GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #5

Assigned: 31-Jan-03
Due Date: Week of 10-Feb-03

Reading: In *SP First*, Chapter 3: *Spectrum Representation*, Sections 3-4, 3-5 and 3-6.
Start reading Chapter 4: *Sampling and Aliasing.*

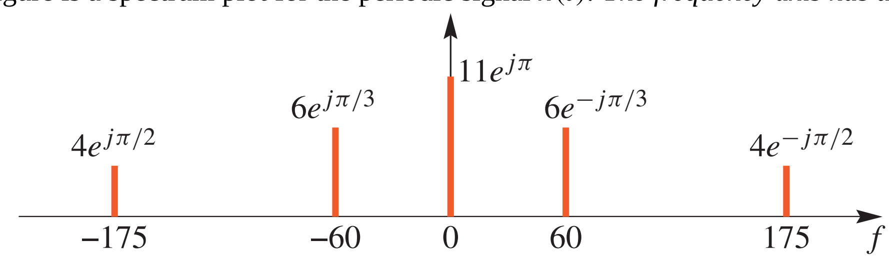
$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

PROBLEM 5.1*:

Shown in the figure is a spectrum plot for the periodic signal $x(t)$. *The frequency axis has units of Hz.*



(a) Determine the period T_0 of $x(t)$.

(b) Determine the fundamental frequency ω_0 of this signal.

(c) Determine the DC value of this signal.

(d) A periodic signal of this type can be represented as a Fourier series of the form

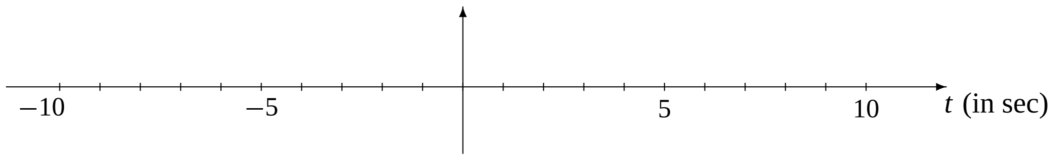
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}.$$

If the Fourier series coefficients of $x(t)$ are denoted by a_k, $\;k = 0, \pm1, \pm2, \pm3, \ldots,$ determine which coefficients have non-zero value. List these Fourier series coefficients and their values in a table.

PROBLEM 5.2*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 35 & \text{for } 0 \le t \le 3 \\ 0 & \text{for } 3 < t < 7 \end{cases}$

(a) Assume that the period of $x(t)$ is 7 sec. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ sec.



(b) Determine the DC value of $x(t)$ from the Fourier series integral.

(c) Determine a general expression for the Fourier series coefficients a_k.

(d) Make a spectrum plot of this signal showing the frequency range $-\frac{1}{2} < f < \frac{1}{2}$ Hz.

PROBLEM 5.3*:

A signal $x(t)$ is periodic with period $T_0 = 3$. Therefore, it can be represented as a Fourier series of the form

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/3)kt}$$

It is known that the Fourier series coefficients for this representation of a particular signal $x(t)$ are given by the integral

$$a_k = \frac{1}{3}\int_{-1}^{0} e^{2t} e^{-j(2\pi/3)kt} dt$$

(a) In the expression for a_k above, the integral and its limits effectively define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

(b) Using your result from part (a), draw a plot of $x(t)$ over the range $-4 \le t \le 4$ seconds. Label it carefully.

(c) Determine a_0, the DC value of $x(t)$ found in part (a).

PROBLEM 5.4*:

A periodic signal $x(t)$ is described over one period $-2 \le t < 2$ by the equation

$$x(t) = e^{-|t|/2} \qquad \text{for } -2 \le t < 2$$

The period of this signal is $T_0 = 4$ sec.

(a) Sketch the periodic function $x(t)$ for $-6 \le t < 6$.

(b) Determine a_0, the DC coefficient for the Fourier series.

(c) Set up the *Fourier analysis* integral for determining a_k for $k \neq 0$. (Insert proper limits and integrand.)

(d) Evaluate the integral in part (c) and obtain an expression for a_k that is valid for all $k \neq 0$.

(e) Make a plot of the spectrum over the range $-3\omega_0 \le \omega \le 3\omega_0$, where ω_0 is the fundamental frequency of the signal in rad/s. Use MATLAB or a calculator to determine the complex numerical values (in polar form) for each of the Fourier coefficients corresponding to this range of frequencies.

PROBLEM 5.5*:

The periodic signal $x(t)$ is described over one period $-2 \le t < 2$ by the equation

$$x(t) = e^{-|t|/2} \qquad \text{for } -2 \le t < 2$$

can be represented by the Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(\pi/2)kt}$$

(a) If we add a constant value of five to $x(t)$, we obtain a new signal $w(t) = 5 + x(t)$. Make a plot of the periodic signal $w(t)$ over the time interval $-6 \le t < 6$.

(b) The new signal $w(t)$ can also be represented by a Fourier series, $w(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$, because it is periodic with period T_0. Explain how b_0 and b_k are related to a_0 and a_k.
Hint: You should not have to evaluate any new integrals explicitly to answer this question.

(c) Sketch the waveform of another new signal $y(t) = 5x(t-2)$ over the time interval $-6 \le t < 6$.

(d) Determine the spectrum for the signal $y(t)$ defined in part (c), and make a plot of its spectrum over the range $-3\omega_0 \le \omega \le 3\omega_0$, where ω_0 is the fundamental frequency of the signal in rad/s. Label all the frequencies and complex amplitudes in the spectrum.
Hint: If you denote the coefficients in the Fourier series for $y(t)$ as c_k, then

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

By substituting the Fourier series expansion for $x(t)$ into the definition of $y(t)$, you should be able to find a simple relationship between c_k and a_k, the Fourier series coefficients of $x(t)$.