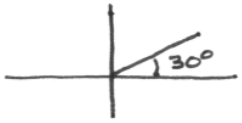
(a) $z = \sqrt{2}\, e^{j3\pi/4}$



$$z = \sqrt{2}\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = -1 + j1$$

(b) $z = 1.6\angle\pi/6 = 1.6\, e^{j\pi/6} = 1.6\left(\cos\frac{\pi}{6} + j\sin\pi/6\right)$

$= 1.6\angle 30°$



$$= 1.6\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = 1.386 + j0.8$$

(c) $z = 3e^{-j\pi/2} = 3\angle -90°$

$z = -3j$

(d) $z = 7\angle 7\pi = 7\angle\pi = 7e^{j\pi} = -7 + j0$ (subtract multiple of 2π)

$= 7\angle 1260°$

PROBLEM 1.3*:

(a) $z = -3 + j4 = 5e^{j0.705\pi}$ (ANGLE = 126.87° or 2.214 rads)

$\frac{1}{z} = \frac{1}{5} e^{-j0.705\pi}$

(b) $z = -2 + j2 = 2\sqrt{2}\, e^{j3\pi/4}$

SUBTRACT 4π

$z^5 = (2\sqrt{2})^5 e^{j15\pi/4} = 128\sqrt{2}\, e^{-j\pi/4}$

(c) $z = -5 + j13$

$|z|^2 = z z^* = (-5+j13)(-5-j13)$

$= 25 + 169 = 194$

(d) $\mathrm{Re}\{(-2+j5)e^{-j\pi/2}\}$

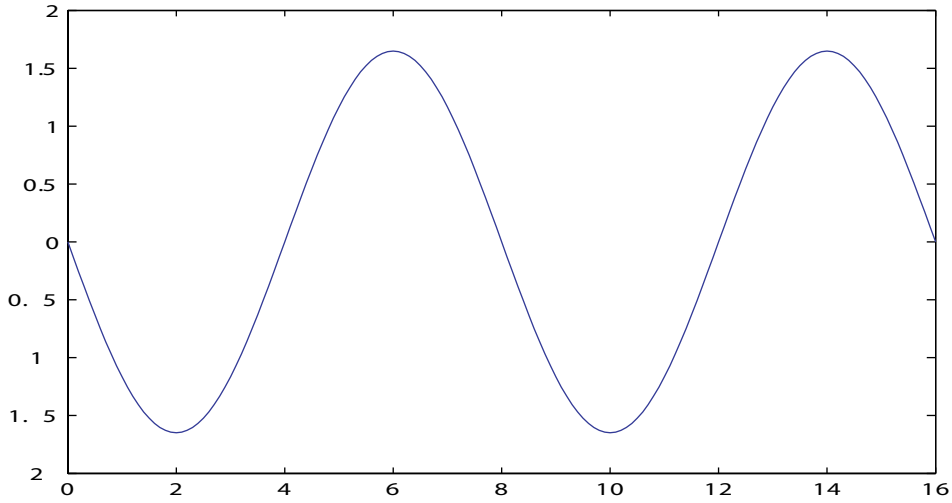
EQUALS $-j$

$= \mathrm{Re}\{(-2+j5)(-j)\}$

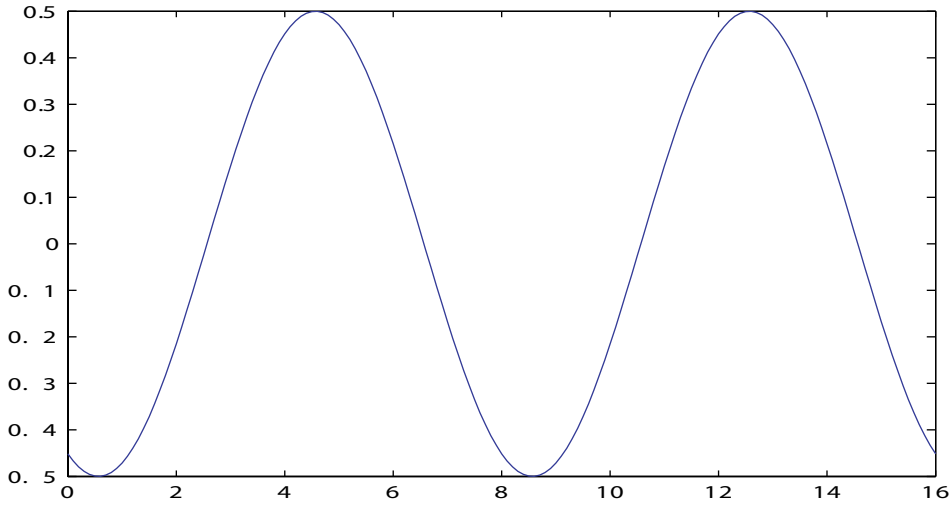
$= \mathrm{Re}\{2j + 5\} = 5$

PROBLEM 1.4*:

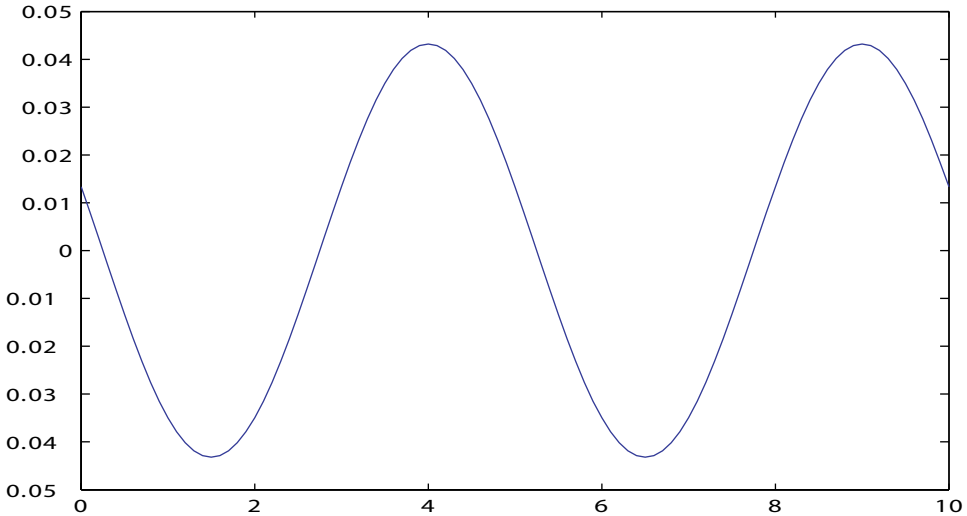
$T = 8$



$T = 8$



$T = 5$



PROBLEM 1.5*:

The amplitude of the sinusoid is: $A = 25$.

The plot shows four periods of the sinusoid over an interval of 0.065 sec., therefore:

$$T_0 = \frac{0.065}{4} = 0.01625 \text{ sec.}$$

$$\omega_0 = \frac{2\pi}{T_0} = 386.7 \text{ rad/sec.}$$

$$f_0 = \frac{1}{T_0} = 61.54 \text{ Hz}$$

The first peak of the sinusoid after $t = 0$ occurs at $t \cong 0.0075$ sec., therefore $t_d = 0.0075$ sec.

$$\phi = -2\pi t_d f_0 = -\omega_0 t_d = -2.9 \text{ rad.}$$