

ECE-2025

Spring-2006

Lecture 15

Zeros of $H(z)$ and the
Frequency Domain

6-Mar-06

Info: Web-CT, Lab, HW

- Lab #7 **ERROR**
 - **Section 4.1 has been rewritten for better clarity**
- Quiz #2 on 17-Mar (Friday)
 - Coverage: HW **#5, #6, #7 and #8**
 - One page of **Hand-written** notes
 - Review Session: ECE Auditorium,
 - Thurs (16-Mar, 6:00 pm)

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COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 7, Section 7-6 to end
- Other Reading:
 - Recitation & Lab: Chapter 7
 - ZEROS (and POLES)
 - Next Lecture: Chapter 8

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LECTURE OBJECTIVES

- ZEROS and POLES
- Relate $H(z)$ to FREQUENCY RESPONSE

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- **THREE DOMAINS:**

- Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

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DESIGN PROBLEM

- Example:

- Design a Lowpass FIR filter (Find b_k)
- Reject completely 0.7π , 0.8π , and 0.9π
 - This is NULLING
- Estimate the filter length needed to accomplish this task. How many b_k ?

- Z POLYNOMIALS provide the TOOLS

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Z-Transform DEFINITION

- POLYNOMIAL Representation of LTI SYSTEM:

$$H(z) = \sum_n h[n]z^{-n}$$

- EXAMPLE:

$$\{h[n]\} = \{2, 0, -3, 0, 2\}$$

$$\begin{aligned} H(z) &= 2z^{-0} + 0z^{-1} - 3z^{-2} + 0z^{-3} + 2z^{-4} \\ &= 2 - 3z^{-2} + 2z^{-4} \\ &= 2 - 3(z^{-1})^2 + 2(z^{-1})^4 \end{aligned}$$

APPLIES to Any SIGNAL

POLYNOMIAL in z^{-1}

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CONVOLUTION PROPERTY

- Convolution in the n -domain
 - SAME AS
- Multiplication in the z -domain

$$y[n] = h[n] * x[n] \leftrightarrow Y(z) = H(z)X(z)$$

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=0}^M h[k]x[n-k] \end{aligned}$$

FIR Filter

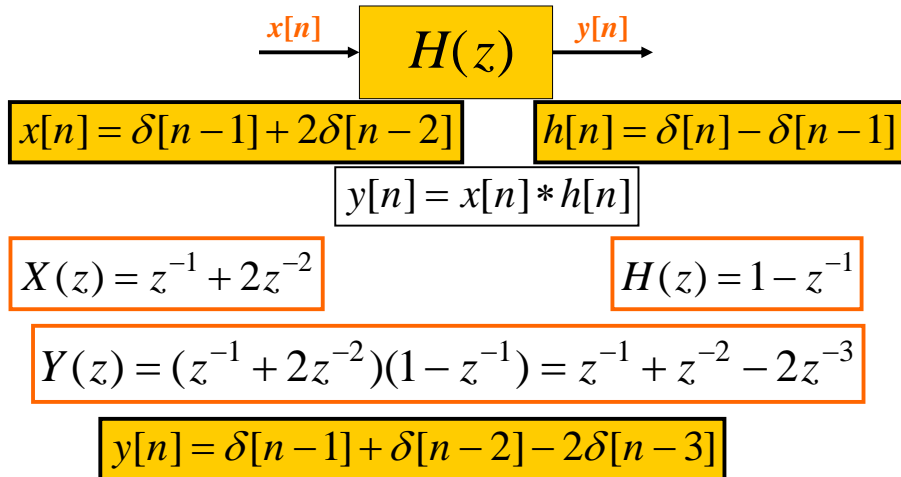
MULTIPLY z-TRANSFORMS

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CONVOLUTION EXAMPLE

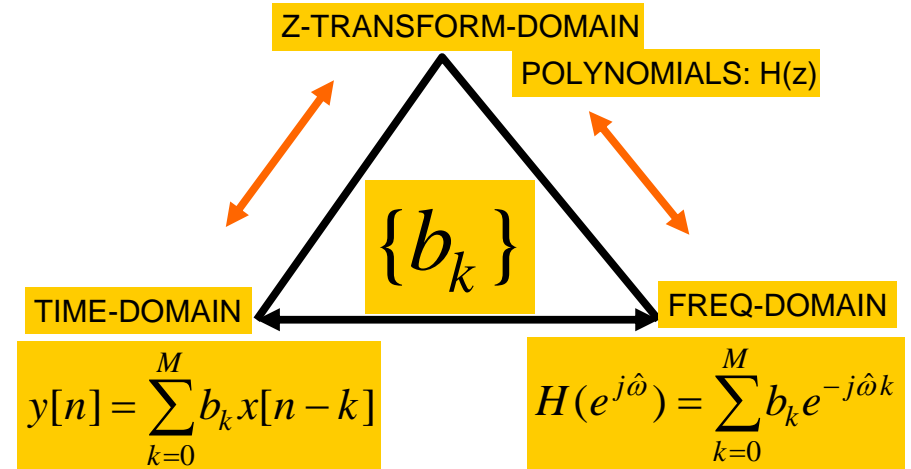


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THREE DOMAINS

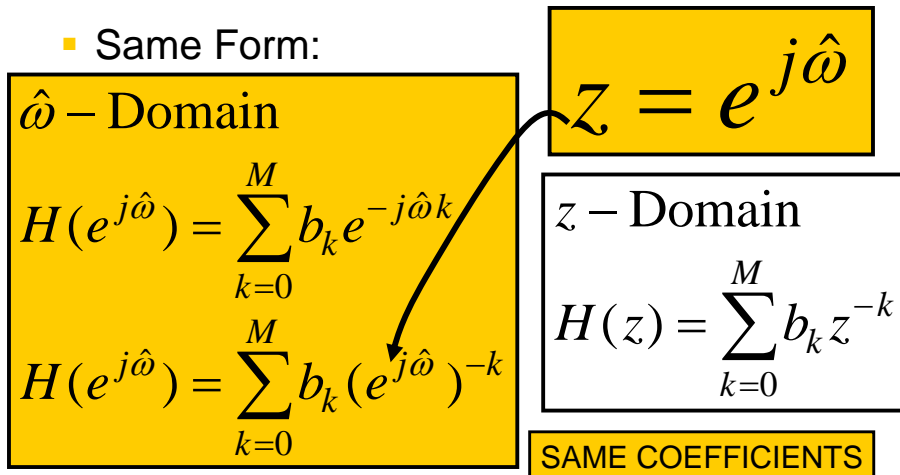


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FREQUENCY RESPONSE ?



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ANOTHER ANALYSIS TOOL

- z-Transform POLYNOMIALS are EASY !
 - ROOTS, FACTORS, etc.
- ZEROS and POLES: where is $H(z) = 0$?**
- The z-domain is COMPLEX
 - $H(z)$ is a COMPLEX-VALUED function of a COMPLEX VARIABLE z .

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ZEROS of $H(z)$

- Find z , where $H(z)=0$

$$H(z) = 1 - \frac{1}{2}z^{-1}$$

$$1 - \frac{1}{2}z^{-1} = 0 ?$$

$$z - \frac{1}{2} = 0$$

$$\text{Zero at : } z = \frac{1}{2}$$

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ZEROS of $H(z)$

- Find z , where $H(z)=0$
 - Interesting when z is ON the unit circle.

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = (1 - z^{-1})(1 - z^{-1} + z^{-2})$$

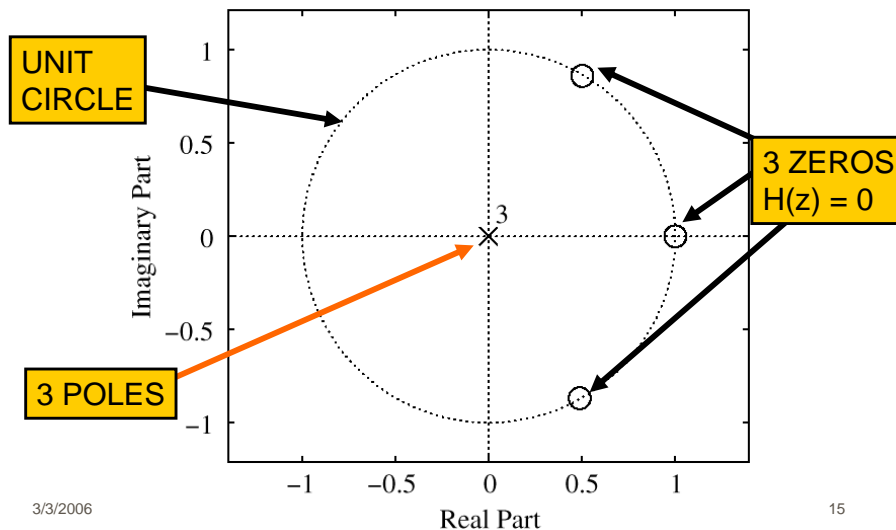
$$\text{Roots : } z = 1, \frac{1}{2} \pm j\frac{\sqrt{3}}{2} \quad e^{\pm j\pi/3}$$

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PLOT ZEROS in z -DOMAIN



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POLES of $H(z)$

- Find z , where $H(z) \rightarrow \infty$
 - Not very interesting for the FIR case

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(z) = \frac{z^3 - 2z^2 + 2z - 1}{z^3}$$

$$\text{Three Poles at : } z = 0$$

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FREQ. RESPONSE from ZEROS

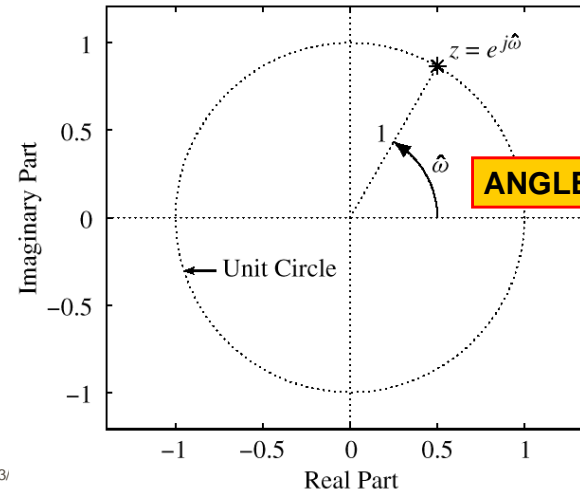
$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

- Relate $H(z)$ to FREQUENCY RESPONSE
- EVALUATE $H(z)$ on the **UNIT CIRCLE**
 - ANGLE is same as FREQUENCY

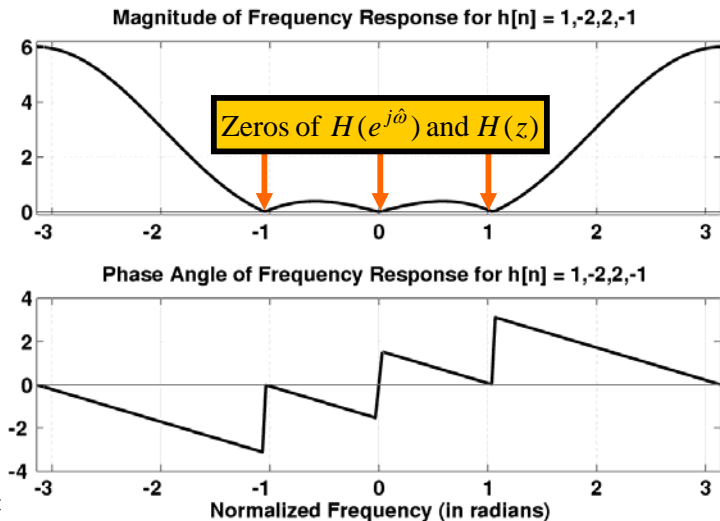
$z = e^{j\hat{\omega}}$ (as $\hat{\omega}$ varies)
defines a CIRCLE, radius = 1

$$H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}}$$

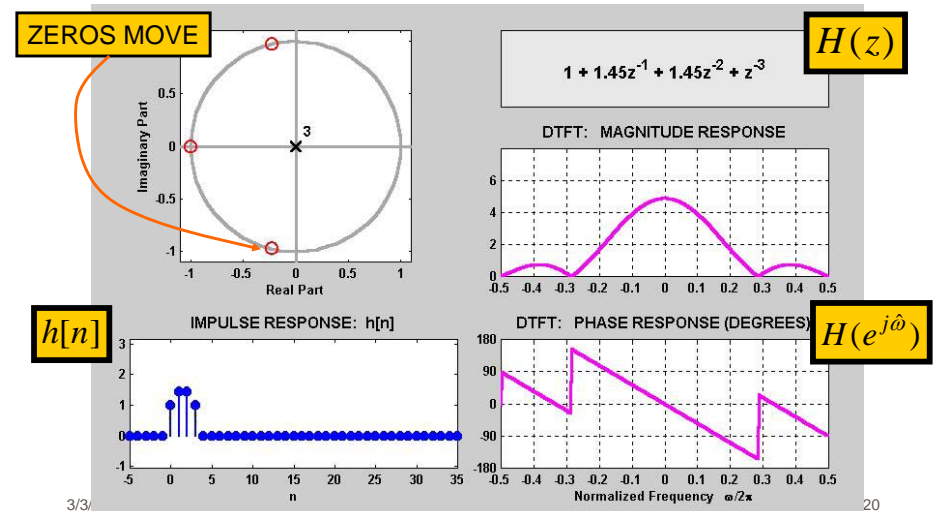
The Complex z -Plane



FIR Frequency Response



3 DOMAINS MOVIE: FIR

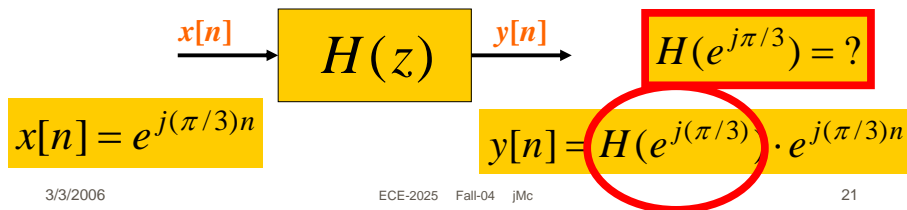


NULLING PROPERTY of H(z)

- When $H(z)=0$ on the unit circle.
 - Find inputs $x[n]$ that give zero output

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

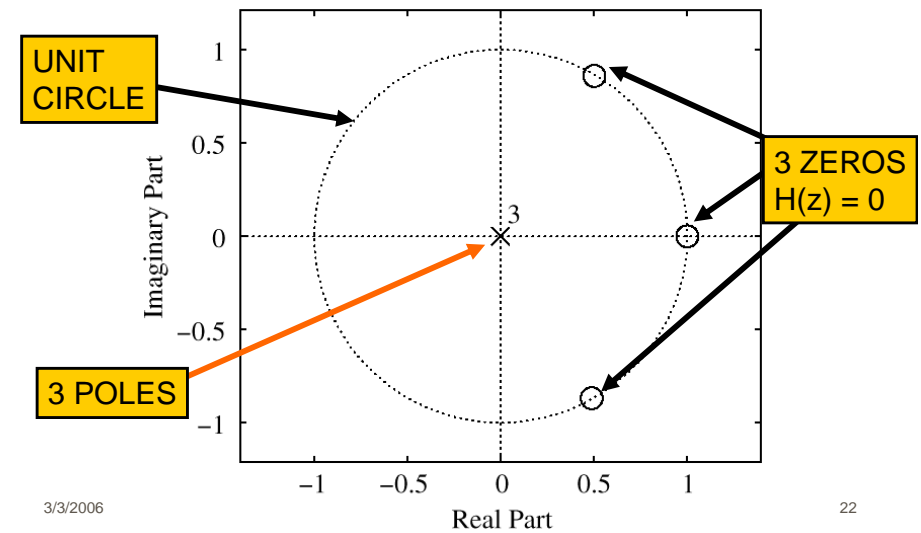


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PLOT ZEROS in z-DOMAIN



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NULLING PROPERTY of H(z)

- Evaluate $H(z)$ at the input "frequency"

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

$$(1 - 2(\frac{1}{2} - j\frac{\sqrt{3}}{2}) + 2(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) - (-1))$$

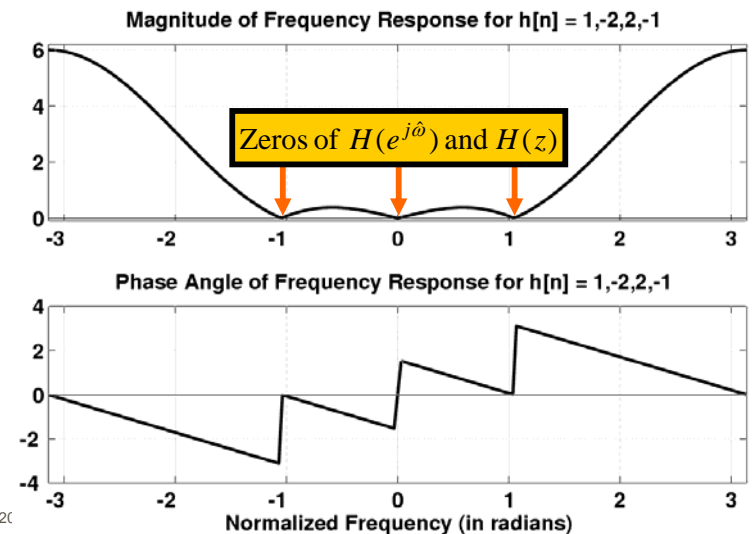
$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

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FIR Frequency Response



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DESIGN PROBLEM

- Example:
 - Design a Lowpass FIR filter (Find b_k)
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 - Estimate the filter length needed to accomplish this task. How many b_k ?
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NULLING FILTER

- PLACE ZEROS to make $y[n] = 0$

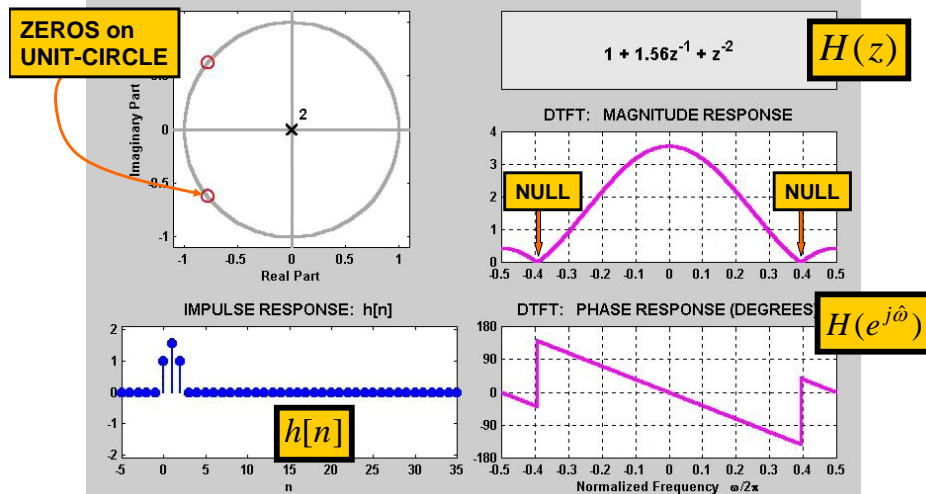
$$H(z) = b_0 + b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5} + b_6z^{-6}$$

Needs 6 ZEROS
where $H(z) = 0$

$$H(z_k) = 0, \text{ for } z_k = e^{\pm j0.7\pi}, e^{\pm j0.8\pi}, e^{\pm j0.9\pi}$$

$$x[n] = e^{j0.8\pi n} \Rightarrow y[n] = H(e^{j0.8\pi})e^{j0.8\pi n}$$

3 DOMAINS MOVIE: FIR



PeZ Demo: Zero Placing

