GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

EE 2025 Spring 2005
Lab #4: FM Synthesis for Musical Instruments

Date: 8 – 14 Feb 2005

FORMAL Lab Report: You must write a formal lab report that describes your approach to FM synthesis (Section 4). ***This lab report will be worth 150 points.***

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section ***before your assigned lab time.***

The Warm-up section of each lab must be completed ***during your assigned Lab time*** and the steps marked *Instructor Verification* must also be signed off ***during the lab time.*** One of the laboratory instructors must verify the appropriate steps by signing on the ***Instructor Verification*** line. When you have completed a step that requires verification, simply raise your hand and demonstrate the step to the TA or instructor. After completing the warm-up section, turn in the verification sheet to your TA.

The exercises in Section 4 should written up as a ***FORMAL*** lab report. More information on the lab report format can be found on Web-CT under the "Information" link. You should ***label*** the axes of your plots and include a title and Figure number for every plot. Every plot should be referenced by Figure number in your text discussion. In order to make it easy to find all the plots, include each plot *inlined* within your report. This can be done easily with MATLAB's `notebook` capability.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

The ***FORMAL*** lab report will be ***due during the period 15 to 21-Feb. at the start of your lab.***

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1 Introduction

The objective of this lab is to introduce more complicated signals that are related to the basic sinusoid. These signals which implement frequency modulation (FM) and amplitude modulation (AM) are widely used in communication systems such as radio and television, but they also can be used to create interesting sounds that mimic musical instruments. There are a number of demonstrations on the CD-ROM that provide examples of these signals for many different conditions.

CD-ROM
FM Synthesis

2 Pre-Lab

We have spent a lot of time learning about the properties of sinusoidal waveforms of the form:

$$x(t) = A\cos(2\pi f_0 t + \phi) = \Re e\left\{ A e^{j\phi} e^{j2\pi f_0 t} \right\} \tag{1}$$

In this lab, we will extend our treatment of sinusoidal waveforms to more complicated signals composed of sums of sinusoidal signals, or sinusoids with changing frequency.

2.1 Frequency Modulated Signals

We will also look at signals in which the frequency varies as a function of time. In the constant-frequency sinusoid (1) the argument of the cosine is $(2\pi f_0 t + \phi)$ which is also the exponent of the complex exponential. We will refer to the argument of the cosine as the ***angle function***. In equation (1), the *angle function* changes *linearly* versus time, and its time derivative is $2\pi f_0$ which equals the constant frequency of the cosine.

A generalization is available if we adopt the following notation for the class of signals with time-varying angle functions:

CD-ROM
FM Synthesis

$$x(t) = A\cos(\psi(t)) = \Re e\{Ae^{j\psi(t)}\} \tag{2}$$

The time derivative of the angle function $\psi(t)$ in (2) gives a frequency

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{(rad/sec)}$$

but if we prefer units of hertz, then we divide by 2π to define the *instantaneous frequency:*

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t) \qquad \text{(Hz)} \tag{3}$$

2.2 Wideband FM

The instantaneous frequency is a useful measure of frequency content when the rate of change of the frequency is slow relative to the frequency analysis of the signal. For example, it is easy to concoct examples where FM signals with various modulation rates sound quite different "to the ear." The examples given on the *SP-First* CD-ROM in Chapter 3 demonstrate this effect.

CD-ROM
Spectrograms & Sounds: Wide-band FM

Please view the examples on the CD-ROM

The fact that we hear different effects depending on the rate of the modulating signal can be explained by modeling the human hearing system as a spectrum analyzer. There is good physiological evidence that the cochlea (part of the inner ear) performs a decomposition of sound signals into frequency bands. Thus, we can think of the first stage of the human hearing system as equivalent to a spectrogram. If so, then the hearing system will analyze the sound signals over some time interval which seems to be in the range of 20 to 50 millisec. If we calculate a window length for the spectrogram that would have the same duration, we get lengths between 160 and 400 at 8000 samples per sec. In the examples below, we use a window length of 512, but 256 should also be a reasonable choice to show the same perceptual phenomenon.

2.3 FM Sound Synthesis

Frequency modulation (FM) can be used to make interesting sounds that mimic musical instruments, such as bells, woodwinds, drums, etc. The goal in this lab is to implement one or two of these FM schemes and hear the results.

CD-ROM
DEMO: FM-Synthesis

We have already seen that FM defines the signal $x(t)$ to have a time-varying phase $\psi(t)$, and that the instantaneous frequency changes according to the derivative of $\psi(t)$. If $\psi(t)$ is linear, $x(t)$ is a constant-frequency sinusoid; whereas, if $\psi(t)$ is quadratic, $x(t)$ is a chirp signal whose frequency changes linearly in time. FM music synthesis uses a more interesting $\psi(t)$, one that is sinusoidal. Since the derivative of a sinusoidal $\psi(t)$ is also sinusoidal, the instantaneous frequency of $x(t)$ will oscillate. This is useful for synthesizing instrument sounds because the proper choice of the modulating frequencies will produce a fundamental frequency and several overtones, as many instruments do.

The general equation for an FM sound synthesizer is:

$$x(t) = A(t)\cos\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \tag{4}$$

where $A(t)$ is the signal's amplitude. It is a function of time so that the instrument sound can be made to fade out slowly or cut off quickly. Such a function is called an *envelope.* The parameter f_c is called the *carrier* frequency. Note that when you take the derivative of $\psi(t)$ to find $f_i(t)$,

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi}\frac{d}{dt}\psi(t) \\ &= \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_c t + I(t)\cos(2\pi f_m t + \phi_m) + \phi_c\right) \\ &= f_c - f_m I(t)\sin(2\pi f_m t + \phi_m) + \frac{1}{2\pi}\frac{dI}{dt}\cos(2\pi f_m t + \phi_m) \end{aligned} \tag{5}$$

f_c will be a constant in that expression. It is the frequency that would be produced without any frequency modulation. The parameter f_m is called the *modulating* frequency. It expresses the rate of oscillation of the instantaneous frequency, $f_i(t)$. The parameters ϕ_m and ϕ_c are arbitrary phase constants, usually both set to $-\pi/2$ so that $x(0) = 0$.

The function $I(t)$ has a less obvious purpose than the other FM parameters in (4). It is technically called the *modulation index envelope.* To see what it does, examine the expression for the instantaneous frequency (5). The quantity $I(t)f_m$ multiplies a sinusoidal variation of the frequency. If $I(t)$ is constant or $\frac{dI}{dt}$ is relatively small, then $I(t)f_m$ gives the maximum amount by which the instantaneous frequency deviates from f_c. Beyond that, however, it is difficult to relate $I(t)$ to the sound made by $x(t)$ without some rather tedious mathematical analysis.

In our study of signals, we would like to characterize $x(t)$ as the sum of several constant-frequency sinusoids instead of a single sinusoidal signal whose frequency changes. In this regard, the following comments are relevant: when $I(t)$ is small (e.g., $I \approx 1$), low multiples of the carrier frequency (f_c) have high amplitudes. When $I(t)$ is large ($I > 4$), both low and high multiples of the carrier frequency have high amplitudes. The net result is that $I(t)$ can be used to vary the harmonic content of the instrument sound (called overtones). When $I(t)$ is small, mainly low frequencies will be produced. When $I(t)$ is large, higher harmonic frequencies can also be produced. Since $I(t)$ is a function of time, the harmonic content will change with time. For more details see the paper by Chowning.[1]

2.4 MATLAB Synthesis of Wideband FM

The following MATLAB code will synthesize FM signals:

```
fsamp = 8000;
dt = 1/fsamp;
dur = 1.1;
tn = 0 : dt : dur;
fc = 900;   fm = 3;   Im = 100/pi;   % <==== FM parameters
psi = 2*pi*fc*tn + Im*cos(2*pi*fm*tn-pi/2) - pi/2;
xx = real( 7.7*exp(j*psi) );
soundsc( xx, fsamp );
plotspec( xx+j*1e-14, fsamp, 512), zoom on, grid on, shg
```

(a) In MATLAB signals can only be synthesized by evaluating the signal's defining formula at discrete instants of time. These are called *samples* of the signal. For FM we do the following:

$$x(t_n) = A\cos(\psi(t_n))$$

[1]Ref: John M. Chowning, "The Synthesis of Complex Audio Spectra by means of Frequency Modulation," *Journal of the Audio Engineering Society*, vol. 21, no. 7, Sept. 1973, pp. 526–534.

where t_n is the n^{th} time sample. From the MATLAB code above, identify the mathematical formula for $\psi(t)$.

(b) Determine the formula for the instantaneous frequency of the FM signal and compare that formula to the result in the spectrogram. Notice that you can hear the frequency variation predicted by the instantaneous frequency. We will refer to this case as *slowly-varying FM*.

(c) Now change the "modulating parameters" f_m and I_m to be $f_m = 100$ Hz and $I_m = 1/\pi$. Listen to the signal and notice that it is no longer possible to hear the instantaneous frequency. This is called *wideband FM*. In addition, try to interpret the spectrogram in terms of what you hear. Think about the following: (i) is $x(t)$ periodic? If so, determine its fundamental period. (ii) does the spectrum appear to be harmonic? If so, determine the spacing of the spectral lines and the fundamental frequency.

3 Warm-up

During the warm-up you will need a general function for synthesizing FM signals with sinusoidal modulation. Therefore, write an M-file **fmsinus.m** that implements Eq. (4) with $A(t) = 1$, $\phi_c = \phi_m = -\pi/2$, and $I(t) = I_m$. Use the following calling arguments:

```
function  [xx,tt] = FMsinus(fc, fm, Im, dur, fsamp)
%FMSINUS     produce FM signal with sinusoidal modulation
%
%     usage:  [xx,tt] = fmsinus(fc, fm, Im, dur, fsamp)
%
%     where:  fc = carrier frequency in Hz
%             fm = modulation frequency in Hz
%             Im = scale factor for modulation index
%            dur = duration (in sec.) of the output signal
%          fsamp = sampling rate
```

Instructor Verification (separate page)

Your function will generate any FM signal with sinusoidal modulation according to the following formula:

$$x(t) = \cos(2\pi f_c t + I_m \cos(2\pi f_m t - \pi/2) - \pi/2)$$

3.1 Slowly Varying FM

Generate an FM signal with $f_c = 900$ Hz, $f_m = 2$ Hz and $I_m = 20$ over the time interval $0 \le t \le 1.25$ secs. Use a sampling rate of 8000 Hz, i.e., `fsamp = 8000`.

(a) Plot the FM signal's spectrogram and listen to the signal to see if it corresponds to the spectrogram.

(b) Assuming that you used a window length of 512 in the spectrogram, then the window length in seconds is equal to $512 \times (1/8000) = 0.064$ secs. Determine how much the instantaneous frequency moves during a time interval of this duration. You will need a formula for the instantaneous frequency, or you can estimate this frequency movement from the "steps" visible in the spectrogram.

(c) Make a sketch (by hand) of the instantaneous frequency for the signal. Determine the period of the signal, and the minimum and maximum values of the instantaneous frequency.

Instructor Verification (separate page)

3.2 Wideband FM

Generate a wideband FM signal with $f_c = 900$ Hz, $f_m = 200$ Hz and $I_m = 1/\pi$ over the time interval $0 \le t \le 1.25$ secs. Use a sampling rate of 8000 Hz, i.e., `fsamp = 8000`.

(a) Plot the FM signal's spectrogram and listen to the signal to see if it corresponds to the spectrogram.

(b) Determine the fundamental period of the FM signal, and then explain why the spectrogram has equally spaced spectral lines, and how that spacing is related to the period.

(c) Derive a formula for the instantaneous frequency for the signal defined above, and then make a sketch (by hand) of $f_i(t)$ versus t.

(d) Assuming that you used a window length of 512 in the spectrogram, then determine how much the instantaneous frequency moves during a time interval of 0.064 secs. You will need a formula for $f_i(t)$, because you cannot estimate this frequency movement from the spectrogram.

(e) Now, change I_m to be $I_m = 1$, but keep $f_c = 900$ Hz and $f_m = 200$ Hz. Generate the signal, plot its spectrogram and listen to the signal. Describe how the sound and spectrogram have changed.
Instructor Verification (separate page)

(f) For an interesting wideband FM signal, listen to the following signal and view its spectrogram:

```
tt = 0 : 1/8000 : 4.4;
xx = exp(-tt).*sin(300*pi*tt+10*exp(-tt).*sin(600*pi*tt));
```

Determine which frequency component is the strongest one in the spectrogram, and try to relate that one to what is heard. (It might only be possible to do this qualitatively.)

4 Lab: FM Synthesis of Instrument Sounds

In the lab exercises, the objective is to write a function that will generate clarinet sounds (or other woodwinds), and then use that function to play a musical scale consisting of eight successive notes on the C-major scale (see Section 3.7 in *SP-First*).

4.1 Generating the Envelopes for Woodwinds

This section shows how to choose the parameters in the FM synthesis formula (4) to obtain a clarinet sound, or other woodwinds.

CD-ROM
woodwenv.m

There is a function on the CD-ROM called `woodwenv` that produces the functions needed to create both the $A(t)$ and $I(t)$ envelopes for a clarinet sound. The file header looks like this:

```
function     [y1, y2] = woodwenv(att, sus, rel, fsamp)
%WOODWENV        produce normalized amplitude and modulation index
%                functions for woodwinds
%
%       usage: [y1, y2] = woodwenv(att, sus, rel, fsamp);
%
%        where  att = attack TIME
%               sus = sustain TIME
%               rel = release TIME
%             fsamp = sampling frequency (Hz)
%        returns:
%             y1 = (NORMALIZED) amplitude envelope
%             y2 = (NORMALIZED) modulation index envelope
%
%  NOTE: attack is exponential, sustain is constant,
%          release consists of two exponentials
```

The parameters `att`, `sus` and `rel` require a bit of explanation, which can be done in terms of the signal's amplitude. When playing a typical musical note, the amplitude rises quickly at the beginning (the attack), then remains nearly constant while the note plays (the sustain), and then dies away quickly at the end (the release). See the plot of $A(t)$ in Fig. 1.

Normalized Outputs: The outputs from `woodwenv` are normalized so that the minimum value is zero and the max is one. Try the following statements to see what the function produces:

```
fsamp = 8000;
Ts = 1/fsamp;
[y1, y2] = woodwenv(0.1, 0.3, 0.1, fsamp);
tt = (0:length(y1)-1)*Ts;
subplot(2,1,1), plot(tt,y1), grid on
subplot(2,1,2), plot(tt,y2), grid on, shg
```

4.1.1 Scaling to Obtain the Clarinet Envelopes

Since the woodwind envelopes produced by `woodwenv.m` range from 0 to 1, some scaling is necessary to make them useful in the FM synthesis equation (4). In this section, we consider the general process of linear re-scaling. If we start with a *normalized* signal $y_{\text{norm}}(t)$ and want to produce a new signal whose max is $y_{\max}$ and whose min is $y_{\min}$, then we must map the value 1 to $y_{\max}$ and the value 0 to $y_{\min}$. Consider the linear mapping:

$$y_{\text{new}}(t) = \alpha\, y_{\text{norm}}(t) + \beta \qquad (6)$$

Determine the values of α and β, so that the max and the min of $y_{\text{new}}(t)$ will be y_{max} and y_{min}, respectively.

(a) Test this idea in MATLAB by doing the following example (where $\alpha = 5$ and $\beta = 3$):

```
ynorm = 0.5 + 0.5*sin( pi*[0:0.01:1]);
subplot(2,1,1),  plot(ynorm)
alpha = 5;
beta = 3;
ynew = alpha*ynorm + beta;          %<------ Linear re-scaling
subplot(2,1,2),  plot(ynew)
max(ynorm),  min(ynorm)      %<--- ECHO the values
max(ynew),  min(ynew)
```

(b) Explain what happens if we make α negative.

(c) Write a short one-line function that implements the scaling in (6) above. Your function should have the following form:

```
function ynew = lin_scale(data, alpha, beta).
```

4.2 Clarinet Envelopes

For the clarinet sound, the amplitude $A(t)$ needs no scaling—the MATLAB function `sound` will automatically scale to the maximum range of the D/A converter. Thus, $A(t)$ can be equal to the vector `y1`. From the plot of `y1` shown in Fig. 1, it should be obvious that this envelope will cause the sound to rise quickly to a certain volume, sustain that volume, and then quickly turn off.

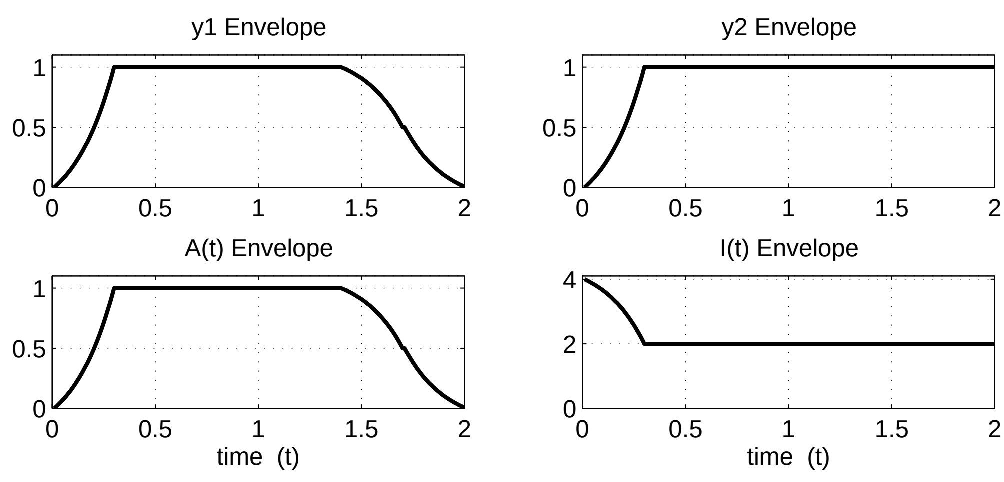

Figure 1: Envelopes for the woodwinds. The functions $A(t)$ and $I(t)$ are produced by scaling `y1` and `y2`, the outputs of `woodwenv`.

The modulation index envelope, $I(t)$, however, does not equal `y2`. The range for $I(t)$ lies between 2 and 4 as in Fig. 1. Furthermore, there is an inversion so that when `y2` is zero, $I(t)$ should equal 4, and when `y2` is one, $I(t)$ should be 2. Using this information solve for the appropriate α and β then use `lin_scale` to produce the modulation index envelope function (`I`) for a clarinet sound.

4.3 Parameters for the Clarinet

So far we have a general equation for FM signals, an amplitude envelope for the clarinet, and a modulation index envelope for the clarinet. To create the actual sound signal for the clarinet, we need to specify the additional parameters in (4). The ratio of carrier to modulating frequency is important in creating the sound of a specific instrument. For the clarinet, this ratio should be 3:2. The actual note frequency will be the greatest common divisor of the carrier and modulating frequencies. For example, when we choose $f_c = 900$ Hz and $f_m = 600$ Hz, the synthesized signal will have a fundamental frequency of $f_0 = 300$ Hz. Consult Chowning's paper for more details.

Write a simple M-file `clarinet.m` that implements the FM synthesis equation (4) to synthesize a clarinet note. Your function should generate the envelopes $A(t)$ and $I(t)$ using calls to `woodwenv` and `lin_scale`. The function header should look like this:

```
function  yy = clarinet(f0, dur, fsamp)
%CLARINET     produce a clarinet note signal
%
%     usage:  yy = clarinet(f0, dur, fsamp)
%
%     where:   f0 = note frequency
%             dur = the amount of time the note lasts
%           fsamp = the sampling rate
```

4.4 Using the Clarinet Sound

4.4.1 One Clarinet Note

Once you have a working `clarinet( )` function, create a 440-Hz clarinet note with $f_s = 11025$ Hz. Play it with the `soundsc( )` function. Does it sound like a clarinet? Experiment with the durations of the attack, sustain and release segments of the sound to obtain the best possible sound. Make three notes with different durations: 0.2 secs, 0.4 secs, and 0.8 secs. Explain how your function adapts to different values of the duration `(dur)`. Explain how can you verify that the note frequency is at 440 Hz (perhaps you can measure the period in the middle of the signal and see if it matches the note frequency).

Explain how the modulation index $I(t)$ will affect the frequency content versus time of the clarinet sound. Describe how you can hear the frequency content changing according to $I(t)$? Plot the instantaneous frequency $f_i(t)$ versus t for comparison. Plot the entire signal and compare it to the amplitude envelope function `y1` in Fig. 1.

4.4.2 C-Major Scale

Read Section 3.7 in the textbook to learn more about the spectrogram and its relationship to music. Then use the **`clarinet`** function to synthesize the eight notes that make up the C-major scale (defined in Fig. 3.23 of *SP-First*) and play the notes in succession so that it sounds like a scale. Pick parameters for the "clarinet" so that each note has the correct frequency and lasts exactly 0.8 seconds. Put 0.1 seconds of silence between successive notes. Use a sampling frequency of $f_s = 11025$ Hz to avoid aliasing.
Show a spectrogram of the entire scale to justify that you generated the correct notes.

4.4.3 (Optional) Make Music

Use your clarinet function to synthesize part of a song. Pick a popular tune, e.g., the well-known passage from Glenn Miller's *In The Mood* or the intro from Gershwin's *Rhapsody in Blue*, and synthesize a a short section. Tweak the FM parameters to make the music sound "realistic." *Be creative.*

Lab #4
ECE-2025
Spring-2005
INSTRUCTOR VERIFICATION SHEET

Turn this page in to your TA before the end of your lab period.

Name: ____ Date of Lab: __________

Part 3: Write the MATLAB function **`fmsinus.m`**

Verified:____________________ Date/Time:__________

Part 3.1: Slowly Varying Sinusoidal FM Signal Generation. Sketch the instantaneous frequency:

Verified:____________________ Date/Time:__________

Part 3.2: Wideband Sinusoidal FM Signal Generation. Show a spectrogram and discuss its qualitative features:

Verified:____________________ Date/Time:__________