

Lecture 5

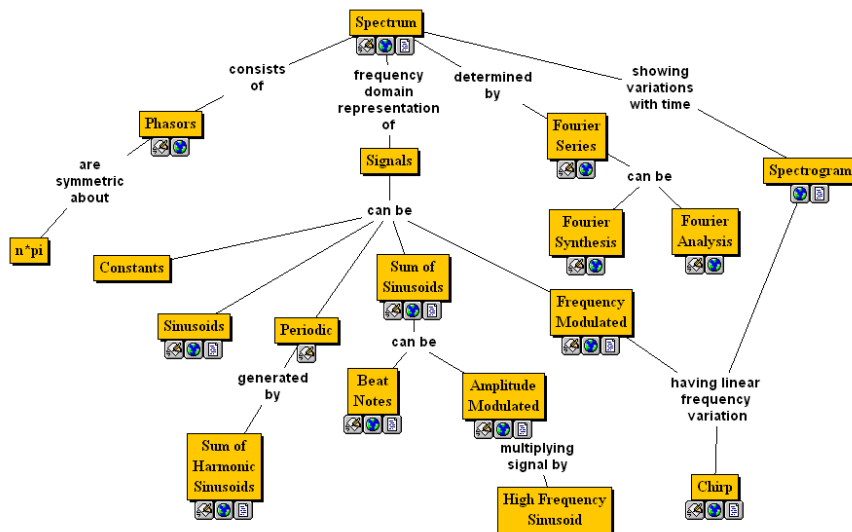
Periodic Signals, Harmonics & Time-Varying Sinusoids

27-Jan-06

General Information

- **Bulletin Board: OFFICIAL ANNOUNCEMENTS**
 - Old Quizzes & Problems are linked via WebCT
- **Quiz #1 on 10-Feb (Friday)**
 - Allowed one page of notes (Handwritten)
 - Calculators permitted
- HW #3 due NEXT WEEK in Recitation
 - Solutions will be posted Thurs evening
- Lab #2 Report
 - Turn in at beginning your lab time
 - Discuss lab report standards with your TA

Concept Map for Chapter 3

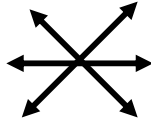


READING ASSIGNMENTS

- This Lecture:
 - Chapter 3, Sections 3-2 and 3-3
 - Chapter 3, Sections 3-7 and 3-8
- Next Lecture:
 - **Fourier Series ANALYSIS**
 - Sections 3-4, 3-5 and 3-6

Problem Solving Skills

- **Math Formula**
 - Sum of Cosines
 - Amp, Freq, Phase
- **Recorded Signals**
 - Speech
 - Music
 - No simple formula
- **Plot & Sketches**
 - S(t) versus t
 - Spectrum
- **MATLAB**
 - Numerical
 - Computation
 - Plotting list of numbers



LECTURE OBJECTIVES

- Signals with **HARMONIC** Frequencies

- Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi k f_0 t + \phi_k)$$

FREQUENCY can change vs. TIME

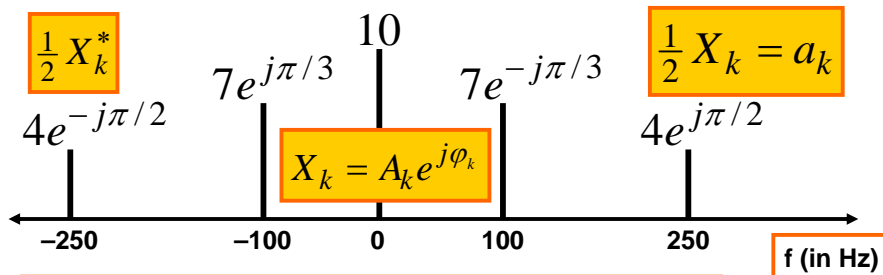
Chirps:

$$x(t) = \cos(\alpha t^2)$$

Introduce Spectrogram Visualization (`specgram.m`) (`plotspec.m`)

SPECTRUM DIAGRAM

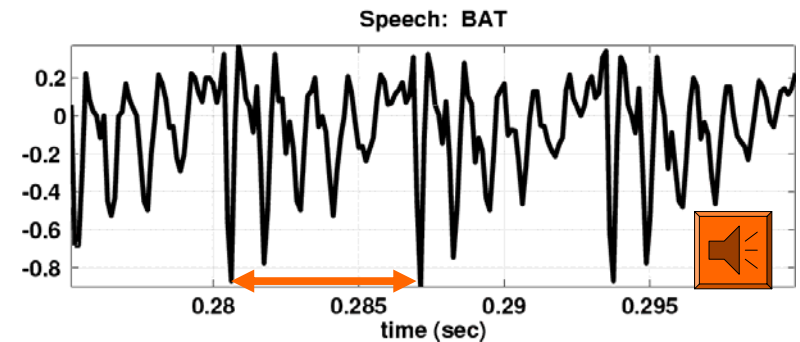
- Recall Complex Amplitude vs. Freq



$$x(t) = 10 + 14 \cos(2\pi(100)t - \pi/3) + 8 \cos(2\pi(250)t + \pi/2)$$

SPECTRUM for PERIODIC ?

- Nearly **Periodic** in the Vowel Region
 - Period is (Approximately) $T = 0.0065$ sec



PERIODIC SIGNALS

- Repeat every T secs

- Definition

$$x(t) = x(t + T)$$

- Example:

$$x(t) = \cos^2(3t)$$

$$T = ?$$

$$T = \frac{2\pi}{3} \quad T = \frac{\pi}{3}$$

- Speech can be “quasi-periodic”

Period of Complex Exponential

$$x(t) = e^{j\omega t}$$

$$x(t + T) = x(t) ?$$

Definition: Period is T

$$e^{j\omega(t+T)} = e^{j\omega t}$$

$$e^{j2\pi k} = 1$$

$$\Rightarrow e^{j\omega T} = 1 \Rightarrow \omega T = 2\pi k$$

$$\omega = \frac{2\pi k}{T} = \left(\frac{2\pi}{T}\right)k = \omega_0 k$$

k = integer

Harmonic Signal Spectrum

Periodic signal can only have: $f_k = k f_0$

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$X_k = A_k e^{j\varphi_k}$$

$$f_0 = \frac{1}{T}$$

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{1}{2} X_k e^{j2\pi k f_0 t} + \frac{1}{2} X_k^* e^{-j2\pi k f_0 t} \right\}$$

Define FUNDAMENTAL FREQ

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi k f_0 t + \varphi_k)$$

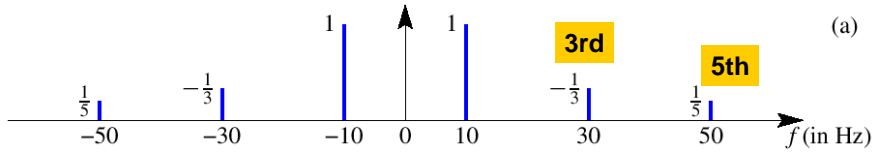
$$f_k = k f_0 \quad (\omega_0 = 2\pi f_0)$$

$$f_0 = \frac{1}{T_0}$$

f_0 = fundamental Frequency

T_0 = fundamental Period

Harmonic Signal (3 Freqs)

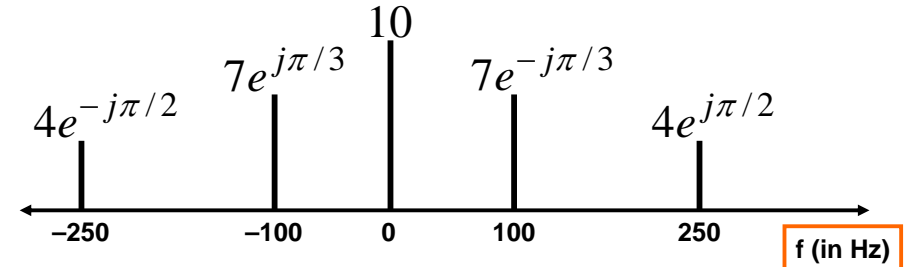


What is the fundamental frequency?

10 Hz

POP QUIZ: FUNDAMENTAL

Here's another spectrum:

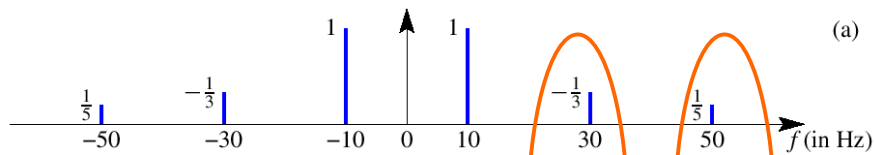


What is the fundamental frequency?

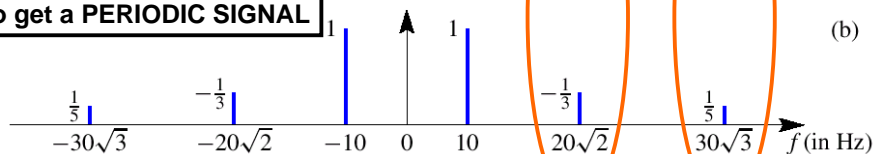
100 Hz ?

50 Hz ?

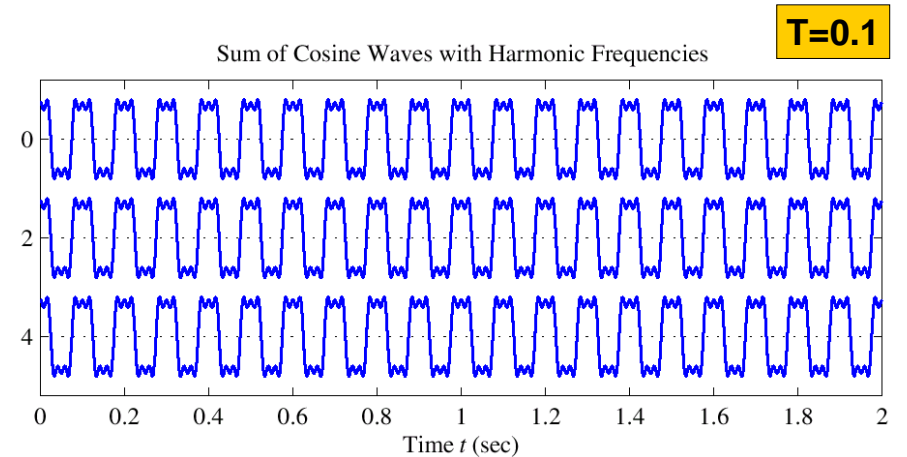
IRRATIONAL SPECTRUM



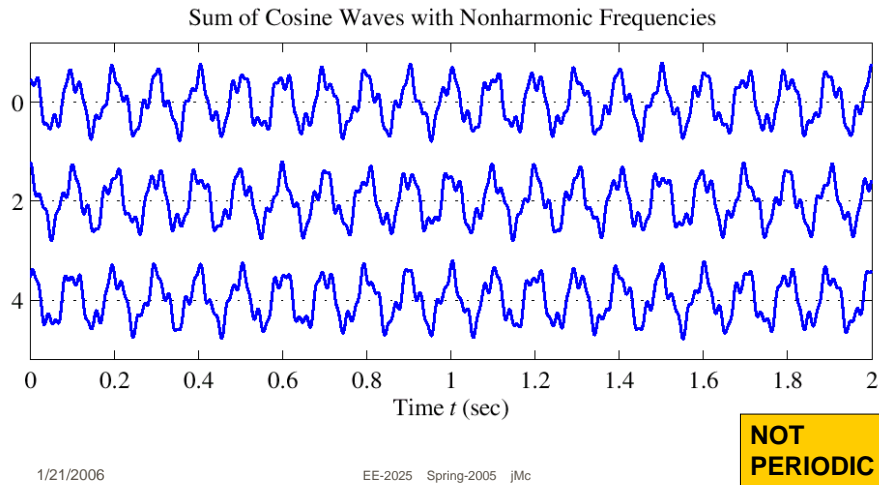
SPECIAL RELATIONSHIP to get a PERIODIC SIGNAL





Harmonic Signal (3 Freqs)



NON-Harmonic Signal



FREQUENCY ANALYSIS

- **Now, a much HARDER problem**
 - Given a recording of a song, have the computer write the music
- 

- Can a machine extract frequencies?
 - Yes, if we COMPUTE the spectrum for $x(t)$
 - During short intervals

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18

Time-Varying FREQUENCIES Diagram

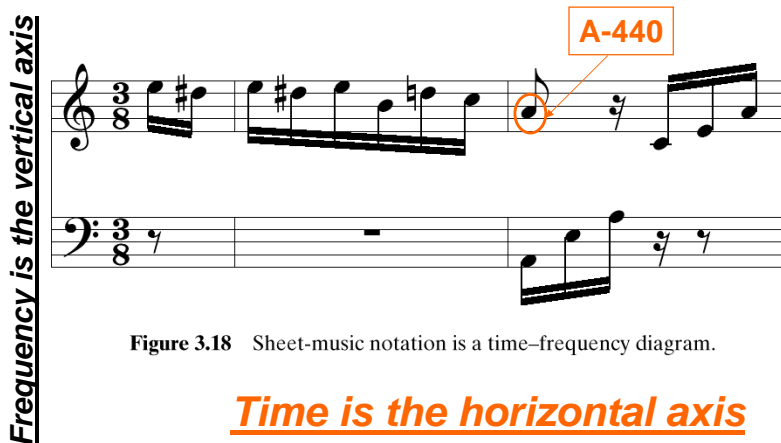


Figure 3.18 Sheet-music notation is a time-frequency diagram.

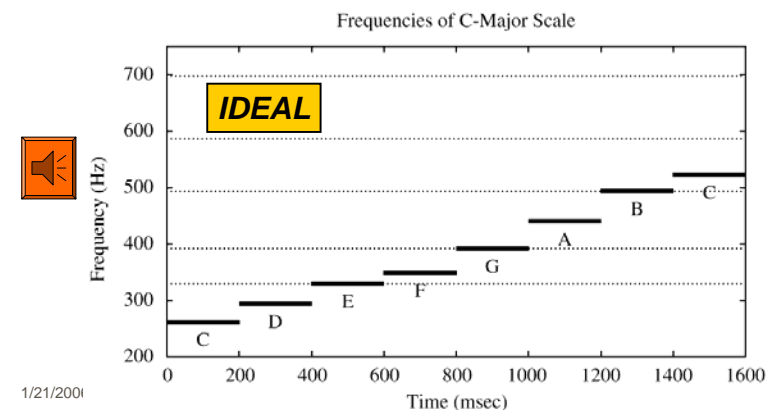
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19

SIMPLE TEST SIGNAL

- C-major SCALE: stepped frequencies
- Frequency is constant for each note



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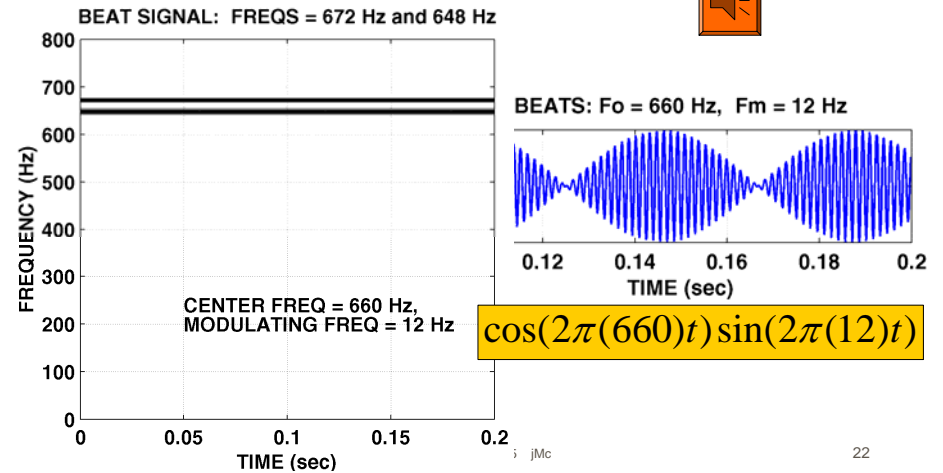
20

R-rated: ADULTS ONLY

- SPECTROGRAM Tool
 - MATLAB function is **specgram.m**
 - SP-First has **plotspec.m** & **spectgr.m**
- **ANALYSIS** program
 - Takes $x(t)$ as input &
 - Produces spectrum values X_k
 - Breaks $x(t)$ into **SHORT TIME SEGMENTS**
 - Then uses the FFT (Fast Fourier Transform)

SPECTROGRAM EXAMPLE

- Two **Constant** Frequencies: Beats



AM Radio Signal

- Same as BEAT Notes

$$\cos(2\pi(660)t) \sin(2\pi(12)t)$$



$$\frac{1}{2} \left(e^{j2\pi(660)t} + e^{-j2\pi(660)t} \right) \frac{1}{2j} \left(e^{j2\pi(12)t} - e^{-j2\pi(12)t} \right)$$

$$\frac{1}{4j} \left(e^{j2\pi(672)t} - e^{-j2\pi(672)t} - e^{j2\pi(648)t} + e^{-j2\pi(648)t} \right)$$

$$\frac{1}{2} \cos(2\pi(672)t - \frac{\pi}{2}) + \frac{1}{2} \cos(2\pi(648)t + \frac{\pi}{2})$$

SPECTRUM of AM (Beat)

- 4 complex exponentials in AM:



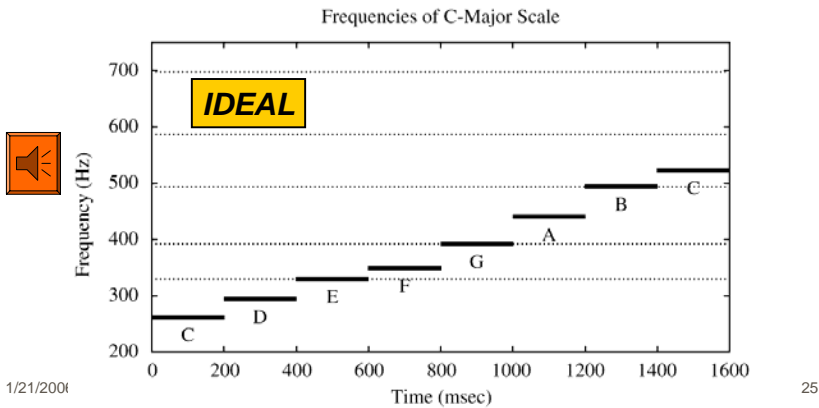
What is the fundamental frequency?

648 Hz ?

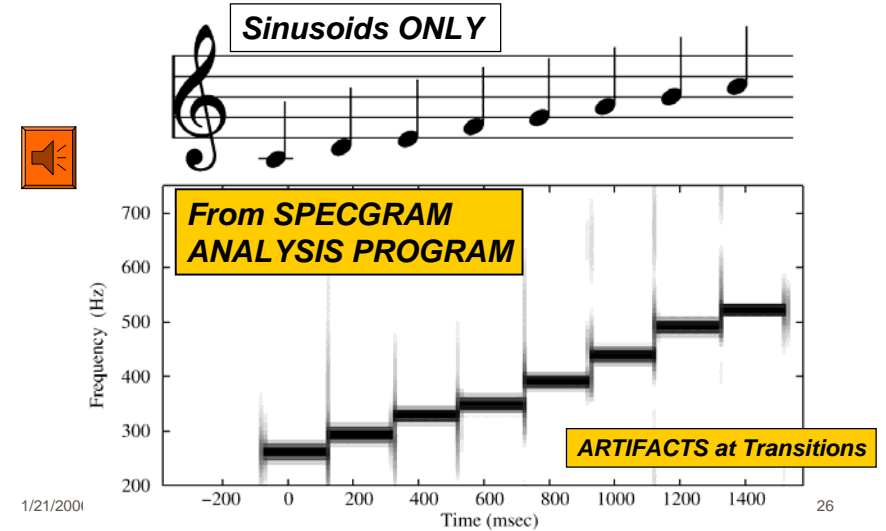
24 Hz ?

STEPPED FREQUENCIES

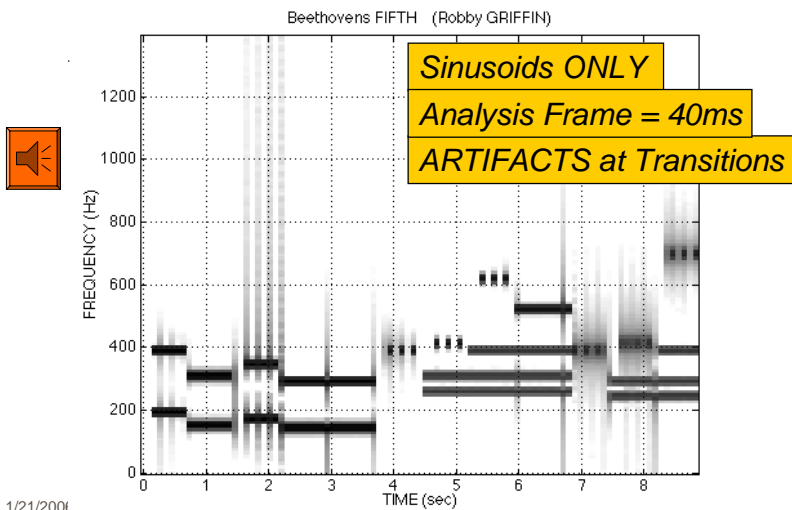
- C-major SCALE: successive sinusoids
 - Frequency is constant for each note



SPECTROGRAM of C-Scale



Spectrogram of LAB SONG




Time-Varying Frequency

- Frequency can change **vs. time**
 - Continuously, not stepped
- **FREQUENCY MODULATION (FM)**

$$x(t) = \cos(2\pi f_c t + v(t))$$

VOICE

- CHIRP SIGNALS 
- Linear Frequency Modulation (LFM)

New Signal: Linear FM

- Called **Chirp** Signals (LFM)

- Quadratic phase

QUADRATIC

$$x(t) = A \cos(\alpha t^2 + 2\pi f_0 t + \varphi)$$

- Freq will change **LINEARLY** vs. time
 - Example of Frequency Modulation (FM)
 - Define “instantaneous frequency”

INSTANTANEOUS FREQ

- Definition

$$x(t) = A \cos(\psi(t))$$
$$\Rightarrow \omega_i(t) = \frac{d}{dt} \psi(t)$$

Derivative
of the “Angle”

- For Sinusoid:

$$x(t) = A \cos(2\pi f_0 t + \varphi)$$

$$\psi(t) = 2\pi f_0 t + \varphi$$

Makes sense

$$\Rightarrow \omega_i(t) = \frac{d}{dt} \psi(t) = 2\pi f_0$$

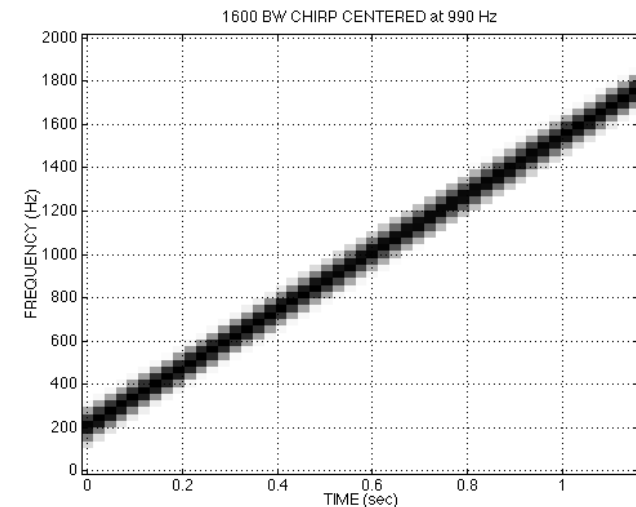
INSTANTANEOUS FREQ of the Chirp

- Chirp** Signals have Quadratic phase
- Freq will change **LINEARLY** vs. time

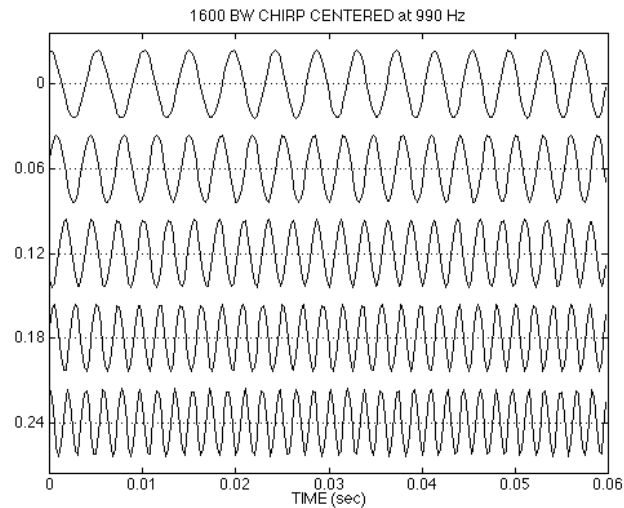
$$x(t) = A \cos(\alpha t^2 + \beta t + \varphi)$$
$$\Rightarrow \psi(t) = \alpha t^2 + \beta t + \varphi$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt} \psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



CHIRP WAVEFORM



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33

OTHER CHIRPS

- $\psi(t)$ can be anything:

$$x(t) = A \cos(\alpha \cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt} \psi(t) = -\alpha \beta \sin(\beta t)$$

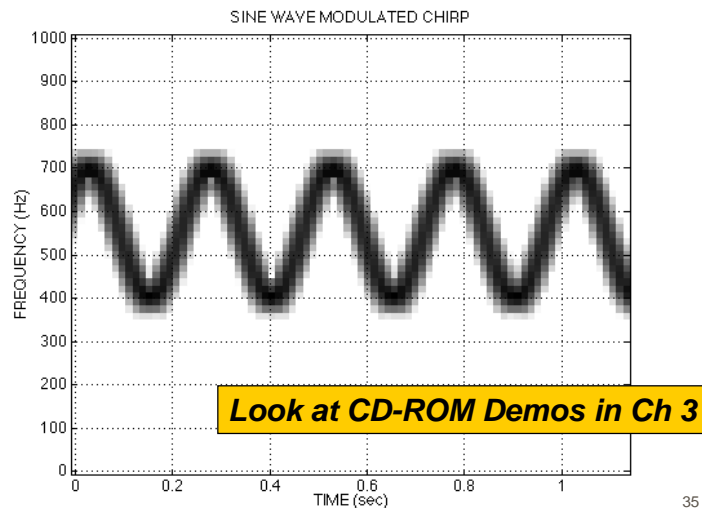
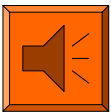
- $\psi(t)$ could be speech or music:
 - FM radio broadcast

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34

SINE-WAVE FREQUENCY MODULATION (FM)



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35