

ECE-2025

Spring-2006

Lecture 16

IIR Filters: Feedback & $H(z)$

10-Mar-06

Info: Web-CT, Lab, HW

- Labs:
 - Lab #8 report should be turned in after Spring Break
 - Lab #9 will start on Wed, 15-March
- Homework:
 - #8 due next week; solutions posted Thursday at 5pm
 - ERRATA: problem 8.2(c) change numerator $H(z) = \frac{1+z^{-11}}{1+z^{-1}}$
- Quiz #2 will be 17-March (Friday)
 - Coverage: HW #5, 6, 7 and 8
 - Chapters 4, 5, 6, and 7
 - Review Session, 16-March, Thurs @ 6:00pm

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Info: Web-CT, Lab, HW

- Labs:
 - New versions of data file & lab write-up
 - posted at 7:45am on Thursday, 9-March
 - New Data File:
 - Both signals have been corrected.
 - The fs of the wav file is now 22050
 - The number of bits in the image is now 8, so the values are 0 for black and 255 for white.
 - Also, in 4.2(a), use a filter length of $L=45$.

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Lecture

- The process whereby the notes of the Professor become the notes of the student with passing through the minds of either
 - Powerpoint slides
 - STREAMED Lectures are available on the web
 - Linked from WebCT

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Lecture

READING ASSIGNMENTS

- This Lecture:
 - Chapter 8, Sects. 8-1, 8-2 & 8-3
- Other Reading:
 - Recitation: Ch. 8, Sects 8-1 thru 8-4
 - POLES & ZEROS
 - Next Lecture: Chapter 8, Sects. 8-4 8-5 & 8-6

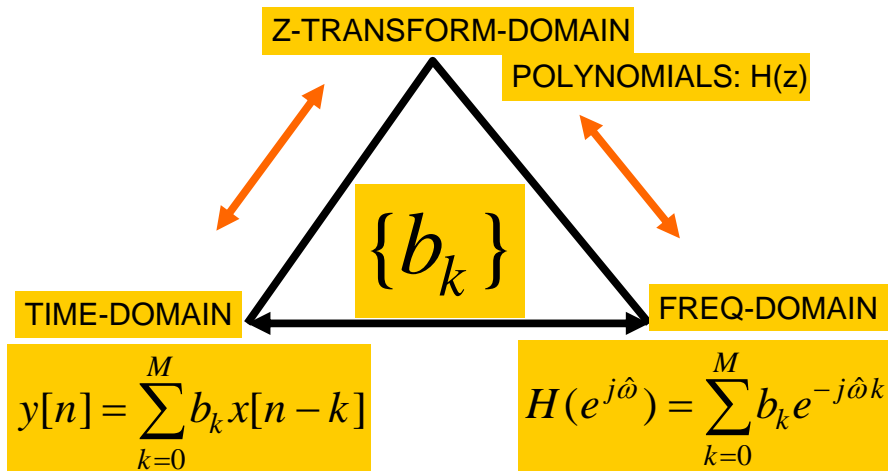
LECTURE OBJECTIVES

- INFINITE IMPULSE RESPONSE FILTERS
 - Define **IIR** DIGITAL Filters
 - Have **FEEDBACK**: use PREVIOUS OUTPUTS

$$y[n] = \sum_{\ell=1}^N a_{\ell} y[n-\ell] + \sum_{k=0}^M b_k x[n-k]$$

- Show how to compute the output $y[n]$
 - FIRST-ORDER CASE ($N=1$)
 - Z-transform: Impulse Response $h[n] \leftrightarrow H(z)$

THREE DOMAINS



Quick Review: Delay by n_d

$$y[n] = x[n - n_d]$$

IMPULSE RESPONSE $h[n] = \delta[n - n_d]$

SYSTEM FUNCTION $H(z) = z^{-n_d}$

FREQUENCY RESPONSE $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}n_d}$

Quick Review: L-pt Averager

$$y[n] = \sum_{k=0}^{L-1} \frac{1}{L} x[n-k]$$

IMPULSE RESPONSE

$$h[n] = \sum_{k=0}^{L-1} \frac{1}{L} \delta[n-k]$$

SYSTEM FUNCTION

$$H(z) = \sum_{n=0}^{L-1} \frac{1}{L} z^{-n}$$

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LOGICAL THREAD

- FIND the IMPULSE RESPONSE, $h[n]$

- INFINITELY LONG

- IIR Filters

$$H(z) = \sum_{n=0}^{\infty} h[n] z^{-n}$$

- EXPLOIT THREE DOMAINS:

- Show Relationship for IIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

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ONE FEEDBACK TERM

- ADD PREVIOUS OUTPUTS

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$



- CAUSALITY

- NOT USING FUTURE OUTPUTS or INPUTS

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FILTER COEFFICIENTS

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB

- `yy = filter([3,-2],[1,-0.8],xx)`

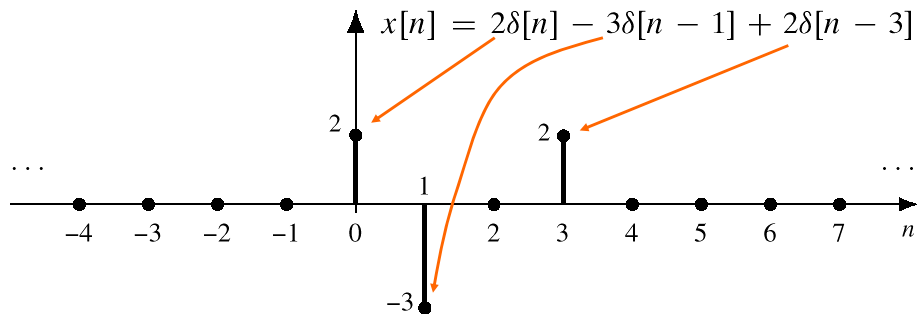
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COMPUTE OUTPUT

$$y[n] = 0.8y[n-1] + 5x[n]$$



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COMPUTE $y[n]$

- FEEDBACK DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 5x[n]$$

- NEED $y[-1]$ to get started

$$y[0] = 0.8y[-1] + 5x[0]$$

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AT REST CONDITION

- $y[n] = 0$, for $n < 0$
- BECAUSE $x[n] = 0$, for $n < 0$

INITIAL REST CONDITIONS

- The input must be assumed to be zero prior to some starting time n_0 , i.e., $x[n] = 0$ for $n < n_0$. We say that such inputs are *suddenly applied*.
- The output is likewise assumed to be zero prior to the starting time of the signal, i.e., $y[n] = 0$ for $n < n_0$. We say that the system is *initially at rest* if its output is zero prior to the application of a suddenly applied input.

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COMPUTE $y[0]$

- THIS STARTS THE RECURSION:

With the initial rest assumption, $y[n] = 0$ for $n < 0$,
 $y[0] = 0.8y[-1] + 5(2) = 0.8(0) + 5(2) = 10$

- SAME with MORE FEEDBACK TERMS

$$y[n] = a_1y[n-1] + a_2y[n-2] + \sum_{k=0}^2 b_k x[n-k]$$

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COMPUTE MORE $y[n]$

- CONTINUE THE RECURSION:

$$y[1] = 0.8y[0] + 5x[1] = 0.8(10) + 5(-3) = -7$$

$$y[2] = 0.8y[1] + 5x[2] = 0.8(-7) + 5(0) = -5.6$$

$$y[3] = 0.8y[2] + 5x[3] = 0.8(-5.6) + 5(2) = 5.52$$

$$y[4] = 0.8y[3] + 5x[4] = 0.8(5.52) + 5(0) = 4.416$$

$$y[5] = 0.8y[4] + 5x[5] = 0.8(4.416) + 5(0) = 3.5328$$

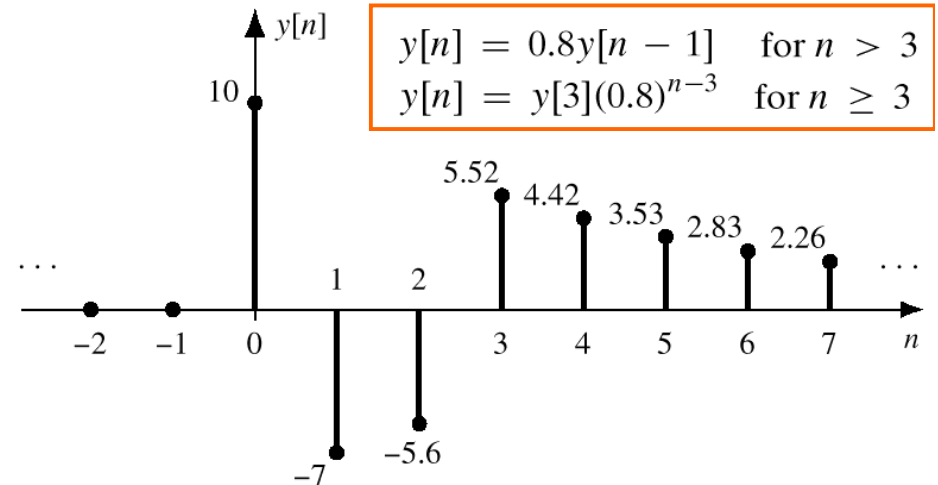
$$y[6] = 0.8y[5] + 5x[6] = 0.8(3.5328) + 5(0) = 2.8262$$

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PLOT $y[n]$



IMPULSE RESPONSE

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

n	$n < 0$	0	1	2	3	4
$\delta[n]$	0	1	0	0	0	0
$h[n-1]$	0	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$
$h[n]$	0	b_0	$b_0(a_1)$	$b_0(a_1)^2$	$b_0(a_1)^3$	$b_0(a_1)^4$

From this table it is obvious that the general formula is

$$h[n] = \begin{cases} b_0(a_1)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$h[n] = b_0(a_1)^n u[n]$$

$$u[n] = 1, \text{ for } n \geq 0$$

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IMPULSE RESPONSE

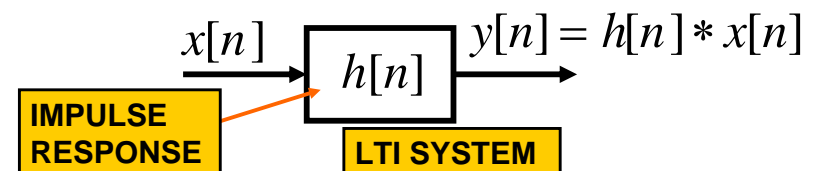
- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n]$$

- Find $h[n]$

$$h[n] = 3(0.8)^n u[n]$$

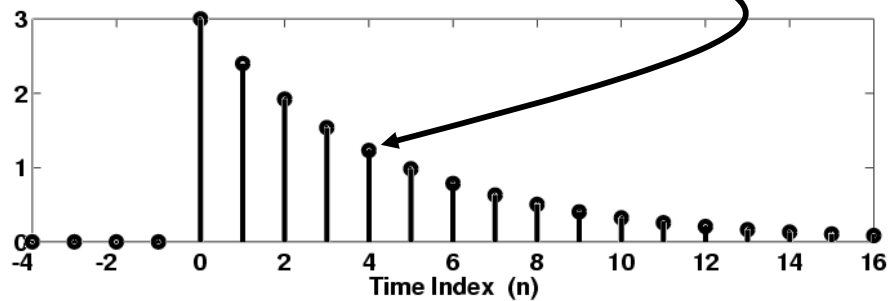
- CONVOLUTION** in TIME-DOMAIN



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PLOT IMPULSE RESPONSE

$$h[n] = b_0(a_1)^n u[n] = 3(0.8)^n u[n]$$



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Infinite-Length Signal: $h[n]$

- POLYNOMIAL Representation

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$$

APPLIES to Any SIGNAL

- SIMPLIFY the SUMMATION

$$H(z) = \sum_{n=-\infty}^{\infty} b_0(a_1)^n u[n]z^{-n} = b_0 \sum_{n=0}^{\infty} a_1^n z^{-n}$$

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Derivation of $H(z)$

- Recall Sum of Geometric Sequence:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

- Yields a COMPACT FORM

$$\begin{aligned} H(z) &= b_0 \sum_{n=0}^{\infty} a_1^n z^{-n} = b_0 \sum_{n=0}^{\infty} (a_1 z^{-1})^n \\ &= \frac{b_0}{1 - a_1 z^{-1}} \quad \text{if } |z| > |a_1| \end{aligned}$$

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$H(z) = z$ -Transform{ $h[n]$ }

- FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$h[n] = b_0(a_1)^n u[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

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$$H(z) = z\text{-Transform}\{ h[n] \}$$

- ANOTHER FIRST-ORDER IIR FILTER:

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$h[n] = b_0 (a_1)^n u[n] + b_1 (a_1)^{n-1} u[n-1]$$

z^{-1} is a shift

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}} + \frac{b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

STEP RESPONSE: $x[n]=u[n]$

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

n	$x[n]$	$y[n]$
$n < 0$	0	0
0	1	b_0
1	1	$b_0 + b_0(a_1)$
2	1	$b_0 + b_0(a_1) + b_0(a_1)^2$
3	1	$b_0(1 + a_1 + a_1^2 + a_1^3)$
4	1	$b_0(1 + a_1 + a_1^2 + a_1^3 + a_1^4)$
...

$u[n] = 1, \text{ for } n \geq 0$

DERIVE STEP RESPONSE

$$y[n] = b_0(1 + a_1 + a_1^2 + \dots + a_1^n) = b_0 \sum_{k=0}^n a_1^k$$

$$\sum_{k=0}^L r^k = \begin{cases} \frac{1 - r^{L+1}}{1 - r} & r \neq 1 \\ L + 1 & r = 1 \end{cases}$$

$$y[n] = b_0 \frac{1 - a_1^{n+1}}{1 - a_1} \quad \text{for } n \geq 0, \quad \text{if } a_1 \neq 1$$

PLOT STEP RESPONSE

