

ECE-2025

Spring-2006

Lecture 13

Digital Filtering of Analog Signals

27-Feb-06

Info: Web-CT, Lab, HW

- Quiz #2 on 17-March (Friday)
 - Coverage: HW #5, #6, #7 and #8
- HW #6 due this week
- Lab #7 is posted

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Perseverance

- **A** lowly virtue whereby mediocrity achieves a glorious success...A. Bierce
- **B**ear in mind, if you are going to amount to anything, that your success does not depend upon the brilliance and the impetuosity with which you take hold, but upon the ever lasting and sanctified bull doggedness with which you hang on after you have taken hold...Dr. A. B. Meldrum

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READING ASSIGNMENTS

- This Lecture:
 - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
 - Recitation: Chapter 6
 - FREQUENCY RESPONSE EXAMPLES
 - Next Lecture: Chapter 7

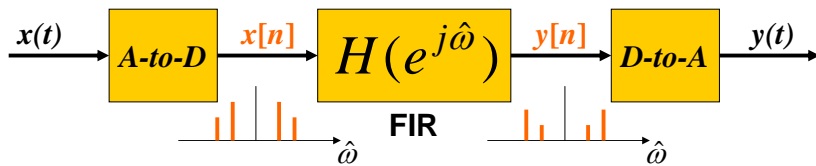
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LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of $x[n]$ thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- UNIFICATION:** How does Frequency Response affect $x(t)$ to produce $y(t)$?



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TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \dots$$

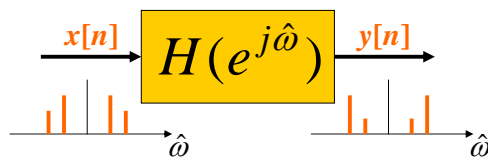
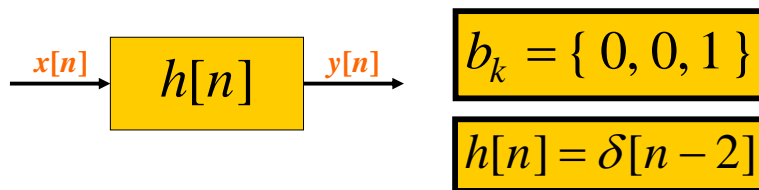
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Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



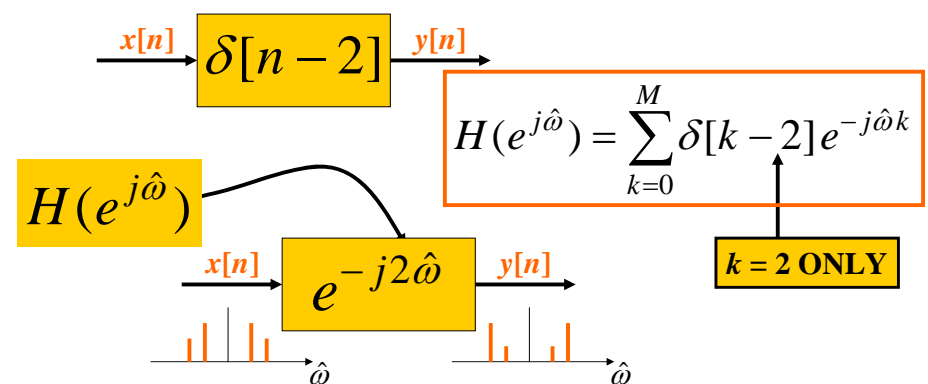
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DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



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GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n - n_d]$

$$h[n] = \delta[n - n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE
non-ZERO TERM
for k at $k = n_d$

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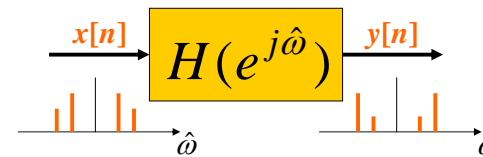
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FREQ DOMAIN --> TIME ??

- START with $H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



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FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER's Formula}$$

$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n - 1] + 3.5\delta[n - 3]$$

$$b_k = \{ 0, 3.5, 0, 3.5 \}$$

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PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
 - OUTPUT has SAME FREQUENCY
 - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
 - MAGNITUDE vs. Frequency
 - PHASE vs. Freq
 - PLOTTING

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

MAG

PHASE

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FREQ. RESPONSE PLOTS

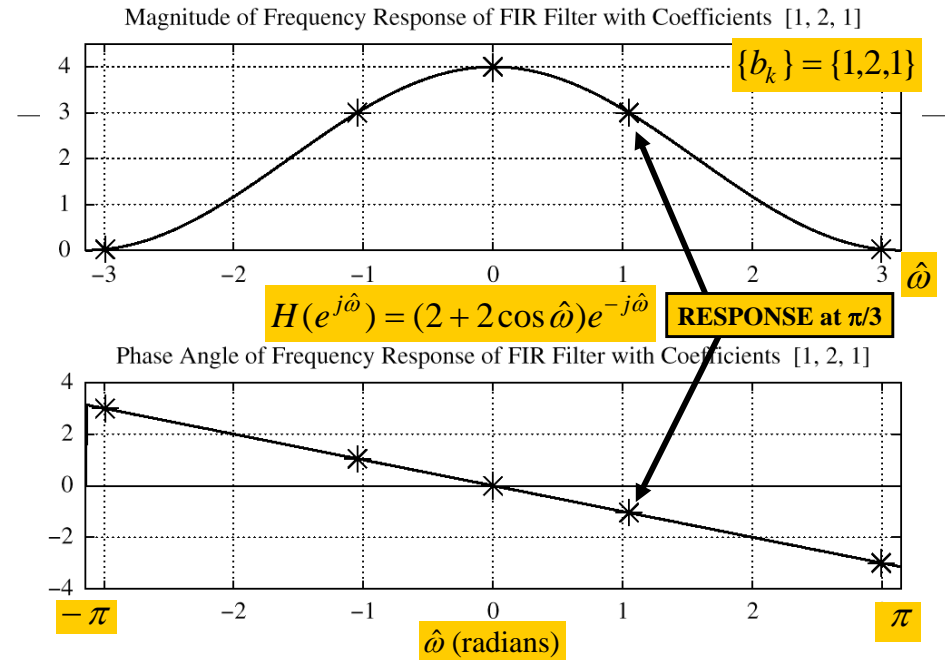
- DENSE GRID (**ww**) from $-\pi$ to $+\pi$
 - **ww** = `-pi:(pi/100):pi;`
- **HH** = `freqz(bb,1,ww)`
 - VECTOR **bb** contains Filter Coefficients
 - DSP-First: **HH** = `freakz(bb,1,ww)`

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

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EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos(\frac{\pi}{3}n - \frac{\pi}{12})$$

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SINUSOID thru FIR

- IF $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A \cos(\hat{\omega}_1 n + \phi)$$

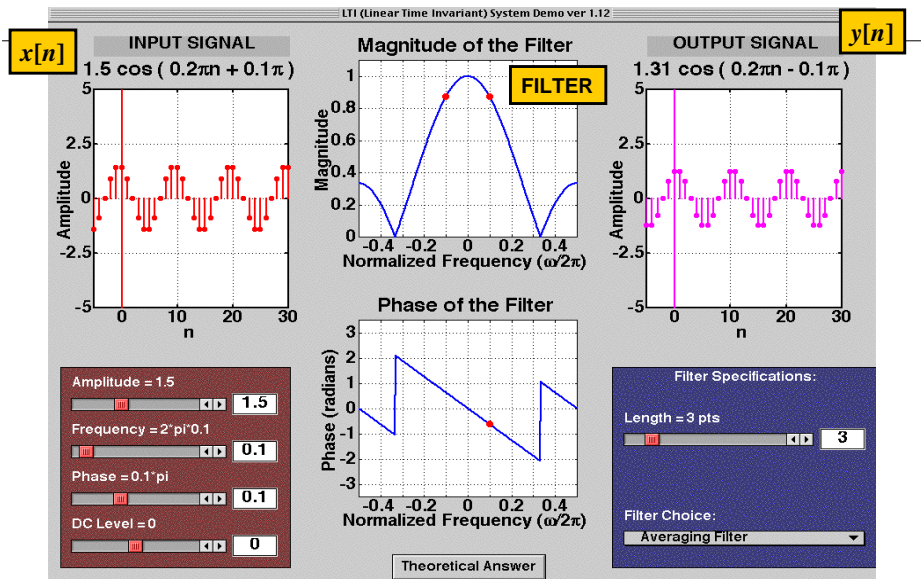
$$\Rightarrow y[n] = A |H(e^{j\hat{\omega}_1})| \cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

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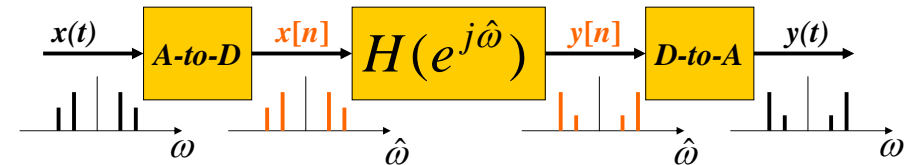
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LTI Demo with Sinusoids

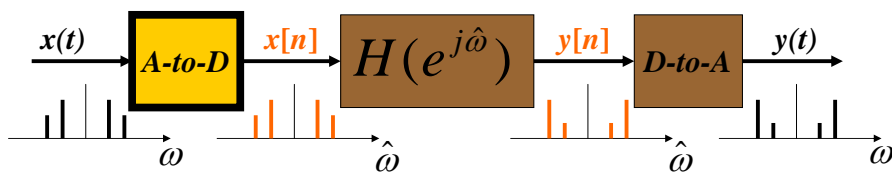


DIGITAL "FILTERING"



- ω ■ SPECTRUM of $x(t)$ (SUM of SINUSOIDS)
- $\hat{\omega}$ ■ SPECTRUM of $x[n]$
 - Is ALIASING a PROBLEM ?
- $\hat{\omega}$ ■ SPECTRUM $y[n]$ (FIR Gain or Nulls)
- ω ■ Then, OUTPUT $y(t)$ = SUM of SINUSOIDS

FREQUENCY SCALING

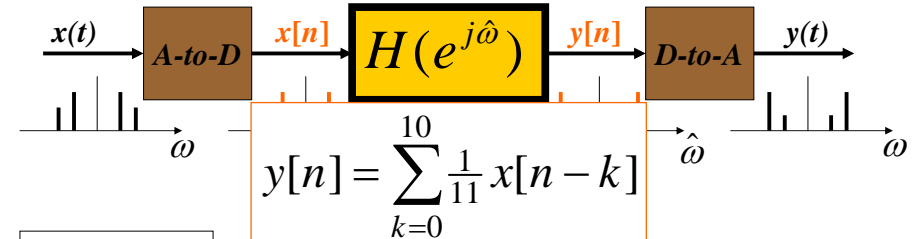


- TIME SAMPLING:
 - IF **NO** ALIASING:
 - FREQUENCY SCALING

$$t = nT_s$$

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

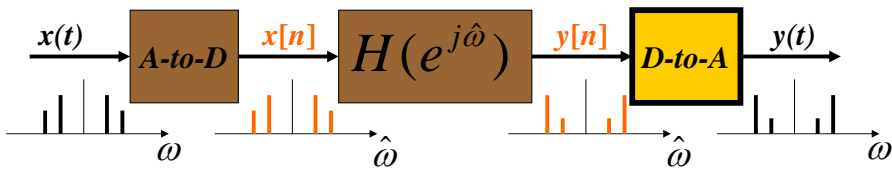
11-pt AVERAGER Example



$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2} \hat{\omega})}{11 \sin(\frac{1}{2} \hat{\omega})} e^{-j5\hat{\omega}} \quad ?$$

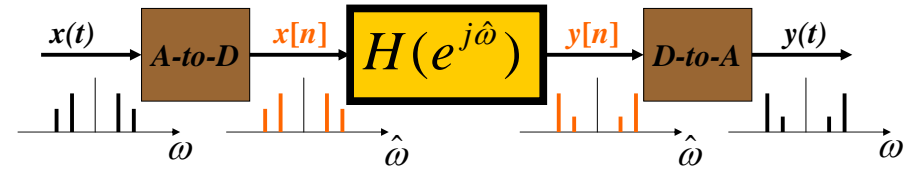
$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$$

D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow tf_s$
- RECONSTRUCT up to $0.5f_s$
 - FREQUENCY SCALING $\omega = \hat{\omega}f_s$

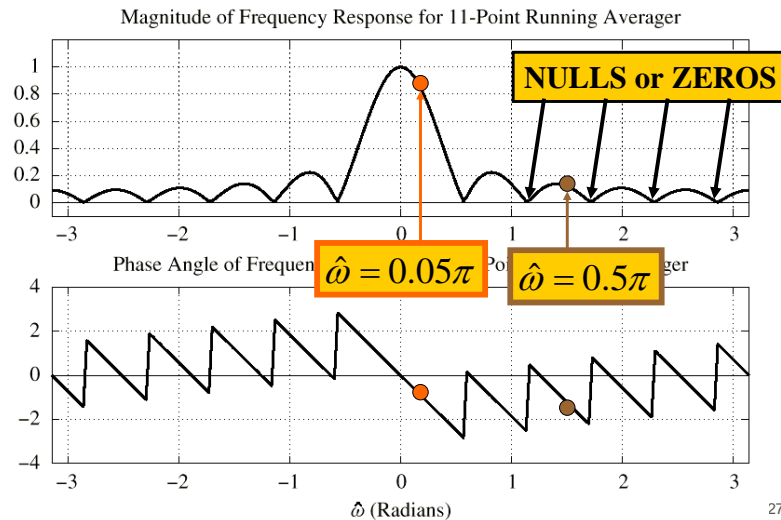
TRACK the FREQUENCIES



250 Hz	0.5π	$H(e^{j0.5\pi})$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(e^{j0.05\pi})$	$.05\pi$	25 Hz

Fs = 1000 Hz **NO new freqs**

11-pt AVERAGER



EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

MAG SCALE

f_s = 1000

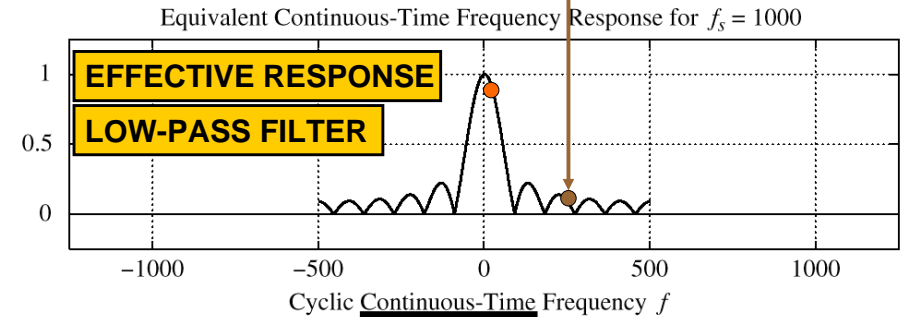
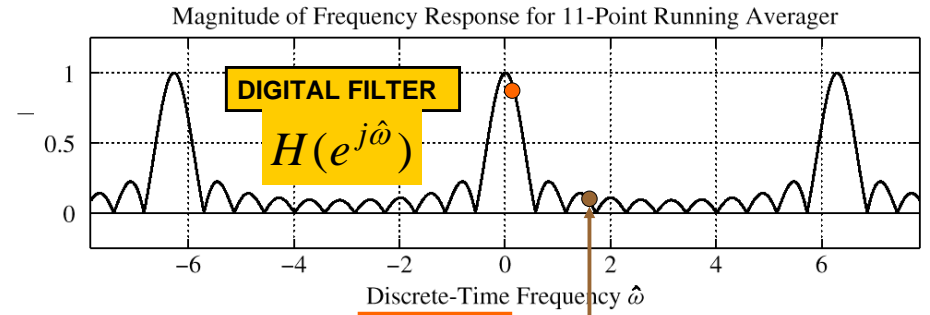
$$= 0.8811 e^{-j\pi/4}$$

PHASE CHANGE

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

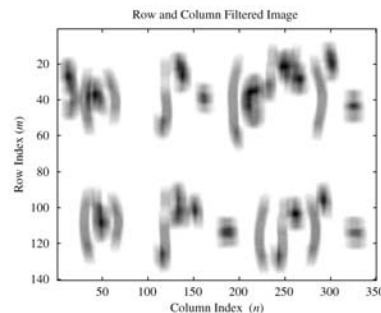
$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = \underline{0.8811} \cos(2\pi(25)t - \underline{\pi/4}) + \underline{0.0909} \sin(2\pi(250)t - \underline{\pi/2})$$

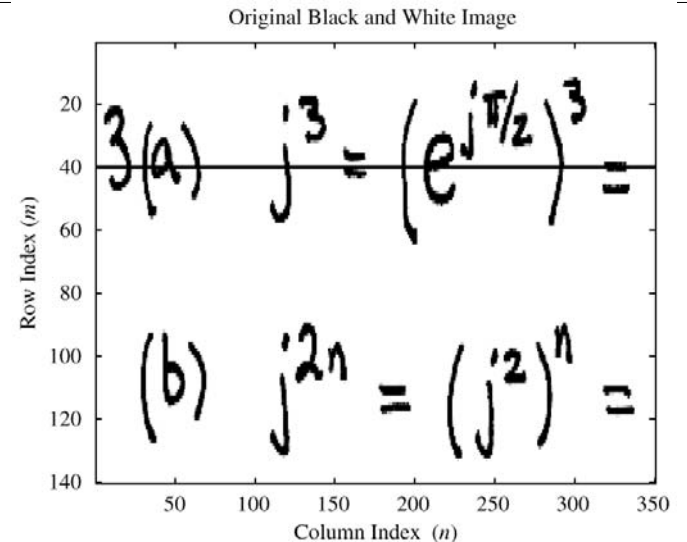


FILTER TYPES

- LOW-PASS FILTER (**LPF**)
 - BLURRING
 - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
 - SHARPENING for IMAGES
 - BOOSTS THE HIGHS
 - REMOVES DC
- BAND-PASS FILTER (**BPF**)



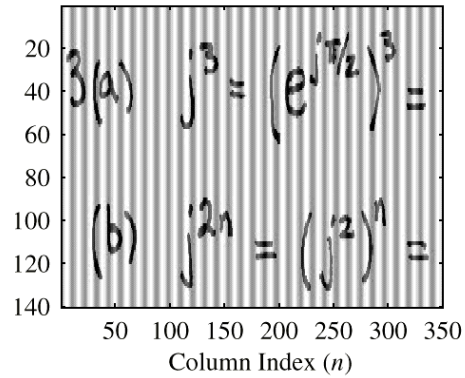
B & W IMAGE



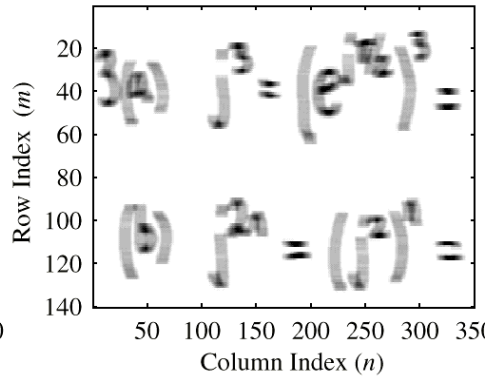
B&W IMAGE with COSINE

FILTERED: 11-pt AVG

Homework plus Cosine

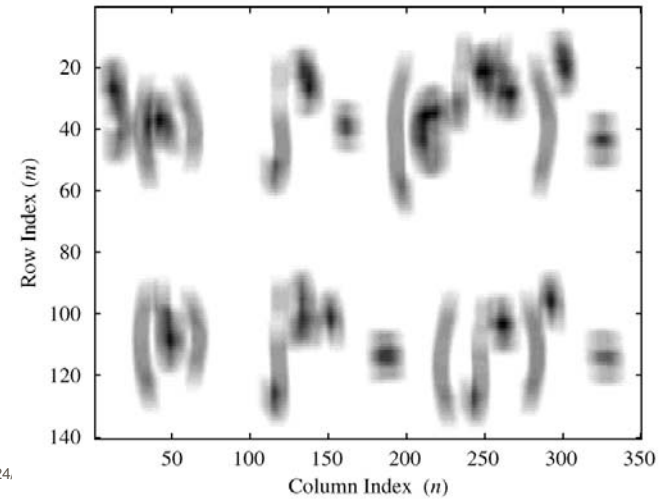


Remove Cosine Stripe with Averaging Fi



FILTERED B&W IMAGE

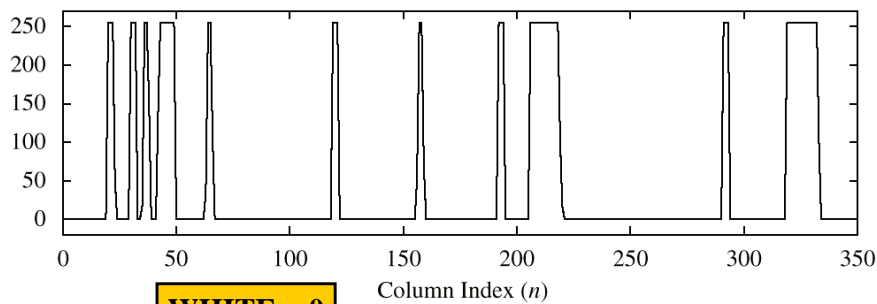
Row and Column Filtered Image



ROW of B&W IMAGE

BLACK = 255

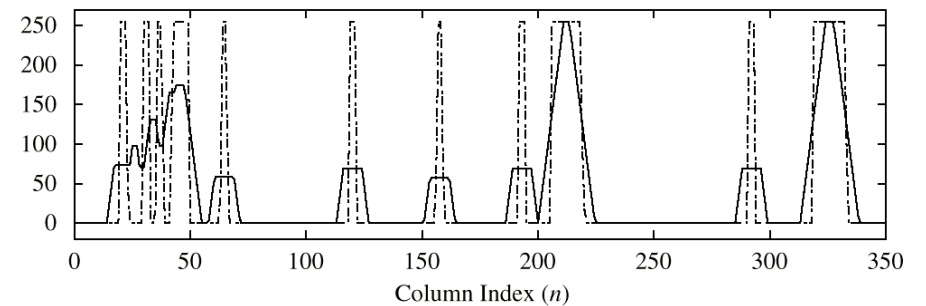
Row 40 of the Image



WHITE = 0

FILTERED ROW of IMAGE

11-Point Averaging: 5-Sample Delay Equalization



ADJUSTED DELAY by 5 samples