GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2005
Lab #9: (B) PeZ - The z, n, and $\hat{\omega}$ Domains

Date: 30-Mar. – 5-Apr. 2005

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

This second part of Lab #9 is worth 100 points; the first part of Lab #9 was 50 pts.
The lab report for this week will be an **Informal Lab Report.** The report will be **due during the period 6–12 April at the start of your lab.**

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1 Introduction

In this part of the lab, you will use **PeZ** to create filters with complex conjugate poles and zeros. These are called *second-order filters* because the denominator polynomial is a quadratic with two roots.

2 PreLab

2.1 PeZ: Introduction

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain), A graphical user interface (GUI) called **PeZ** was written in MATLAB for doing interactive explorations of the three domains.[1] **PeZ** is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\,\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{\ell=1}^{N}(1 - p_\ell z^{-1})} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{\ell=1}^{N} a_\ell z^{-\ell}} \tag{1}$$

To run **PeZ**, type `pezdemo` at the command prompt and you will see the GUI shown in Fig. 1.[2]

2.1.1 Controls for PeZ using `pezdemo`

The **PeZ** GUI is controlled by the `Pole-Zero Plot` where the user can add (or delete) poles and zeros, as well as move them around with the pointing device. For example, Fig. 1 shows a case where two (complex-

[1] The original **PeZ** was written by Craig Ulmer; a later version by Koon Kong is the one that we will use in this lab. Recent modifications by Greg Krudysz have added new features such as movie-making capability.

[2] The command `pez` will invoke the older version of **PeZ** which is distinguished by a black background in all the plot regions.

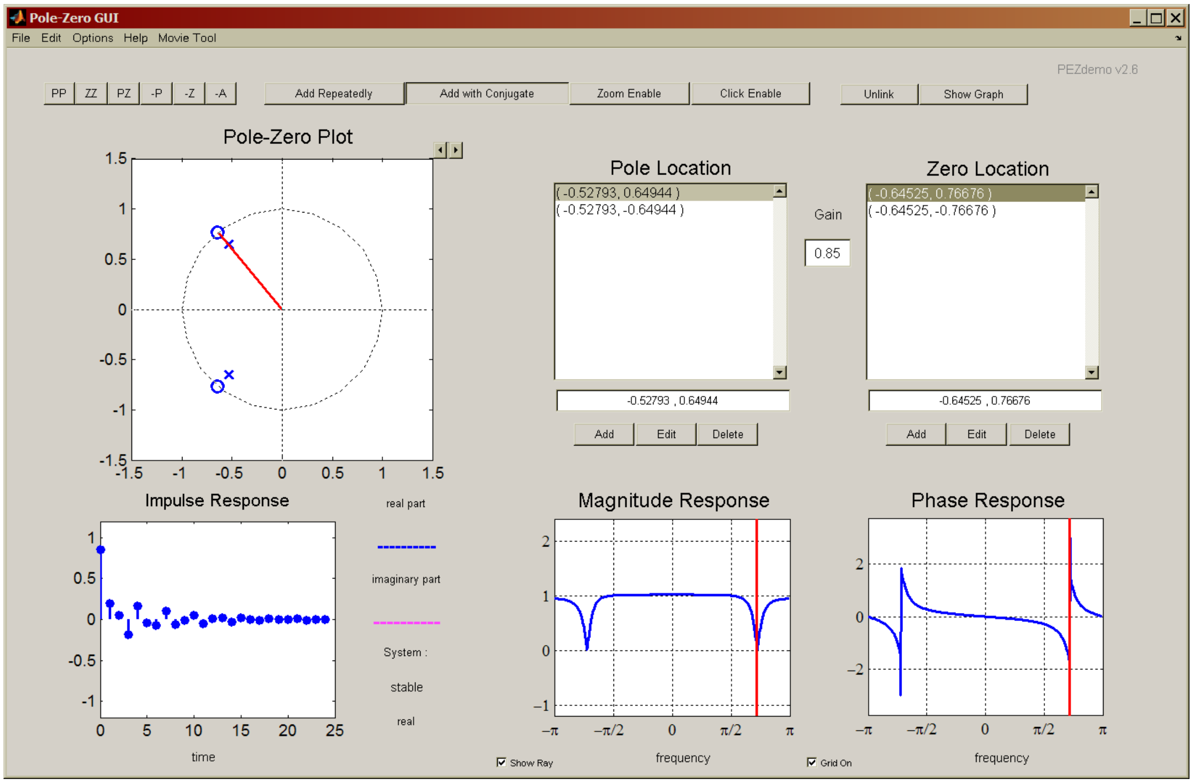

Figure 1: GUI interface for `pezdemo` showing a second-order filter. Pole and zero locations are given in rectangular coordinates. The "Gain" was adjusted to $G = 0.85$ in order to make the frequency response equal to one in the passbands.

conjugate) poles have been added, along with two (complex-conjugate) zeros on the unit circle. The buttons named `PP` and `ZZ` were used to add these poles and zeros. By default, the `Add with Conjugate` property is turned on, so poles and zeros are typically added in pairs to satisfy the complex-conjugate property:

A polynomial with real coefficients has roots that are real, or occur in complex-conjugate pairs.

To learn about the other controls in `pezdemo`, access the menu item called "Help" for extensive information about all the **PeZ** controls and menus. Here are a few things to try. You can use the `Pole-Zero Plot` to selectively place poles and zeros in the z-plane, and then observe (in the other plots) how their placement affects the impulse and frequency responses. In **PeZ** an individual pole/zero pair can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero). The red ray in the z-domain window is tied to the red vertical lines on the frequency responses, and they move together. This helps identify frequency domain features that are caused by pole locations or zero locations, because the angle around the unit circle corresponds to frequency $\hat{\omega}$. Since exact placement of poles and zeros with the mouse is difficult, an `Edit` button is provided for numerical entry of the real and imaginary parts. Before you can edit a pole or zero, however, you must first select it in the list of `Pole Locations` or `Zero Locations`. Removal of individual poles or zeros can also be performed by using the `-P` or `-Z` buttons, or with the `Delete` button. Note that all poles and/or zeros can be easily cleared by clicking on the `-A` button.

2.1.2 Create an IIR Filter with PeZ

Use the **PeZ** interface to implement the following second-order system:

$$H(z) = \frac{1 - z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the z-plane. First try placing the poles and zeros with the mouse, and then use the `Edit` feature to get exact locations. Since **PeZ** wants to add complex-conjugate pairs, you should only have to add one of the poles; for the zeros, the `Add with Conjugate` feature should be turned off because you will be adding two real-valued zeros.

Look at the frequency response and determine what kind of filter you have.

2.2 Not Bandpass Filters

It is tempting to think that with two poles the frequency response ends up always having a peak, but there are two interesting cases where that doesn't happen: (1) all-pass filters where $|H(e^{j\hat{\omega}})| = \text{constant}$, and (2) IIR notch filters that null out one frequency, but are relatively flat across the rest of the frequency band.

• Implement the following second-order system:

$$H(z) = \frac{64 + 80z^{-1} + 100z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

by determining where the two poles and two zeros are located and then placing the poles and zeros at the correct locations in the z-plane.

⇒ Look at the frequency response and determine what kind of filter you have.[3]

• Now, use the mouse to "grab" the zero-pair and move the zeros to be exactly on the unit-circle at the same angle as the poles. Observe how the frequency response changes. In addition, determine the $H(z)$ for this filter.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

⇒ Describe the type of filter that you have now created.

3 Warm-up

3.1 Relationships between z, n, and $\hat{\omega}$ domains

The lab verification requires that you write down your observations on the verification sheet when using the PeZ GUI. These written observations will be graded.

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

- How does $h[n]$ change with respect to its rate of decay? For example, when $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when a is smaller.
- If $h[n]$ exhibits an oscillating component, what is the period of oscillation? Also, estimate the decay rate of the "envelope" that overlays the oscillation. How are the period and decay rate related to the pole location?
- How does $|H(e^{j\hat{\omega}})|$ change with respect to peak location and peak width?
- If $|H(e^{j\hat{\omega}})|$ doesn't have a peak, is it a notch filter or an all-pass filter ?

Note: review the "Three-Domains - FIR" under the Demos link for Chapter 7 and "Three-Domains - IIR" under the Demos link for Chapter 8 for movies and examples of these relationships.

[3] The relationship between the poles and zeros of an all-pass filter is zero = 1/(pole)*; this situation where two poles and two zeros are linked together can be done with the `PZ` option in **PeZ**.

3.2 Complex Poles and Zeros

PeZ assumes real coefficients for the numerator and denominator polynomials when the `Add with Conjugate` mode is on (which it is by default). Therefore, if we enter a complex pole or zero, **PeZ** will automatically insert a second root at the conjugate location, i.e., $z = \frac{1}{3} + j\frac{1}{2}$ would be accompanied by $z = \frac{1}{3} - j\frac{1}{2}$.

(a) State the property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ that will guarantee that the two roots of $A(z)$ are either both real, or are complex conjugates of each other.

(b) Clear all the poles and zeros from **PeZ**. Now place a pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Determine the radius and angle of the poles. Note that **PeZ** automatically places a conjugate pole in the z-domain. The frequency response has a peak—record the frequency (location) of this peak.

(c) Change the angle of the pole and observe $h[n]$ and $H(e^{j\hat{\omega}})$: move the pole-pair angles to $(\pm 90^\circ)$, then to $(\pm 60^\circ)$ and $(\pm 45^\circ)$. Describe the changes in $|H(e^{j\hat{\omega}})|$. Concentrate on the height and location of the peak versus frequency $\hat{\omega}$.

(d) Start again with the pole pair at $-0.3 \pm j0.8$, and zeros at $z = \pm 1$. Decrease the radial distance of the poles from the origin (by dragging), e.g., try $-0.2 \pm j0.533$, and then $-0.1 \pm j0.266$. If you use the `Pole Location` edit window to change the values, the two poles will be "unlinked" and you will have to edit them separately. Therefore, dragging is a more informative way to do this even though it's less precise. Describe the changes in both $h[n]$ and $|H(e^{j\hat{\omega}})|$, as you reduce the pole radius.

(e) Increase the magnitude of the poles by pushing them closer to the unit circle, and then move the poles outside the unit circle. When the pole-pair is outside the unit circle, describe what happens to $h[n]$.

(f) Clear all the poles and zeros from **PeZ**, using the `-A` button. Then place two poles at $-0.3 \pm j0.8$ again, and place two zeros exactly on the unit circle at the same angles as the poles. Judging from the frequency response what type of filter have you just implemented? Is the system IIR or FIR?

(g) Write out the expression for $H(z)$ created in part (f). *Hint:* use MATLAB's `poly` function.

(h) Once again clear all the zeros from **PeZ** keeping the two poles at $-0.3 \pm j0.8$. Now place zeros at $\frac{1}{-0.3 \pm j0.8}$. It would be helpful to get the polar representation of these complex-valued zeros. Judging from the frequency response what type of filter have you just implemented? Is the system IIR or FIR?

(i) Write out the expression for $H(z)$ created in part (h).

4 Bandpass Filter Design for IIR

It is easy to design a narrow passband IIR filter by putting a complex pole-pair near the unit circle.

4.1 Complex Poles

The first exercise is to move one pole-pair around and obtain formulas for how the frequency response changes as a function of the pole-pair radius and angle.

(a) Place a single pole-pair at $z = 0.9e^{\pm j\pi/3}$, and zeros at $z = \pm 1$. Then determine the coefficients of the numerator and denominator of the resulting $H(z)$.

(b) Make a plot of the frequency response (magnitude only) with `freqz` and measure the width of the peak versus frequency. This presents a problem because we must define how to measure width. The usual definition is to measure the width at the "3-dB level." In order to do this, the measurement must be made with respect to the peak value of the frequency response. If the peak value is $H_{\max}$, then the "3-dB level" is at $0.707 H_{\max}$.[4]

(c) Move the pole-pair so that the angles remain fixed at $\pm\pi/3$, but the radius is $r = 0.95$ and $r = 0.975$. In each case, measure the 3-db width of the peak. Using these measured values, create a formula for the width that is proportional to $(1 - r)$, e.g., the following works quite well

$$\text{PeakWidth} \approx K(1 - r)/\sqrt{r}$$

where K is a constant of proportionality.

(d) Move the pole-pair so that its radius remains fixed and the angles change from $\pm\pi/3$ to $\pm\pi/4$ and then to $\pm\pi/2$. State a formula for the peak location as a function of the pole location.

4.2 Passband and Stopband

We can characterize general bandpass filters if we define the passband width to be equal to the 3-dB width (as in the previous section). We also need a a definition for the stopband, and we will arbitrarily define the stopbands of a BPF to be those regions where the frequency response (magnitude) is below -20 db, which is equivalent to 10% of the peak value.

(a) Determine the stopband regions for three of the filters designed in the previous section. Use the cases where the pole angles are $\pm\pi/3$ and the radii are $r = 0.9$, 0.95 and 0.975. In each case, measure the frequency regions of the two stopbands. There is one lower stopband for $0 \le \hat{\omega} \le \hat{\omega}_{s1}$ and one upper stopband for $\hat{\omega}_{s2} \le \hat{\omega} \le \pi$.

(b) For the same three filters, record the passband edges. The passband will be the peak width at the 3-dB level, so it will occupy a region such as $\hat{\omega}_{p1} \le \hat{\omega} \le \hat{\omega}_{p2}$, where $\hat{\omega}_{p1}$ and $\hat{\omega}_{p2}$ are the band edges.

(c) Usually, filter design becomes difficult when we want the passband and stoband edges to be very close to one another. The difference between neighboring passband and stopband edges is called the *Transition Width.* Therefore, summarize the measurements of the previous two parts in a table that lists the two transition widths for each filter versus r. Can you state a simple formula for the transition width? Does it depend on r?

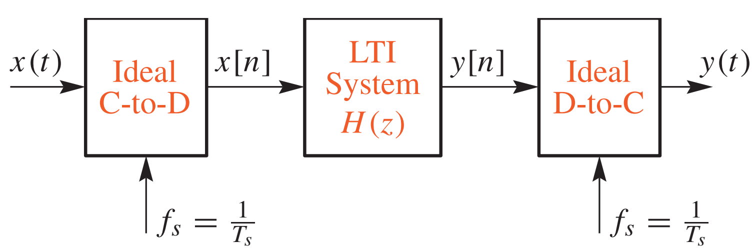

Figure 2: Filtering the analog signal $x(t)$ with a digital filter.

4.2.1 Design a Filter Based on Analog Frequencies

One last question that relates to your understanding of sampling as well as digital filtering. Design a BPF whose passband is $2000 \le f \le 2200$ Hz when the sampling rate is 8000 Hz using the IIR method above, i.e., determine the value of r. As a reminder, you are designing a digital filter to be used as the system $H(z)$ in Fig. 2. Once you have the filter, determine its stopbands and give the stopband edges in hertz.

[4] The frequency response is often plotted on a logarithmic scale using decibels, i.e., $20\log_{10}|H(e^{j\omega})|$. If you compute $20\log_{10}(1/\sqrt{2})$ you get -3.01 dB, and $1/\sqrt{2} \approx 0.707$.

Lab #9 (B)
ECE-2025 Spring-2005
WORKSHEET & VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ____ Date of Lab: __________

Part	Observations from **PeZ**
3.2(a)	
3.2(b)	
3.2(c)	
3.2(d)	
3.2(e)	
3.2(f)	
3.2(g)	
3.2(h)	
3.2(i)	