GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #6

Assigned: 7-Feb-03
Due Date: Week of 17-Feb-03

Reading: In *SP First*, Chapter 4: *Sampling and Aliasing*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.
Please follow the format guidelines (cover page, etc.) for homework.

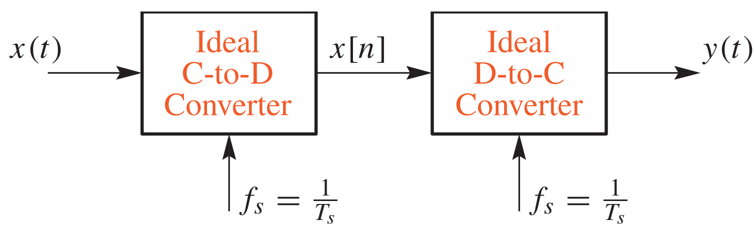


Figure 1: Ideal sampling and reconstruction systems. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$.

PROBLEM 6.1*:

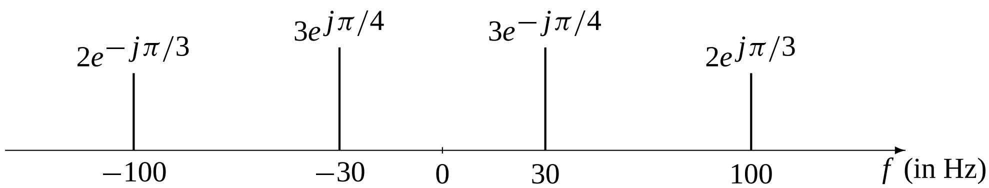
Consider the ideal sampling and reconstruction system shown in Fig. 1.

(a) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = 7\cos(0.5\pi n + \pi/4)$$

If the sampling rate of the C-to-D converter is $f_s = 400$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency between 400 and 800 Hz; i.e., find $x_1(t) = A_1\cos(\omega_1 t + \phi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/400$ secs.

(b) Now if the input $x(t)$ to the system in Fig. 1 has the two-sided spectrum representation shown below, what is the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$?



(c) Using the signal $x(t)$ from part (b), determine the spectrum for $x[n]$ when $f_s = 400$ samples/sec. Make a plot for your answer, but label the frequency, and complex amplitude (magnitude and phase) of each spectral component.

PROBLEM 6.2*:

Assume that $x(t)$ is the input to an ideal C-to-D converter; $x[n]$ is its output, and $y(t)$ is the output of an ideal C-to-C converter when $x[n]$ is the input (as in Fig. 1).

(a) Suppose that the input $x(t)$ is given by

$$x(t) = 3 + 4\cos(2\pi(2000)t - \pi) + 5\cos(2\pi(7000)t - 3\pi/4)$$

Determine the spectrum for $x[n]$ when $f_s = 8000$ samples/sec. Make a plot for your answer, making sure to label the frequency, amplitude and phase of each spectral component.

(b) Using the discrete-time spectrum for $x[n]$ from part (a), determine the analog frequency components in the spectrum of the output $y(t)$ when the sampling rate of the D-to-C converter is $f_s = 8000$ Hz.

(c) It is possible to choose a sampling rate so that the output is a constant. Determine the *largest* value of f_s for which $y(t)$ will be a constant. Furthermore, determine the numerical value of the constant.

PROBLEM 6.3*:

Chirps are very useful signals for probing the behavior of sampling and reconstruction systems (such as Fig. 1). In particular, a linear-FM chirp can exhibit the "folding" type of aliasing.

(a) Suppose that the input signal is a chirp signal defined as follows:

$$x(t) = \cos(-4000\pi t^2) \qquad \text{for } 0 \le t \le 5 \text{ sec.}$$

If the sampling rate is $f_s = 4000$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction**.
Hint: this could be done in MATLAB by putting a sampled chirp signal into the MATLAB function `specgram()`, or the *SP-First* function `plotspec()`.

(b) Suppose that the input signal is a chirp signal defined as follows:

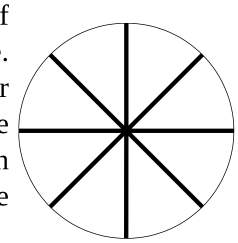
$$x(t) = \cos(4000\pi t + 200\sin(6\pi t)) \qquad \text{for } 0 \le t \le 1 \text{ sec.}$$

If the sampling rate is $f_s = 4000$ Hz, then draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction.**

PROBLEM 6.4*:

When watching old TV movies, all of us have seen the phenomenon where a wagon wheel appears to move backwards. The same illusion can also be seen in automobile commercials, when the car's hubcaps have a spoked pattern. Both of these are due to the 30 frames/sec sampling used in transmitting TV images.

In the figure to the right, an eight-spoked wheel is shown. Assume that the diameter of this wheel is two feet, which is almost exactly the tire diameter of a typical automobile. In addition, assume that the wheel is rotating CCW, so that if attached to a car, the car would be traveling to the left *at a constant speed.* However, when seen on TV the spoke pattern of the car wheel appears to stand still. How fast is the car traveling (in miles per hour)? Derive a general equation that will make it easy to give all possible answers.



PROBLEM 6.5*:

In all parts below, the sampling rates of the C/D and D/C converters are **equal,** and the input to the Ideal C/D converter is

$$x(t) = 2e^{j(44\pi t - \pi/3)} + 2e^{j(20\pi t + \pi/3)}$$

(a) If the sampling rate is $f_s = 16$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-\pi \le \hat{\omega} \le \pi$.

(b) Determine the output of the ideal D-to-C Converter for this input signal. Explain why the output $y(t)$ is a purely real signal, even though the input is complex.

PROBLEM 6.6:

Shown in Fig. 1 above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [10 + 10\cos(500\pi t - \pi/2)]\cos(2000\pi t)$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate f_s that can be used in the system of Fig. 1 so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 5000$. Your plot should include labels on the frequency axis ($\hat{\omega}$), as well as the amplitude and phase of each spectrum component.

PROBLEM 6.7:

A non-ideal D-to-C converter takes a sequence $y[n]$ as input and produces a continuous-time output $y(t)$ according to the relation

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

where $T_s = 0.1$ second. The input sequence is given by the formula

$$y[n] = \begin{cases} 1 - (.5)^n & 0 \le n \le 4 \\ (15/16)(.5)^{n-4} & 5 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $y[n]$ versus n.

(b) For the pulse shape

$$p(t) = \begin{cases} 1 & -0.05 \le t \le 0.05 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its nonzero region.

(c) For the pulse shape

$$p(t) = \begin{cases} 1 - 10|t| & -0.1 \le t \le 0.1 \\ 0 & \text{otherwise} \end{cases}$$

carefully sketch the output waveform $y(t)$ over its nonzero region.