ECE2025 Spring 2005

HW#12, Solutions

problem 12.1

(a) $[\pi e^{-10t}\cos(10\pi t)u(t)+u(t-0.09)]\delta(t-0.03)$

$= [\pi e^{-10(0.03)}\cos(10\pi(0.03))\underbrace{u(0.03)}_{=1}+\underbrace{u(0.03-0.09)}_{=0}]\delta(t-0.03)$

$=[\pi e^{-0.3}\cos(0.3\pi)]\delta(t-0.03)$

(b) $[3\delta(t+2)-7\delta(t)]*[\delta(t+2)+\delta(t-2)]$

$= 3\delta(t+4)+3\delta(t)-7\delta(t+2)-7\delta(t-2)$

(c) $\frac{d}{dt}[e^{(t-3)}\cos(10\pi(t-3))u(t-3)]$

$= e^{(t-3)}\cos(10\pi(t-3))u(t-3)-e^{(t-3)}10\pi\sin(10\pi(t-3))u(t-3)$

$+e^{(t-3)}\cos(10\pi(t-3))\delta(t-3)$

$= e^{(t-3)}[\cos(10\pi(t-3))-10\pi\sin(10\pi(t-3))]u(t-3)$

$+\delta(t-3)$

(d) $\int_{-\infty}^{t-0.1}\delta(\tau+0.2)\cos(4\pi\tau)u(-\tau)\,d\tau$

$=\int_{-\infty}^{t-0.1}\cos(4\pi(-0.2))\delta(\tau+0.2)u(-(-0.2))\,d\tau$

$= \cos(-0.8\pi)\int_{-\infty}^{t-0.1}\delta(\tau+0.2)\,d\tau$

$= \cos(0.8\pi)\,u(\tau+0.2)\Big|_{-\infty}^{t-0.1}$

$= \cos(0.8\pi)\,u(t-0.1+0.2)$

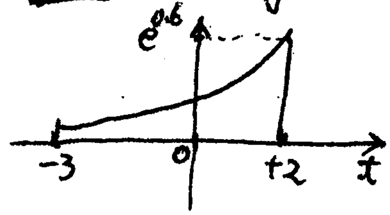
$= \cos(0.8\pi)\,u(t+0.1)$

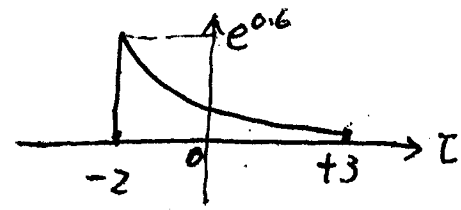
problem 12.3

$$h(t) = e^{0.3t}[u(t+3)-u(t-2)] = \begin{cases} e^{0.3t}, & -3 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

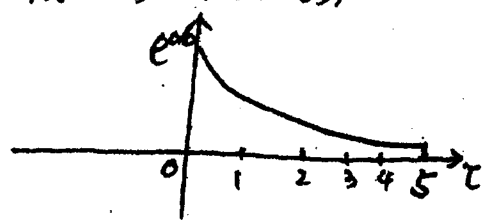
(a) plot $h(t-\tau)$ for $t = 0, 2, 4$

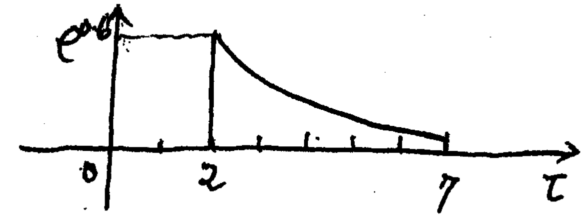
first the original $h(t)$, $h(t-\tau) = h(-\tau)$, when $t=0$





$h(t-\tau) = h(2-\tau)$, when $t=2$ $\qquad$ $h(t-\tau) = h(4-\tau)$, $t=4$





(b)
$$y(t) = \delta(t-1) * e^{0.3t}[u(t+3)-u(t-2)]$$
$$= e^{0.3(t-1)}[u(t+2)-u(t-3)]$$

(C)
$$x(t) = e^{-0.2t}[u(t)-u(t-6)] = \begin{cases} e^{-0.2t}, & 0 \le t < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \int_a^b e^{0.3\tau} e^{-0.2(t-\tau)} d\tau$$
$$= e^{-0.2t} \int_a^b e^{0.5\tau} d\tau = e^{-0.2t} \frac{(e^{0.5b} - e^{0.5a})}{0.5}$$

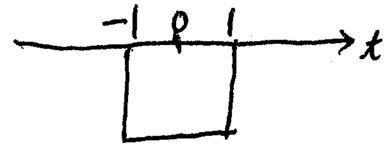
$$y(t) = \begin{cases} 0 & \text{for } t < -3 \\ e^{-0.2t} \int_{-3}^{t} e^{\tau/2} d\tau = 2e^{-0.2t}(e^{0.5t} - e^{-1.5}) & \text{for } -3 \le t < 2 \\ e^{-0.2t} \int_{-3}^{2} e^{\tau/2} d\tau = 2e^{-0.2t}(e - e^{-1.5}) & \text{for } 2 \le t < 3 \\ e^{-0.2t} \int_{t-6}^{2} e^{\tau/2} d\tau = 2e^{-0.2t}(e - e^{0.5t-3}) & \text{for } 3 \le t < 8 \\ 0 & \text{for } 8 \le t \end{cases}$$

problem 12.4

a linear time-invariant system, $h(t) = u(t-1) - u(t+1)$

(a) plot $h(t-\tau)$ vs. τ, for $t=-3$ and $t=2$, Label the plot

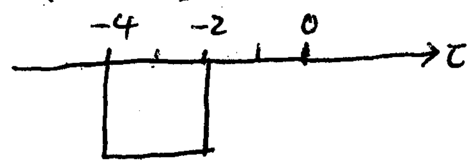
$h(t) = u(t-1) - u(t+1)$



$h(t-\tau)$ for $t=-3$

$= h(-3-\tau) = u(-3-\tau-1) - u(-3-\tau+1)$

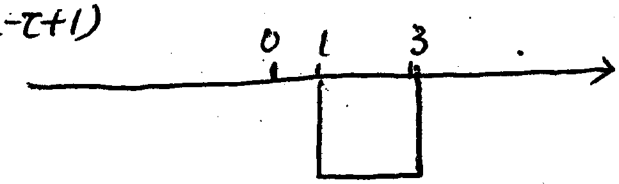
$= u(-4-\tau) - u(-2-\tau)$



$h(t-\tau)$ for $t=2$

$= h(2-\tau) = u(2-\tau-1) - u(2-\tau+1)$

$= u(1-\tau) - u(3-\tau)$

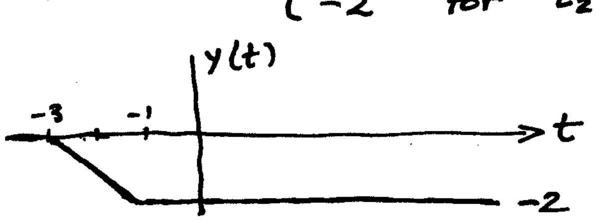


(b) no, the system is not causal. Because $h(t) \neq 0$, when $t<0$

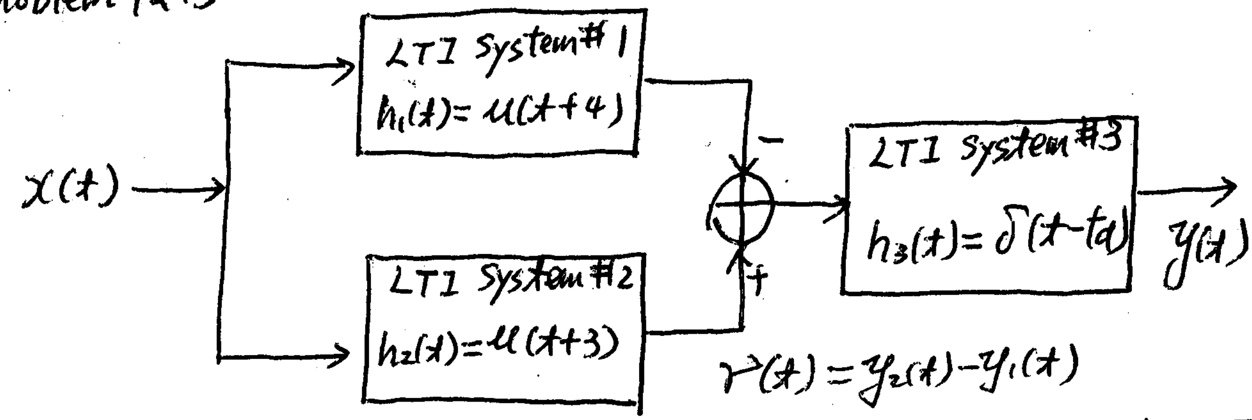
(c) The system is stable. Because $\int_{-\infty}^{\infty} |h(t)|\,dt$ is finite

(d) $x(t) = u(t+2)$

$$y(t) = u(t+2) * (u(t-1) - u(t+1))$$
$$= u(t+2) * u(t-1) - u(t+2) * u(t+1)$$
$$= (t+1)\,u(t+1) - (t+3)\,u(t+3)$$
$$= \begin{cases} 0 & \text{for } t \le t_1 = -3 \\ -(t+3) & \text{for } t_1 < t < t_2 = -1 \\ -2 & \text{for } t_2 \le t \end{cases}$$



problem 12.5

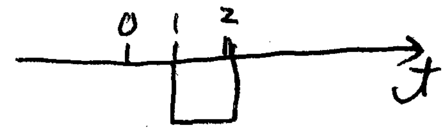


$r(t) = y_2(t) - y_1(t)$

(a) If $t_d = 5$, what is the overall impulse response of the LTI System

$$h(t) = [-h_1(t) + h_2(t)] * h_3(t)$$
$$= [-u(t+4) + u(t+3)] * \delta(t-5)$$
$$= -u(t-1) + u(t-2)$$

$h(t)$ plot



(b) $t_d \geq 4$, so that $h(t) = 0$ when $t < 0$

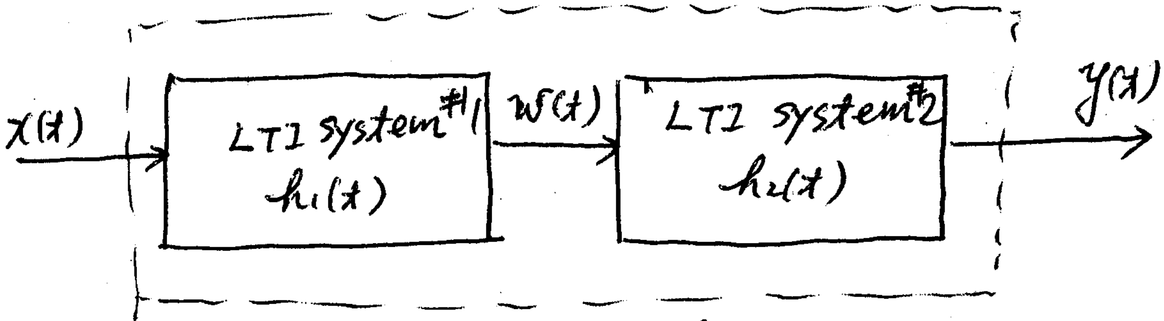
(c) - systems #1 and #2 are unstable because

$\int_{-\infty}^{\infty} |h_i(t-\tau)| d\tau$ is proportional to t
when $t \to \infty$ the integral value is approaching infinite

(d) - system #3 is stable because the value is finite and non-zero at $t = t_d$

- the overall system is stable because the non-zero value existed for finite period of time.

problem 12.6



$$h_1(t) = \delta(t) - e^{-3t}u(t)$$

the second system has

$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau) w(t-2-\tau)\, d\tau$$

(a) Find the impulse response of the second system

$$h_2(t) = e^{-2(t-2)} u(t-2)$$

proof: $y(t) = \int_{-\infty}^{\infty} e^{-2(\tau'-2)} u(\tau'-2)\, w(t-\tau')\, d\tau'$

set $\tau = \tau' - 2 \rightarrow d\tau = d\tau'$ and $\tau' = \tau + 2$

$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau} u(\tau)\, w(t-\tau-2)\, d\tau$$

(b)

$$y(t) = [\delta(t) - e^{-3t}u(t)] * [e^{-2(t-2)} u(t-2)]$$

$$= e^{-2(t-2)} u(t-2) - \int_0^2 e^{-3\tau} e^{-2(t-2-\tau)}\, d\tau$$

$$= e^{-2(t-2)} u(t-2) - \int_0^2 e^{-\tau} e^{-2(t-2)}\, d\tau$$

$$= e^{-2(t-2)} u(t-2) - e^{-2(t-2)} \left. \frac{e^{-\tau}}{-1} \right|_0^2$$

$$= e^{-2(t-2)} u(t-2) + (e^{-2}-1)\, e^{-2(t-2)}$$

$$= e^{-2(t-2)} u(t-2) + (e^{-2}-1)\, e^{2(2-t)}$$