GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Problem Set #12

Assigned: 4-April-05
Due Date: Week of 11-April-05

Quiz #4 will be given on 22-April. One page ($8\frac{1}{2} \times 11$ in.) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 9: *Continuous-Time Signals & Systems*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 12.1*:

Express each of the following in a simpler form:

(a) $\left[\pi e^{-10t}\cos(10\pi t)u(t) + u(t-0.09)\right]\delta(t-0.03) =$

(b) $[3\delta(t+2) - 7\delta(t)] * [\delta(t+2) + \delta(t-2)] =$

(c) $\dfrac{d}{dt}\left\{e^{(t-3)}\cos(10\pi(t-3))u(t-3)\right\} =$

(d) $\displaystyle\int_{-\infty}^{t-0.1} \delta(\tau+0.2)\cos(4\pi\tau)u(-\tau)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution. Convolution is denoted by a "star", as in $x(t) * \delta(t-2) = x(t-2)$ and multiplication is usually indicated as in $x(t)\delta(t-2) = x(2)\delta(t-2)$.

PROBLEM 12.2:

This is Problem 9.2 of Problem Set #9 of ECE2025 from the Fall of 2000. It is helpful to understand this problem. The best way to work it is to draw pictures with "typical" input and impulse response signals.

The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \leq T_1$ and for $t \geq T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \leq T_3$ and for $t \geq T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \leq T_5$ and for $t \geq T_6$. In the process of proving this result you should obtain expressions for T_5 (the starting time) and T_6 (the ending time) in terms of T_1, T_2, T_3, and T_4.

PROBLEM 12.3*:
A linear time-invariant system has impulse response:

$$h(t) = e^{0.3t}\{u(t+3) - u(t-2)\} = \begin{cases} e^{0.3t} & -3 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t-\tau)$ as a function of τ for $t = 0$, 2, and 4.

(b) Find the output $y(t)$ when the input is $x(t) = \delta(t-1)$, and make a sketch of $y(t)$.

(c) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = e^{-0.2t}\{u(t) - u(t-6)\} = \begin{cases} e^{-0.2t} & 0 \le t < 6 \\ 0 & \text{otherwise} \end{cases}$$

PROBLEM 12.4*:
A linear time-invariant system has impulse response: $h(t) = u(t-1) - u(t+1)$

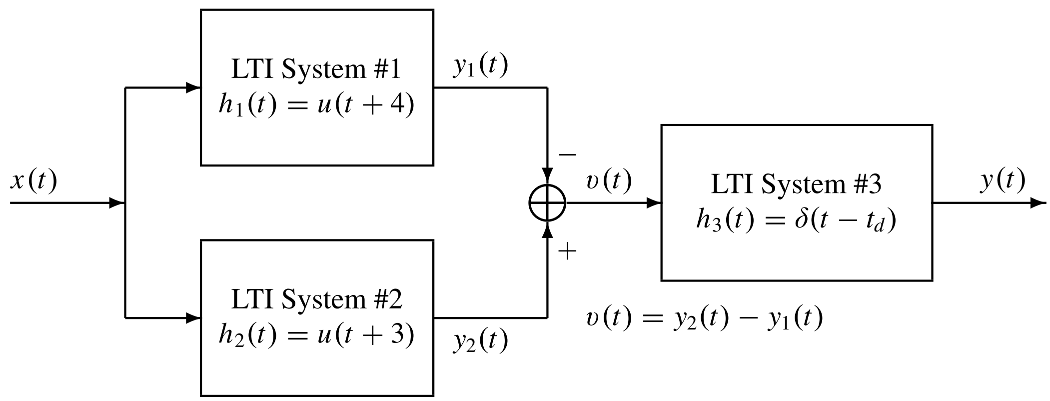
(a) Plot $h(t-\tau)$ versus τ, for $t = -3$ and $t = 2$. Label your plot.

(b) Is the LTI system causal? Give a reason to support your answer.

(c) Is the system stable? Explain with a proof or counter-example.

(d) If the input is $x(t) = u(t+2)$, then it will be true that the output $y(t)$ is zero for $t \le t_1$ and $y(t) = K$ for $t \ge t_2$. Find t_1, t_2 and the constant K.

PROBLEM 12.5*:

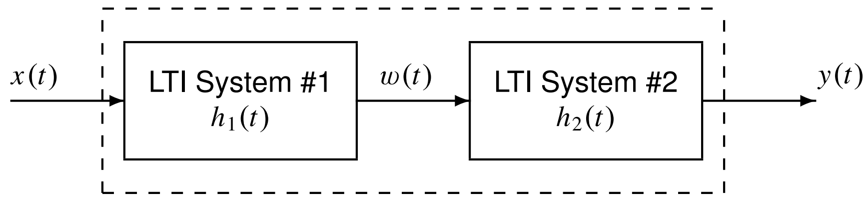


(a) If $t_d = 5$, what is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer as a carefully labeled sketch.

(b) How should the time delay t_d be chosen so that the overall system is causal?

(c) Which systems (#1, #2, #3) are stable? Is the overall system a stable system? Explain to receive credit.

PROBLEM 12.6*:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = \delta(t) - e^{-3t}u(t)$$

and the second system is described by the input/output relation

$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)w(t-2-\tau)d\tau$$

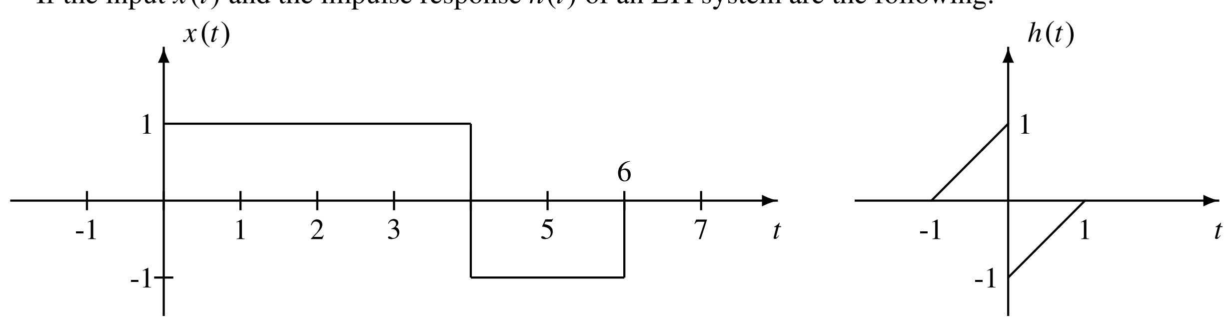
(a) Find the impulse response of the second system.

(b) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

PROBLEM 12.7:

This is a problem from Problem Set #9 of Fall 2000. Try working it first before checking the answer.

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:



(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Find all the values of t for which the output $y(t) = 0$. *Note: You do not need to find $y(t)$ at any other values of t.*