GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2003
Problem Set #14

Assigned: 12-Apr-03
Due Date: 25-Apr-03

This Homework can be turned at the last lecture on ***Friday, 25-April before Noon,*** or earlier that week.

Final Exam will be given on 28-April at 2:50 PM. One page ($8\frac{1}{2} \times 11''$) of **handwritten** notes allowed.

Reading: In *SP First*, Chapter 11: *Continuous-Time Fourier Transform*
Chapter 12: *Filtering, Modulation and Sampling,* (applications of the Fourier Transform).

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

PROBLEM 14.1*:

In the following, either the time-domain signal $x(t)$ or the Fourier transform is given. Use the tables of Fourier transforms and Fourier transform properties to determine the Fourier transform for each of the signals or the inverse Fourier transform for each of the given Fourier transforms. You may give your answer either as an equation or a carefully labelled plot, whichever is most convenient.

(a) $x(t) = \frac{1}{2}u(t+2)u(2-t)$

(b) $x(t) = \dfrac{d}{dt}\left(\dfrac{\sin(20\pi t)}{10\pi t}\right)$

(c) $x(t) = \dfrac{\sin(20\pi t)}{10\pi t}\cos(100\pi t)$

(d) $X(j\omega) = \dfrac{e^{-j2\omega}}{3 + j4\omega}$

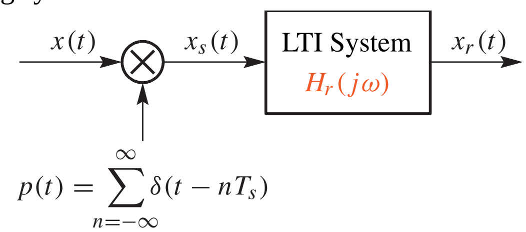
(e) $X(j\omega) = \dfrac{j2\omega}{3 + j4\omega}$

(f) $X(j\omega) = \displaystyle\sum_{k=-\infty}^{\infty} \pi\, e^{j\pi k}\delta(\omega - 20\pi k)$ ***(Make a plot of $x(t)$ obtained in this part.)***

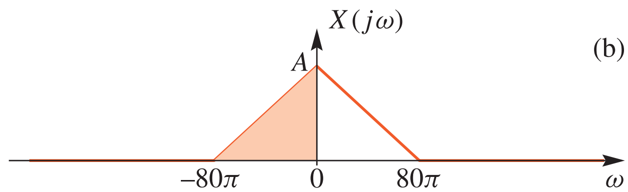
PROBLEM 14.2*:

The derivation of the Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:

(a)



(b)



A "typical" bandlimited Fourier transform of the input is also shown above.

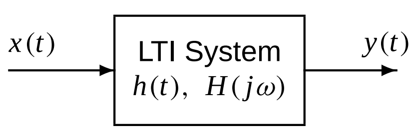
(a) For the input with Fourier transform depicted above, determine the smallest sampling rate $\omega_s = 2\pi/T_s$ so that $x_r(t) = x(t)$. Plot $X_s(j\omega)$ for the value of $\omega_s = 2\pi/T_s$ that is equal to the Nyquist rate.

(b) If $\omega_s = 2\pi/T_s = 120\pi$ in the above system and $X(j\omega)$ is as depicted above, plot the Fourier transform $X_s(j\omega)$ and show that aliasing occurs. There will be an infinite number of shifted copies of $X(j\omega)$, so indicate the general pattern versus ω.

(c) For the conditions of part (b), i.e., $T_s = 1/60$, determine and sketch the Fourier transform of the output, $X_r(j\omega)$, if the frequency response of the LTI system is

$$H_r(j\omega) = \begin{cases} T_s & |\omega| \le \pi/T_s \\ 0 & |\omega| > \pi/T_s \end{cases}$$

PROBLEM 14.3*:



The impulse response of the above system is $h(t) = \dfrac{10\sin(\omega_{co}(t-0.1))}{\pi(t-0.1)}$,

and the input to this system is a periodic signal (with period $T_0 = 2$) given by the following equation:

$$x(t) = \sum_{n=-\infty}^{\infty} \tfrac{1}{3}\delta(t-2n)$$

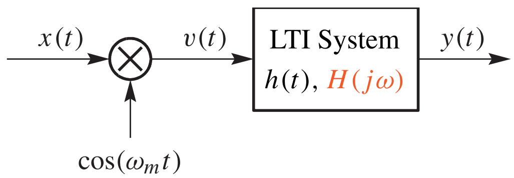
(a) Determine the Fourier transform $X(j\omega)$ of the input signal. Plot $X(j\omega)$ over the range $|\omega| < 6\pi$.

(b) For the case $\omega_{co} = 2.2\pi$, determine $H(j\omega)$ and plot $|H(j\omega)|$ on the same graph as $X(j\omega)$

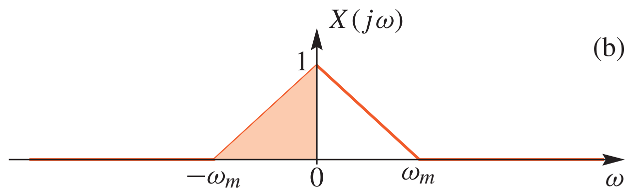
(c) For the case $\omega_{co} = 2.2\pi$, use the plot in (b) to determine $y(t)$, the output of the LTI system for the given input $x(t)$ above.

(d) Determine ω_{co} so that the output is a constant; i.e., $y(t) = C$ for all t, and determine the value of the constant C.

PROBLEM 14.4*:
The system below involves a multiplier followed by a filter:



(a)



(b)

The Fourier transform of the input is also shown above. For all parts below, assume that $\omega_m = 100\pi$.

(a) If the filter is an ideal HPF defined by

$$H(j\omega) = \begin{cases} 0 & |\omega| < 100\pi \\ 1 & |\omega| \geq 100\pi \end{cases}$$

Make a sketch of $Y(j\omega)$, the Fourier transform of the output $y(t)$ when the input is $X(j\omega)$ shown above.

(b) From the output, $Y(j\omega)$, found in the previous part, it is possible to "recover" the original input, $X(j\omega)$. Define a "signal recovery" system where the input is denoted by $w(t)$ and the output by $r(t)$. This system would consist of filters and mixers, and when its input is $W(j\omega) = Y(j\omega)$, its output would be $R(j\omega) = X(j\omega)$, where $X(j\omega)$ is the Fourier transform shown above and $Y(j\omega)$ is the Fourier transform determined in the previous part.

PROBLEM 14.5*:
The frequency response of an ideal lowpass LTI system is

$$H(j\omega) = \begin{cases} 7e^{-j\omega/25} & |\omega| < 20\pi \\ 0 & |\omega| > 20\pi \end{cases}$$

In each of the following cases, determine the Fourier transform of the input signal and then use frequency-domain methods to determine the corresponding output signal.

(a) Using frequency-domain methods, determine the output of the system when the input signal is

$$x(t) = \cos(15\pi t) + \frac{\sin(30\pi t)}{2\pi t}$$

(b) Determine the output if the input signal is

$$x(t) = \cos(15\pi t) + \frac{\sin(30\pi t)}{2\pi t} + \cos(50\pi t)$$

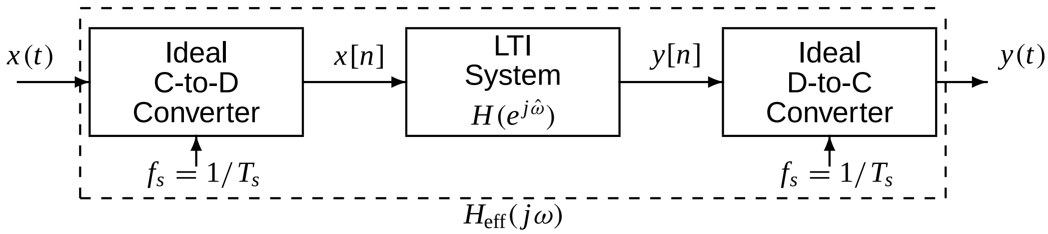
(c) Determine the output if the input signal is $x(t) = \cos(15\pi t) + \frac{1}{2}\delta(t)$

(d) Explain why the output can be the same, even though the input signals are different.

PROBLEM 14.6:

This type of problem has often appeared on the Final Exam.

Consider the following system for discrete-time filtering of a continuous-time signal:

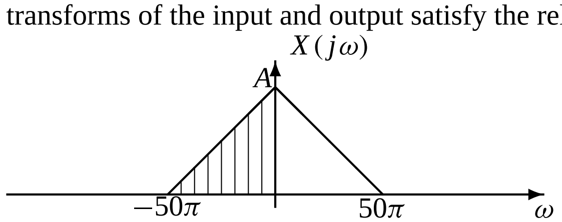


(a) Suppose that the discrete-time system is defined by the difference equation

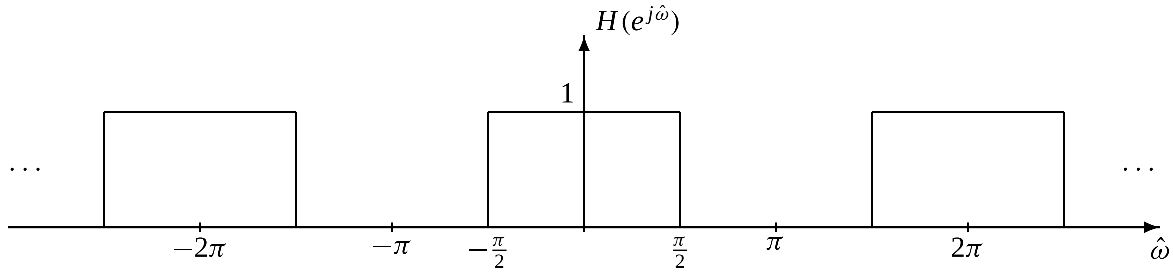
$$y[n] = 0.8y[n-1] + x[n] + x[n-2],$$

and the sampling rate of the input is $f_s = 200$ samples/second. Determine an expression for $H_{\text{eff}}(j\omega)$, the overall effective frequency response of the above system. Use this result to find the output $y(t)$ when the input to the overall system is $x(t) = 2\cos(100\pi t)$.

(b) Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that the Fourier transforms of the input and output satisfy the relation $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$?



(c) Assume that the discrete-time system has frequency response $H(e^{j\hat{\omega}})$ defined by the following plot:



Now, if $\boxed{f_s = 200 \text{ samples/sec}}$, make a carefully labeled plot of $H_{\text{eff}}(j\omega)$, the effective frequency response of the overall system. Also plot $Y(j\omega)$, the Fourier transform of the output $y(t)$, when the input has Fourier transform $X(j\omega)$ as depicted in the graph of part (b).

(d) For the input in part (b) and the system in part (c), what is the smallest sampling rate such that the input signal passes through the lowpass filter unaltered; i.e., what is the minimum f_s such that $Y(j\omega) = X(j\omega)$?