GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #1

Assigned: 6-Jan-03
Due Date: Week of 13-Jan-03

Reading: In *SP First* notes: App. A on *Complex Numbers*, pp. 430–451; and Ch. 2 on *Sinusoids*, pp. 8–43. In *DSP First*, Appendix A is pp. 378–398; and Ch. 2 is pp. 9–43.

The web site for the course uses Web-CT: `webct.gatech.edu`
The login for Web-CT is your PRISM login `(gtxxxx)`.

$\Longrightarrow$ Please check the "Bulletin Board" daily. All official course announcements will be posted there.

ALL of the **STARRED** problems should be turned in for grading.

Some of the problems have solutions that are similar to those found on the CD-ROM or under "Word" on WebCT. After this assignment is handed in by everyone, a solution to all the starred problems will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

A complex number is just an ordered pair of real numbers. Several different mathematical notations can be used to represent complex numbers. In *rectangular form* we will use all of the following notations:

$$\begin{aligned} z &= (x, y) \\ &= x + jy \qquad\qquad \text{where } j = \sqrt{-1} \\ &= \Re e\{z\} + j\Im m\{z\} \end{aligned}$$

Note that $i = \sqrt{-1}$ in most math courses. The pair (x, y) can be drawn as a vector, such that x is the horizontal coordinate and y the vertical coordinate in a two-dimensional space. Addition of complex numbers is the same as vector addition; i.e., add the real parts and add the imaginary parts.

In *polar form* we will use these notations:

$$\begin{aligned} z &= |z|e^{j \arg z} \\ &= re^{j\theta} \\ &= r\angle\theta \end{aligned}$$

where $|z| = r = \sqrt{x^2 + y^2}$ and $\arg z = \theta = \arctan(y/x)$. In a vector drawing, r is the length and θ the direction of the vector.

Euler's Formula:

$$re^{j\theta} = r\cos\theta + j\,r\sin\theta$$

can be used to convert between Cartesian and polar forms.

Some of these problems should be a review of complex numbers learned in high school. In these problems you will manipulate some complex numbers. A calculator will be useful for this purpose, especially if it is one with complex arithmetic capability. It is convenient to learn how to use this feature. However, it is also worthwhile to be able to do the calculations by hand; i.e., it is important to *understand* what your calculator is doing!

PROBLEM 1.1*:
Convert the following to polar form:

(a) $z = -20$ (b) $z = 1 - j\sqrt{3}$ (c) $z = (-4, 4)$
(d) $z = -33 - j33$ (e) $z = -3 + j4$ (f) $z = -j8$

Give numerical values for the magnitude, and the angle (or phase) in radians.

PROBLEM 1.2*:
Convert the following to rectangular form (by using Euler's formula):

(a) $z = 3\,e^{-j(5\pi/4)}$ (c) $z = 10\,\angle\,(-5\pi/6)$
(b) $z = 8e^{-j(3\pi/2)}$ (d) $z = \sqrt{3}\,\angle\,(31\pi)$

Give numerical values for the real and imaginary parts.

PROBLEM 1.3*:
Evaluate the following and give the answer in both rectangular and polar form. In all cases, assume that the complex numbers are $z_1 = -5 + j5$ and $z_2 = 5\sqrt{2}e^{-j(7\pi/2)}$.

(a) Conjugate: z_1^* (d) z_2^2 (g) $z_1 + z_2^*$
(b) jz_2 (e) $z_1^{-1} = 1/z_1$ (h) $|z_2|^2 = z_2 z_2^*$
(c) z_2/z_1 (f) $z_1 z_2$ (i) $z_2 + z_2^*$

Note: z^* means the "conjugate" of z. Part (h) is the *magnitude-squared.*

PROBLEM 1.4*:
Plot two periods of the following sinusoids with $t = 0$ in the middle (i.e., for $-T \le t \le T$):

(a) $x(t) = 7\cos(100\pi t)$

(b) $x(t) = 3\cos(100\pi t - 0.25\pi)$

(c) $x(t) = 100\cos(0.2\pi t + 11.4\pi)$

PROBLEM 1.5*:

The waveform in the following figure (generated from the MATLAB GUI `sindrill`) can be expressed as

$$x(t) = A\cos[\omega_0(t - t_d)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_d, and ϕ. Choose the value of ϕ such that $-\pi < \phi \le \pi$.