GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #4

Assigned: 24-Jan-03
Due Date: Week of 3-Feb-03

Quiz #1 will be held in lecture on Friday 31-Jan-03. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2 and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP First*, Chapter 3: *Spectrum Representation*, all.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

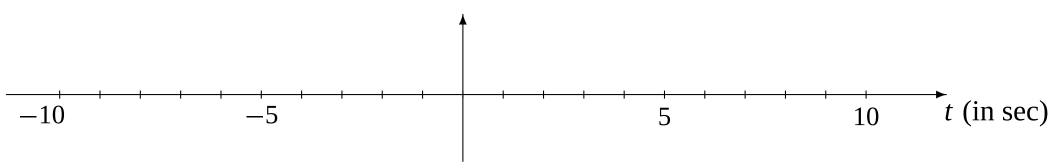
ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 4.1*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} t & \text{for } 0 \le t \le 1 \\ 0 & \text{for } 1 < t < 7 \end{cases}$

(a) Assume that the period of $x(t)$ is 7 sec. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ sec.



(b) Determine the DC value of $x(t)$ from the Fourier series integral.

(c) Write the Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. Set up all the specifics of the integral (e.g., limits of integration, integrand), but do not evaluate the integral.

PROBLEM 4.2*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. Determine the mathematical formula for a chirp whose instantaneous frequency sweeps from $f_1 = 8000$ Hz down to $f_2 = 1000$ Hz as time goes from $t = 0$ to $t = 1.5$ sec.

PROBLEM 4.3*:

A signal composed of sinusoids is given by the equation

$$x(t) = 10\cos(1.2\pi t + 0.2\pi) + 15\cos(2.8\pi(t - 0.2)) - 5$$

(a) Is $x(t)$ periodic? If so, what is the smallest period?

(b) Determine the fundamental frequency of the signal.

(c) Determine the DC value of the signal.

PROBLEM 4.4*:

We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. In musical notation the tones are called notes; the names of the notes in the octave starting with middle-C and ending with high-C are:

note name	C	$C^\#$	D	$E^\flat$	E	F	$F^\#$	G	$G^\#$	A	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

PROBLEM 4.5*:

The following MATLAB program makes a plot of the *amplitude modulated* signal that is “cosine-times-sine.” (Actually the plot is of a finite time segment of the signal.)

```
tt = -1:0.01:1;
xx = cos(20*pi*tt) .* sin(2*pi*tt);
plot(tt,xx)
```

(a) Make a *sketch* of the plot that will be done by MATLAB. Label the time axis carefully.
Note: this can be done *without* running the MATLAB commands.

(b) The “spectrum” diagram gives the frequency content of a signal. Draw a sketch of the spectrum of the signal represented by `xx`. Label the frequencies and the complex amplitudes of each component.