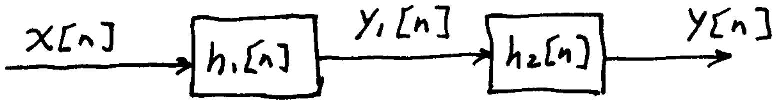
ECE 2025 Spring 2005

HW # 8

Prob. 8.1



$$y_1[n] = -\frac{1}{2}x[n] + 3x[n-1] - \frac{1}{2}x[n-2]$$

$$h_2[n] = \delta[n-2]$$

a) $$H_1(e^{j\hat{\omega}}) = -\frac{1}{2} + 3e^{-j\hat{\omega}} - \frac{1}{2}e^{-j2\hat{\omega}}$$

b) $$H_2(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}$$

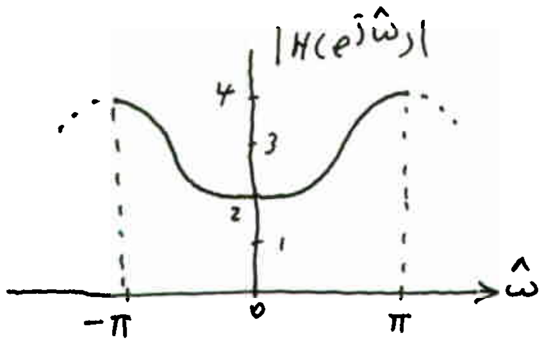
$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})\,H_2(e^{j\hat{\omega}})$$

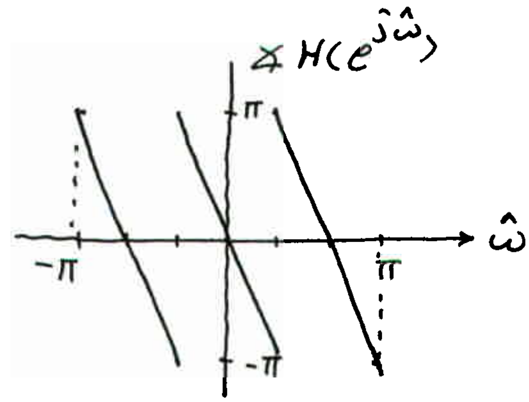
$$= -\frac{1}{2}e^{-j2\hat{\omega}} + 3e^{-j3\hat{\omega}} - \frac{1}{2}e^{-j4\hat{\omega}}$$

$$= e^{-j3\hat{\omega}}(3 - \cos\hat{\omega})$$ ← Freq. response of the overall system

c) $$|H(e^{j\hat{\omega}})| = 3 - \cos\hat{\omega} \quad > 0$$

$$\measuredangle H(e^{j\hat{\omega}}) = -3\hat{\omega}$$





Prob. 8.2

a) $x_1[n] = \cos\frac{\pi n}{2} = \begin{cases} (-1)^{n/2}, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

$y_1[n] = (-1)^n x_1[n] = (-1)^n \cos\frac{\pi n}{2} = \left\{ \begin{matrix} (-1)^{n/2}, & n \text{ even} \\ 0, & n \text{ odd} \end{matrix} \right\} = x_1[n]$

or

$(-1)^n = \cos \pi n$ $\qquad \cos\frac{3\pi n}{2} = \cos(-\frac{\pi}{2}n) = \cos\frac{\pi n}{2}$

$y_1[n] = \cos\pi n \cos\frac{\pi n}{2} = \frac{1}{2}\left[\cos\frac{3\pi}{2}n + \cos\frac{\pi n}{2}\right] = \cos\frac{\pi n}{2} = x_1[n]$

The output is identical to the input; i.e. no new freq. component present at the output — therefore, we cannot conclude that the system is not linear.

We can still examine the time invariance part. See c).

b) Any (non-DC) sinusoid with freq. other than $\pi/2$ will produce new freq. components at the output.

For example.

$x_2[n] = A\cos\frac{\pi n}{4}$

$y_2[n] = \frac{A}{2}\left[\cos\frac{3\pi n}{4} + \cos\frac{\pi n}{4}\right]$

Two sinusoids are present at the output with $\hat{\omega} = \frac{3\pi}{4}$ and $\frac{\pi}{4}$; $\hat{\omega} = \frac{3\pi}{4}$ is new, not in the input.

c) Part b) shows that the system is not linear.

For time invariance:

$x[n] \rightarrow x[n-1]$

$(-1)^n x[n-1] = (-1)(-1)^{n-1}x[n-1] = -y[n-1] \neq y[n-1]$

The system is not time invariant.

Prob. 8.3

a) $H(e^{j\hat{\omega}}) = (1-je^{-j\hat{\omega}})(1+je^{-j\hat{\omega}})(1-e^{j\frac{3\pi}{4}}e^{-j\hat{\omega}})(1-e^{-j\frac{3\pi}{4}}e^{-j\hat{\omega}})$

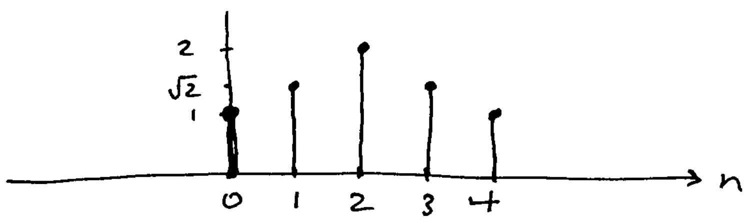
$= (1+e^{-j2\hat{\omega}})(1-2\cos\frac{3\pi}{4}e^{-j\hat{\omega}}+e^{-j2\hat{\omega}})$

$= 1+\sqrt{2}e^{-j\hat{\omega}}+2e^{-j2\hat{\omega}}+\sqrt{2}e^{-j3\hat{\omega}}+e^{-j4\hat{\omega}}$

$\therefore \quad y[n] = x[n]+\sqrt{2}x[n-1]+2x[n-2]+\sqrt{2}x[n-3]+x[n-4]$

b) Impulse response of the system

$h[n] = \delta[n]+\sqrt{2}\delta[n-1]+2\delta[n-2]+\sqrt{2}\delta[n-3]+\delta[n-4]$



Prob. 8.4

a) $H(e^{j\hat{\omega}}) = (1+e^{-j2\hat{\omega}})(1-2\cos\frac{3\pi}{4}e^{-j\hat{\omega}}+e^{-j2\hat{\omega}})$

$x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$ a sinusoid

$H(e^{j\hat{\omega}}) = \underline{(1+e^{-j2\hat{\omega}}})(1+\sqrt{2}e^{-j\hat{\omega}}+e^{-j2\hat{\omega}})$

goes to 0 if $\underline{\hat{\omega} = \pm\frac{\pi}{2}}$

$1+e^{\pm j\pi} = 0$

Therefore, $y[n]=0$ for all n if $x[n] = Ae^{j\phi}e^{\pm j\frac{\pi}{2}n}$

b) $x[n] = 5+9\delta[n-4]+7\cos(\frac{\pi n}{2}-\frac{3\pi}{4}) \qquad -\infty<n<\infty$

Let $x_1[n]=5$, DC $\Leftrightarrow \hat{\omega}=0$, and $H(e^{j0}) = 4+2\sqrt{2}$

Therefore, $y_1[n] = 5\cdot(4+2\sqrt{2}) = 20+10\sqrt{2}$

Let $x_2[n] = 9\delta[n-4]$, $y_2[n] = 9h[n-4]$

Let $x_3[n] = 7\cos(\frac{\pi n}{2}+\frac{3\pi}{4})$, $y_3[n]=0$ due to result in a.

Prob 8.4 (cont'd)

$$x[n] = x_1[n] + x_2[n] + x_3[n]$$
$$y[n] = y_1[n] + y_2[n] + y_3[n]$$
$$= 20 + 10\sqrt{2} + 9\delta[n-4] + 9\sqrt{2}\,\delta[n-5] + 18\delta[n-6] + 9\sqrt{2}\,\delta[n-7] + 9\delta[n-8]$$

Prob. 8.5

$$H(e^{j0}) = \tfrac{1}{2}, \quad H(e^{j\pi}) = 2e^{j11\pi} = 2e^{j\pi} = -2 \quad \text{(real)}$$

a) $x_1[n] = \begin{cases} 8 & \text{for even } n \\ 4 & \text{odd } n \end{cases}$

$x_1[n] = 6 + 2\cos\pi n$

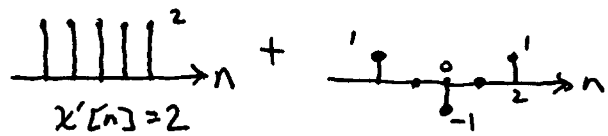
$6 \xrightarrow{@\,\hat{\omega}=0} H(e^{j0})\cdot 6 = \frac{6}{2} = 3$

$2\cos\pi n \xrightarrow{@\,\hat{\omega}=\pi} H(e^{j\pi})\,2\cos\pi n = -4\cos\pi n$

Thus, output is

$y_1[n] = 3 - 4\cos\pi n$

b) $x_2[n] = 2 - \cos\frac{\pi n}{2}$



$x'[n] = 2 \quad + \quad x''[n] = -\cos\frac{\pi n}{2}$

$2 \xrightarrow{@\,\hat{\omega}=0} H(e^{j0})\cdot 2 = 1$

$\cos\frac{\pi n}{2} \xrightarrow{@\,\hat{\omega}=\pi/2} |H(e^{j\pi/2})|\left\{\cos\left[\frac{\pi n}{2} + \angle H(e^{j\pi/2})\right]\right\}$

$H(e^{j\pi/2}) = 3e^{+j\pi/2}$

$= 3\cdot\left[\cos\left(\frac{\pi n}{2} + \frac{\pi}{2}\right)\right] = 3\cos\left(\frac{\pi n}{2} + \frac{\pi}{2}\right)$

The output is therefore $y_2[n] = 1 - 3\cos\left(\frac{\pi n}{2} + \frac{\pi}{2}\right)$
$= 1 + 3\sin\frac{\pi n}{2}$

Prob. 8-6

a) $x[n] = 2e^{j\pi/2}e^{j0.3\pi n} + 2e^{-j\pi/2}e^{-j0.3\pi n} + 3e^{j\pi}$

$= 2j\,e^{j0.3\pi n} - 2j\,e^{-j0.3\pi n} - 3 = -4\sin(0.3\pi n) - 3$

b) 9-pt. running average filter

$$H(e^{j\hat{\omega}}) = \frac{\sin(9\hat{\omega}/2)}{9\sin(\hat{\omega}/2)}e^{-j\hat{\omega}4}, \qquad H(e^{j0}) = 1$$

$$H(e^{j0.3\pi}) = \frac{\sin(\frac{27}{20}\pi)}{9\sin(\frac{3\pi}{20})}e^{-j1.2\pi} = 0.218\,e^{-j0.2\pi}$$

The result is

$y[n] = -3 - 0.872\sin(0.3\pi n - 0.2\pi)$

or $= -3 + 0.872\cos(0.3\pi n + 0.3\pi)$

$\left(\sin(\theta - \frac{\pi}{2}) = -\cos\theta\right)$