

Lecture 8
Sampling & Aliasing
6-Feb-06

Info

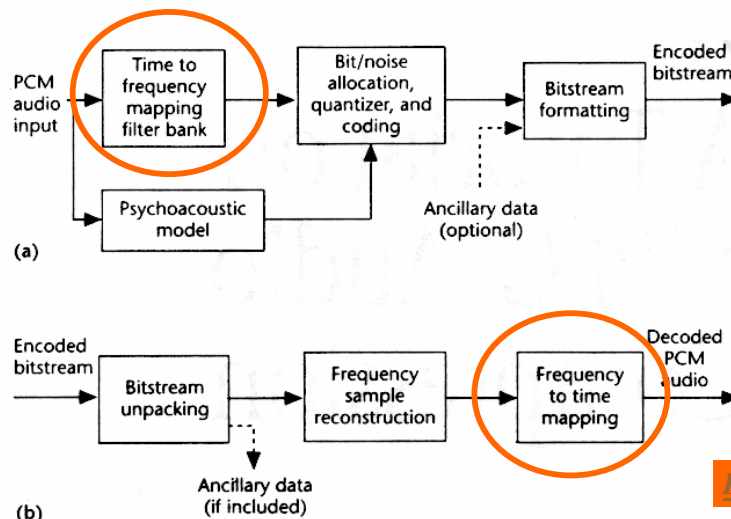
- Lab #4
 - **Formal** Lab Report will be worth 150 pts
- **NO Lab on Tuesday, 14-Feb**
 - **Instead, Open lab, early afternoon, for help on Lab #4**
 - Lab #5 will start on Wed, 15-Feb
- Honor Code !!!
 - **Don't exchange anything written or any electronic files**
- Quiz #1:
 - Coverage: HW #1, #2, #3, and #4
 - Review Session 7pm Thurs ECE Aud
 - No Make-ups; Written Excuse with documentation for excuse
 - Calculators OK; no wireless
 - One page of **handwritten** notes: 8.5 by 11 in, write on both sides

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MP-3 Block Diagram



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Education

- “Education is an admirable thing, but it is well to remember from time to time that **nothing that is worth knowing can be taught.**”Oscar Wilde
- So, Labs are one way to approximate knowledge acquisition in the “real world”

Lecture

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READING ASSIGNMENTS

- This Lecture:
 - **Chap 4, Sections 4-1 and 4-2**
 - Replaces Ch 4 in DSP First, pp. 83-94
- Other Reading:
 - Recitation: Strobe Demo (Sect 4-3)
 - Next Lecture: **Chap. 4 Sects. 4-4 and 4-5**

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LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
 - **Sampling Theorem**
 - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, $x[n]$
 - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi \ell$$

↑
ALIASING

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SYSTEMS Process Signals



- PROCESSING GOALS:
 - Change $x(t)$ into $y(t)$
 - For example, more BASS
 - Improve $x(t)$, e.g., image deblurring
 - Extract Information from $x(t)$

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System IMPLEMENTATION

- ANALOG/ELECTRONIC:
 - Circuits: resistors, capacitors, op-amps



- DIGITAL/MICROPROCESSOR
 - Convert $x(t)$ to **numbers** stored in memory



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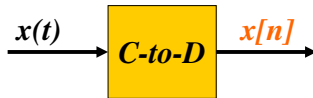
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SAMPLING $x(t)$

SAMPLING PROCESS

- Convert $x(t)$ to **numbers** $x[n]$
- "n" is an integer; $x[n]$ is a sequence of values
- Think of "n" as the storage address in memory
- UNIFORM SAMPLING** at $t = nT_s$
 - IDEAL: $x[n] = x(nT_s)$



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SAMPLING RATE, f_s

SAMPLING RATE (f_s)

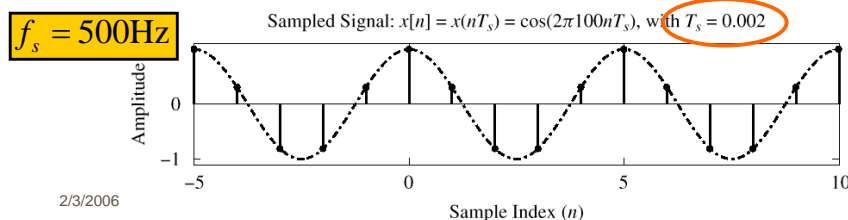
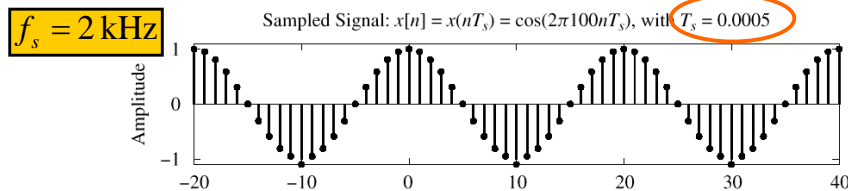
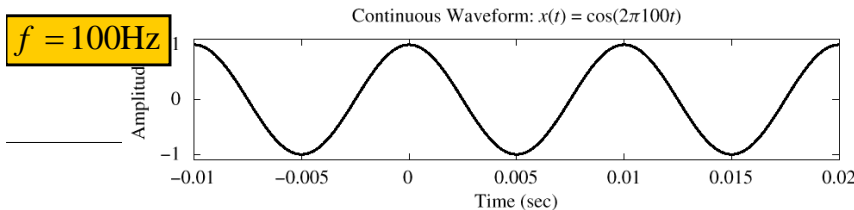
- $f_s = 1/T_s$
 - NUMBER of SAMPLES PER SECOND
- $T_s = 125 \text{ microsec} \rightarrow f_s = 8000 \text{ samples/sec}$
 - UNITS ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING** at $t = nT_s = n/f_s$
 - IDEAL: $x[n] = x(nT_s) = x(n/f_s)$



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Sample Index (n)

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SAMPLING THEOREM

HOW OFTEN ?

- DEPENDS on FREQUENCY of SINUSOID
- ANSWERED by SHANNON/NYQUIST Theorem
- ALSO DEPENDS on **RECONSTRUCTION**

Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{\max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

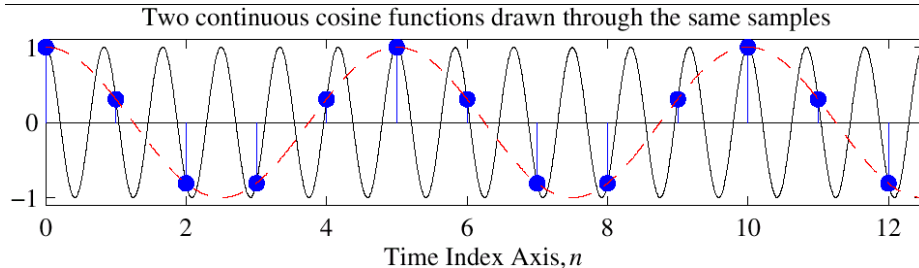
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Reconstruction? Which One?

Given the samples, draw a sinusoid through the values



$$x[n] = \cos(0.4\pi n)$$

When n is an integer
 $\cos(0.4\pi n) = \cos(2.4\pi n)$

STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
 - A list of numbers stored in memory
- EXAMPLE: audio CD
- CD rate is 44,100 samples per second
 - 16-bit samples
 - Stereo uses 2 channels
- Number of bytes for 1 minute is
 - $2 \times (16/8) \times 60 \times 44100 = 10.584$ Mbytes

DISCRETE-TIME SINUSOID

- Change $x(t)$ into $x[n]$ **DERIVATION**

$$x(t) = A \cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A \cos(\omega nT_s + \varphi)$$

$$x[n] = A \cos((\omega T_s)n + \varphi)$$

$$x[n] = A \cos(\hat{\omega}n + \varphi)$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} \quad \text{DEFINE DIGITAL FREQUENCY}$$

DIGITAL FREQUENCY $\hat{\omega}$

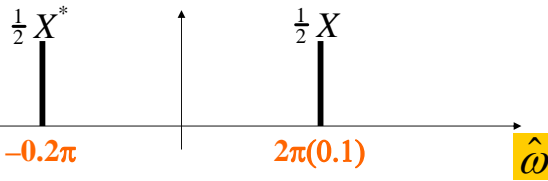
- $\hat{\omega}$ VARIES from **0** to **2π** , as f varies from 0 to the sampling frequency
- UNITS are radians, **not** rad/sec
 - DIGITAL FREQUENCY is NORMALIZED

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

SPECTRUM (DIGITAL)

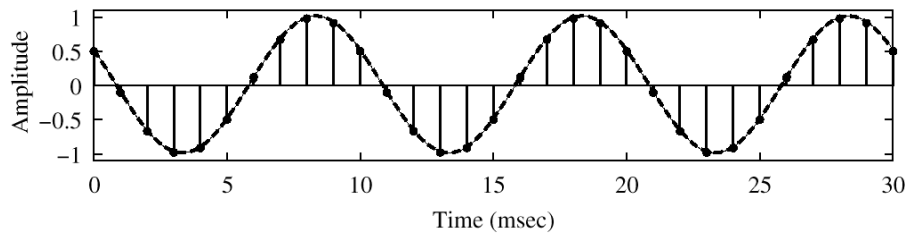
$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f_s = 1 \text{ kHz}$$



$$x[n] = A \cos(2\pi(100)(n/1000) + \varphi)$$

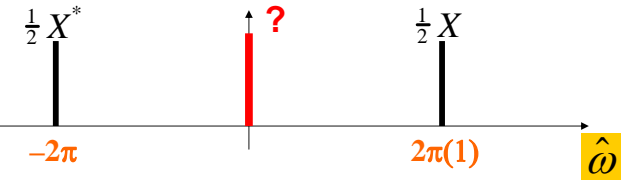
100-Hz Cosine Wave: Sampled with $T_s = 1$ msec (1000 Hz)



SPECTRUM (DIGITAL) ???

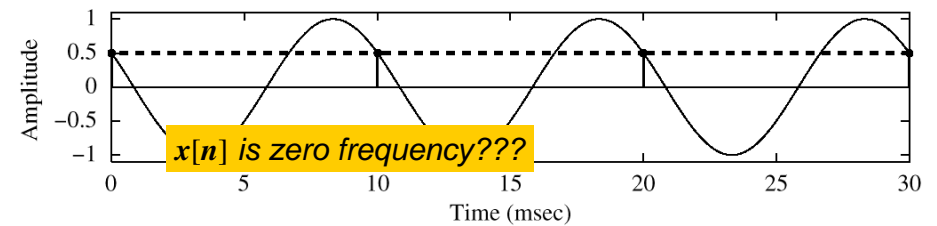
$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f_s = 100 \text{ Hz}$$



$$x[n] = A \cos(2\pi(100)(n/100) + \varphi)$$

100-Hz Cosine Wave: Sampled with $T_s = 10$ msec (100 Hz)



The REST of the STORY

- Spectrum of $x[n]$ has more than one line for each complex exponential
 - Called **ALIASING**
 - **MANY SPECTRAL LINES**
- SPECTRUM is PERIODIC with period = 2π
 - Because

$$A \cos(\hat{\omega}n + \varphi) = A \cos((\hat{\omega} + 2\pi)n + \varphi)$$

ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$

$$x_1(t) = \cos(400\pi t) \quad \text{sampled at } f_s = 1000 \text{ Hz}$$

$$x_1[n] = \cos(400\pi \frac{n}{1000}) = \cos(0.4\pi n)$$

$$x_2(t) = \cos(2400\pi t) \quad \text{sampled at } f_s = 1000 \text{ Hz}$$

$$x_2[n] = \cos(2400\pi \frac{n}{1000}) = \cos(2.4\pi n)$$

$$x_2[n] = \cos(2.4\pi n) = \cos(0.4\pi n + 2\pi n) = \cos(0.4\pi n)$$

$$\Rightarrow x_2[n] = x_1[n] \quad \text{2400}\pi - 400\pi = 2\pi(1000)$$

ALIASING DERIVATION-2

- Other Frequencies give the same $\hat{\omega}$

$$\text{If } x(t) = A \cos(2\pi(f + lf_s)t + \varphi)$$

$$t \leftarrow \frac{n}{f_s}$$

$$\text{and we want: } x[n] = A \cos(\hat{\omega}n + \varphi)$$

$$\text{then: } \hat{\omega} = \frac{2\pi(f + lf_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi lf_s}{f_s}$$

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

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ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ or $-f_s$ to the FREQ of $x(t)$ gives exactly the same $x[n]$
 - The samples, $x[n] = x(n/f_s)$ are EXACTLY THE SAME VALUES
- GIVEN $x[n]$, WE CAN'T DISTINGUISH f_o FROM $(f_o + f_s)$ or $(f_o + 2f_s)$

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NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi l$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

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SPECTRUM for $x[n]$

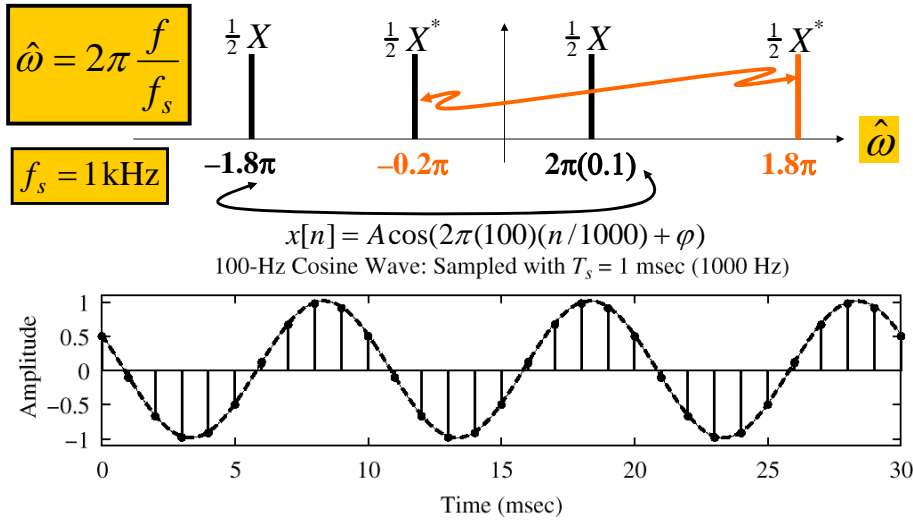
- PLOT versus NORMALIZED FREQUENCY
- INCLUDE **ALL** SPECTRUM LINES
 - ALIASES
 - ADD MULTIPLES of 2π
 - SUBTRACT MULTIPLES of 2π
 - FOLDED ALIASES
 - (to be discussed later)
 - ALIASES of NEGATIVE FREQS

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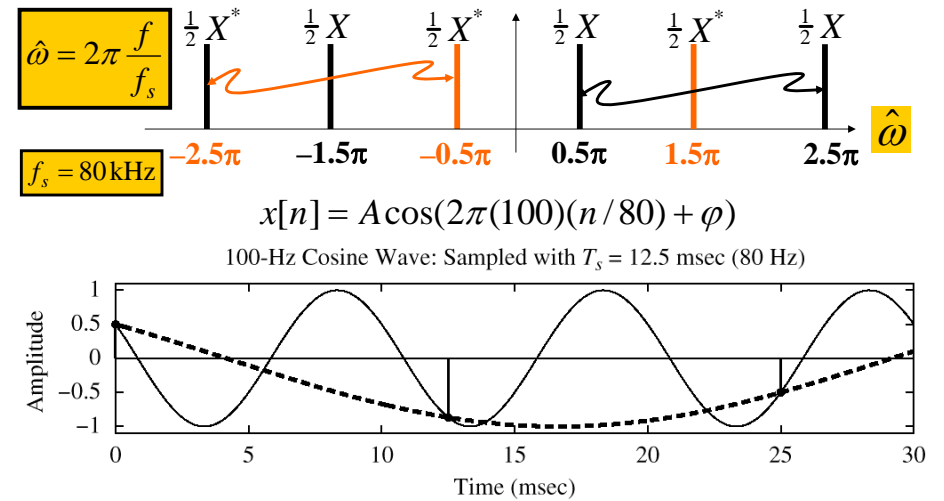
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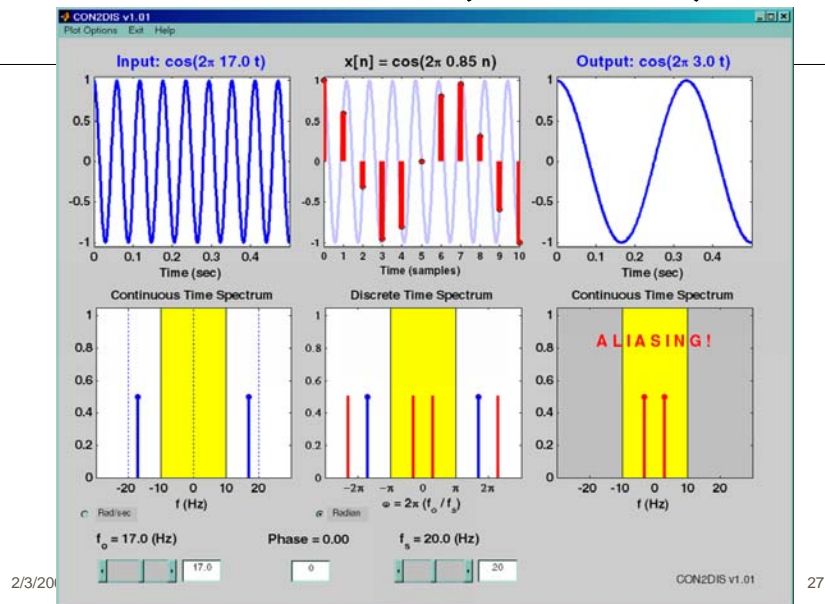
SPECTRUM (MORE LINES)



SPECTRUM (ALIASING CASE)



SAMPLING GUI (con2dis)



SPECTRUM (FOLDING CASE)

