HW 13

13.1

a)

$$x(t) = 13\cos(77\pi t - \pi/3) = \frac{13}{2}\left[e^{j(77\pi t-\frac{\pi}{3})} + e^{-j(77\pi t-\frac{\pi}{3})}\right]$$

$$\mathcal{F}\{x(t)\} = 13\pi\left[\delta(\omega-77\pi)\,e^{-j\pi/3} + \delta(\omega+77\pi)\,e^{j\pi/3}\right]$$

b) $x(t) = 7e^{-3t}u(t-2) - 7e^{-3t}u(t)$

$$7e^{-3t}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{7}{3+j\omega},$$

$$7e^{-3t}u(t-2) = 7e^{-3(t-2)}\cdot e^{-6}u(t-2)$$

$$7\cdot e^{-6}\,e^{-3(t-2)}u(t-2) \longleftrightarrow 7e^{-6}\frac{e^{-j2\omega}}{3+j\omega}$$

$$\therefore\quad x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{7}{3+j\omega}\left[1 - e^{-2(3+j\omega)}\right]$$

c) $$x(t) = \frac{\sin(3\pi(t-\frac{1}{2}))}{2t-1} = \frac{\pi}{2}\,\frac{\sin(3\pi(t-\frac{1}{2}))}{(t-\frac{1}{2})\pi}$$

$$\frac{\pi}{2}\,\frac{\sin(3\pi(t-\frac{1}{2}))}{\pi(t-\frac{1}{2})} \overset{\mathcal{F}}{\longleftrightarrow} \frac{\pi}{2}e^{-j\frac{\omega}{2}}\left[u(\omega+3\pi)-u(\omega-3\pi)\right]$$

d) $x(t) = u(t-8) - u(t-3)$

Let $x'(t) = u(t+\frac{5}{2}) - u(t-\frac{5}{2}) \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{\sin(\omega\frac{5}{2})}{\omega/2}$

$$x(t) = -x'(t-\tfrac{11}{2}) \longleftrightarrow -e^{-j\frac{11\omega}{2}}\,\frac{\sin(\omega 5/2)}{\omega/2}$$

13.2

a) $X(j\omega) = j\sin(4\omega)$

$$= \frac{1}{2}\left[e^{j4\omega} - e^{-j4\omega}\right] \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \frac{1}{2}\left[\delta(t+4) - \delta(t-4)\right]$$

b) $X(j\omega) = \dfrac{10}{4+3j\omega}\, e^{-j\omega/4}$

$$\frac{10}{4+3j\omega} \longleftrightarrow \frac{10}{3} e^{-4t/3} u(t) \quad , \quad e^{-j\omega/4} \longleftrightarrow \delta\left(t-\frac{1}{4}\right)$$

$$X(j\omega) \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \frac{10}{3} e^{-\frac{4}{3}(t-\frac{1}{4})} u\left(t-\frac{1}{4}\right)$$

c) $X(j\omega) = \underset{\downarrow \atop \delta(t+\frac{1}{5})}{e^{j\omega/5}} \underbrace{\{\delta(\omega-7\pi) + \delta(\omega+7\pi)\}}_{\frac{1}{\pi}\cos(7\pi t)}$

$$\therefore \quad X(j\omega) \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \frac{1}{\pi}\cos\left[7\pi\left(t+\frac{1}{5}\right)\right]$$

d) $X(j\omega) = u(\omega-7\pi) - u(\omega+7\pi)$

$$= -\left[u(\omega+7\pi) - u(\omega-7\pi)\right]$$

$$\overset{\mathcal{F}^{-1}}{\longleftrightarrow} \; -\frac{\sin(7\pi t)}{\pi t}$$

13.3

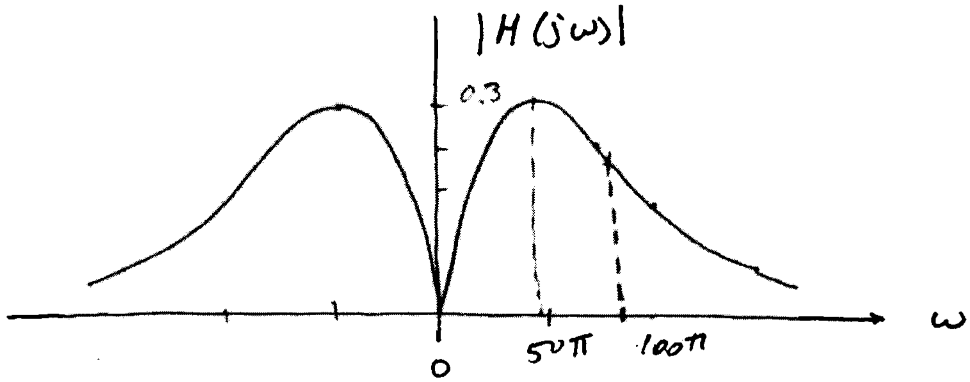
a) $h(t) = \frac{1}{2}\{ a e^{-at} u(t) - b e^{-bt} u(t) \}$

$$a e^{-at} u(t) \longleftrightarrow \frac{a}{a+j\omega}, \quad b e^{-bt} u(t) \longleftrightarrow \frac{b}{b+j\omega}$$

$$H(j\omega) = \frac{1}{2}\left(\frac{a}{a+j\omega} - \frac{b}{b+j\omega} \right)$$

b) $a = 25\pi$, $b = 100\pi$

$$H(j\omega) = \frac{1}{2}\left[\frac{25\pi}{25\pi + j\omega} - \frac{100\pi}{100\pi + j\omega} \right]$$



c) BPF

d) $\measuredangle H(j0) = 0$, $\measuredangle H(j50\pi) = -\pi$, $\measuredangle H(j100\pi) = 2.6$

$$H(j50\pi) = \frac{1}{2}\left(\frac{1}{1+2j} - \frac{2}{2+j} \right) = -\frac{3}{10}, \quad H(j0) = 0$$

$$H(j100\pi) = \frac{1}{2}\left(\frac{1}{1+4j} - \frac{1}{1+j} \right) = \frac{3j}{6-10j} = \frac{-15+9j}{68} = 0.2572 e^{j2.601}$$

e) $x(t) = 77 + 33\cos(50\pi t - \pi/4) + 44\cos(100\pi t)$

$77 \to 0$

$33\cos(50\pi t - \pi/4) \longrightarrow -9.9\cos(50\pi t - \pi/4)$

$44\cos(100\pi t) \longrightarrow 11.319\cos(100\pi t + 2.6)$

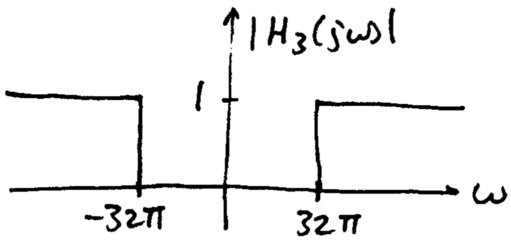
Output $y(t) = -9.9\cos(50\pi t - \pi/4) + 11.319\cos(100\pi t + 2.601)$

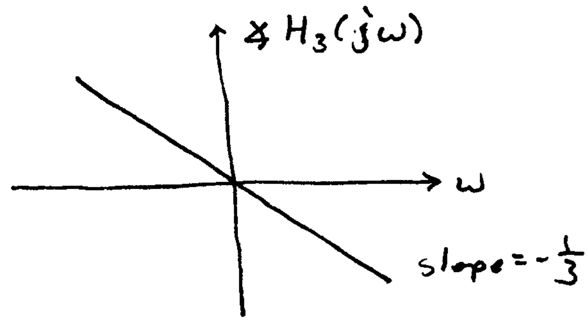
13.4 $\quad H_1(j\omega) = e^{-j\omega/3}$

$$H_2(j\omega) = [u(\omega+32\pi) - u(\omega-32\pi)]\, e^{-j\omega/3}$$

a) $\quad H_3(j\omega) = H_1(j\omega) - H_2(j\omega)$

$$= [1 - u(\omega+32\pi) + u(\omega-32\pi)]\, e^{-j\omega/3}$$





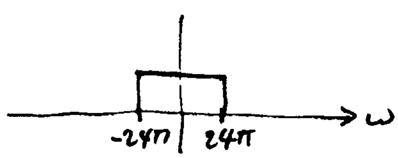
b) HPF

c) Input $\quad x(t) = \sin(44\pi t) + \dfrac{\sin(24\pi t)}{\pi t}$

$\sin(44\pi t)$ in passband without attenuation except phase change

$$\frac{\sin(24\pi t)}{\pi t} \longleftrightarrow u(\omega+24\pi) - u(\omega-24\pi)$$

entirely in stop band



Answer: output

$$y(t) = \sin\left(44\pi t - \frac{44\pi}{3}\right)$$

$$= \sin\left(44\pi t - \frac{2}{3}\pi\right)$$

d) $\quad H_3(j\omega) = [1 - u(\omega+32\pi) + u(\omega-32\pi)]\, e^{-j\omega/3}$

$$H_1(j\omega) \Leftrightarrow \delta\left(t - \tfrac{1}{3}\right)$$

$$H_2(j\omega) \Leftrightarrow \frac{\sin\left(32\pi\left(t-\frac{1}{3}\right)\right)}{\pi\left(t-\frac{1}{3}\right)}$$

$$h_3(t) = \delta\left(t - \tfrac{1}{3}\right) - \frac{\sin\left[32\pi\left(t-\frac{1}{3}\right)\right]}{\pi\left(t-\frac{1}{3}\right)}$$

13.5

$$h(t) = \delta(t) - \frac{\sin(\omega_{00} t)}{\pi t} \qquad T_0 = 1/5, \quad f_0 = 5$$

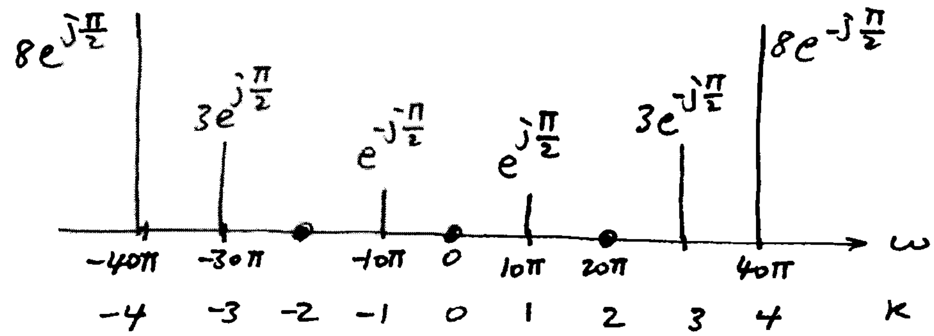
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi f_0 k t} = \sum_{k=-\infty}^{\infty} a_k e^{j10\pi k t}$$

$$a_k = jk(2-|k|), \quad k = 0, \pm 1, \pm 2, \ldots$$

$$a_0 = 0, \quad a_1 = j, \quad a_{-1} = -j, \quad a_2 = a_{-2} = 0$$

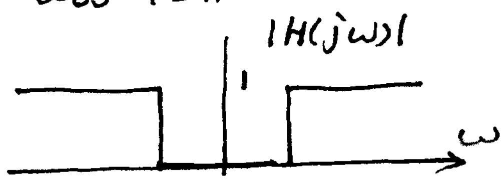
$$a_3 = -3j, \quad a_{-3} = 3j, \quad a_4 = -8j, \quad a_{-4} = 8j$$

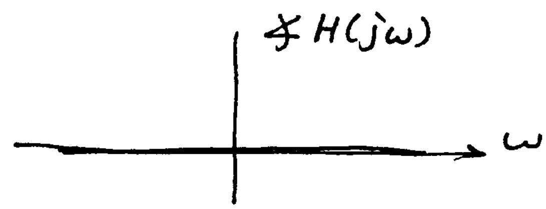
a) X spectrum



b) $H(j\omega) = 1 - u(\omega + \omega_{00}) + u(\omega - \omega_{00})$ (See Prob 13.4)

$\omega_{00} = 12\pi$





c) $a_{\pm 1} e^{j(\pm 10\pi)t}$ are totally suppressed

Output spectrum