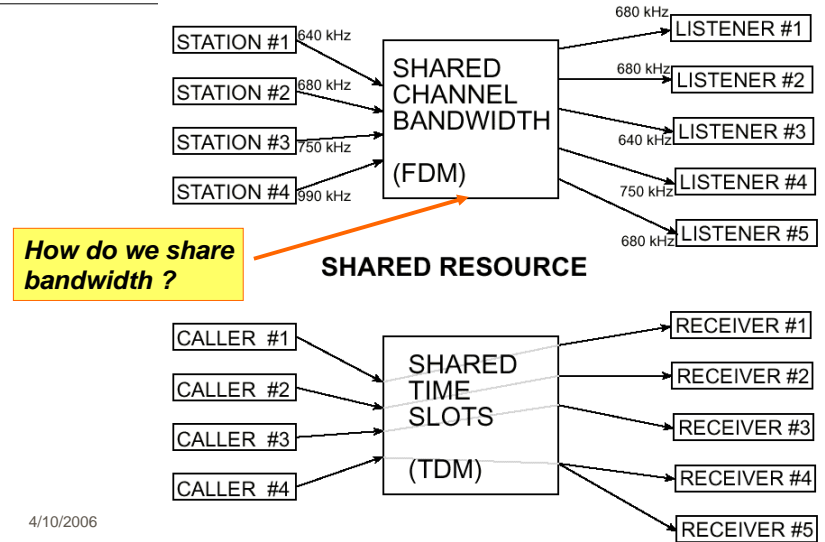


Lecture 23  
Amplitude Modulation (AM)  
14-Apr-06

# The way communication systems work



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## Frequency Allocation (Govt)

<http://www.ntia.doc.gov/osmhome/allochrt.pdf>



## Info: Web-CT, Lab, HW

- Lab #10 due starting on Monday, 17-April
- Lab #11 due starting on Monday 24-April
  - Can be turned in early
- Lab #12 during the last week
  - Done completely in-Lab**
- CHECK YOUR GRADES !!!**
  - Web-CT is the OFFICIAL gradebook
- Quiz #3 will be 21-April (Friday)**
  - Coverage: HW #9, 10, and 11
  - Chapters 8, 9, 10, and 11
  - Review Session, 20-April, Thurs @ 6:00pm

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**Lecture**

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# LECTURE OBJECTIVES

- Review of FT properties
  - Convolution  $\leftrightarrow$  multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM
- Reading: Chapter 12, Section 12-2

# Table of Easy FT Properties

## Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

## Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

## Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

## Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j(\frac{\omega}{a}))$$

# Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

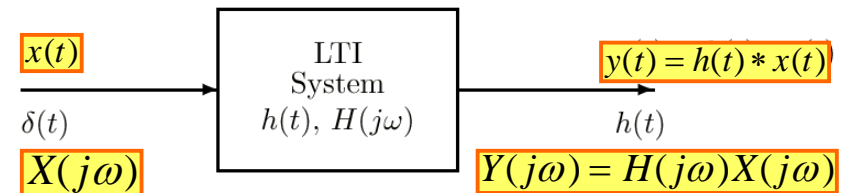
$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

## Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

# Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

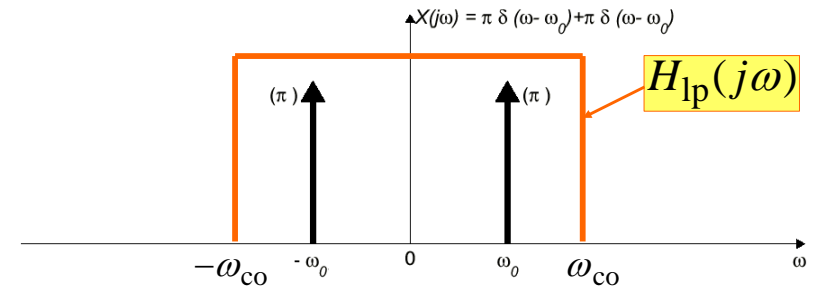
corresponds to **MULTIPLICATION** in the frequency-domain

$$Y(j\omega) = H(j\omega)X(j\omega)$$

# Cosine Input to LTI System

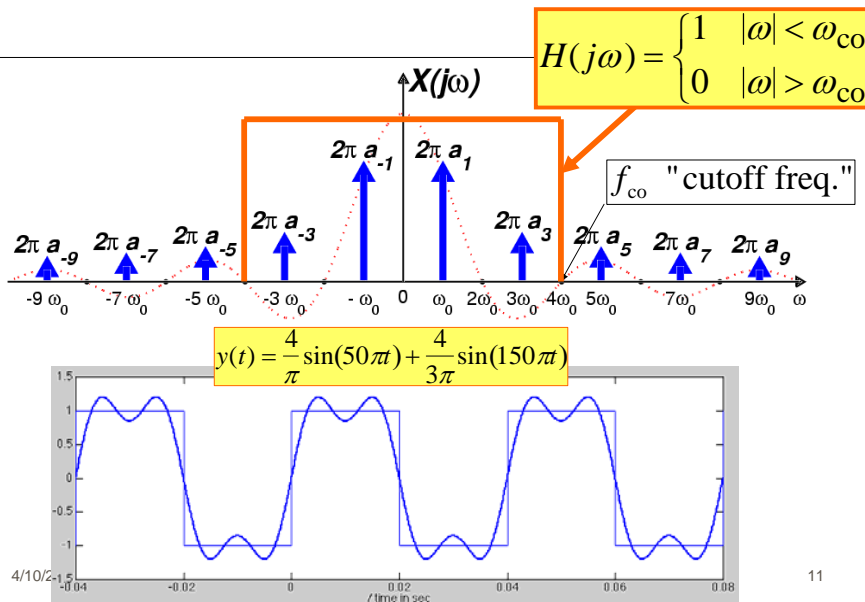
$$\begin{aligned}
 Y(j\omega) &= H(j\omega)X(j\omega) \\
 &= H(j\omega)[\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)] \\
 &= H(j\omega_0)\pi\delta(\omega - \omega_0) + H(-j\omega_0)\pi\delta(\omega + \omega_0) \\
 y(t) &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\
 &= H(j\omega_0)\frac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\frac{1}{2}e^{-j\omega_0 t} \\
 &= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0))
 \end{aligned}$$

# Ideal Lowpass Filter

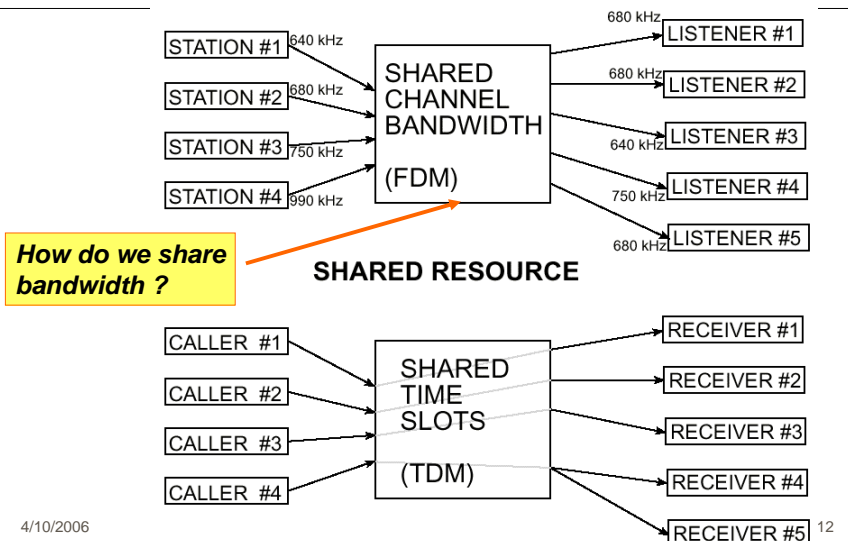


$$\begin{aligned}
 y(t) &= x(t) \quad \text{if } \omega_0 < \omega_{co} \\
 y(t) &= 0 \quad \text{if } \omega_0 > \omega_{co}
 \end{aligned}$$

# Ideal LPF: Fourier Series



# The way communication systems work



How do we share bandwidth?

## Frequency Shifting Property

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt &= \int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt \\ &= X(j(\omega - \omega_0)) \end{aligned}$$

$$y(t) = \frac{\sin 7t}{\pi} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & \text{elsewhere} \end{cases}$$

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## Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

### Differentiation Property

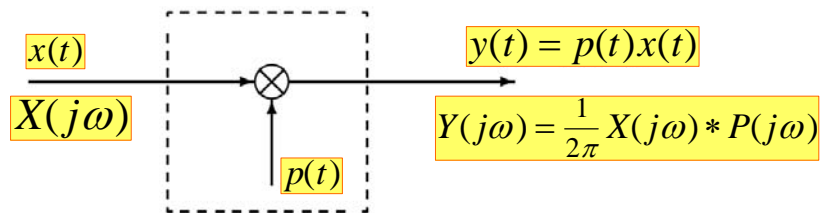
$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

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## Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)P(j(\omega - \theta))d\theta$$

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$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$p(t) = \cos(\omega_c t) \Leftrightarrow$$

$$P(j\omega) = \pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * [\pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)]$$

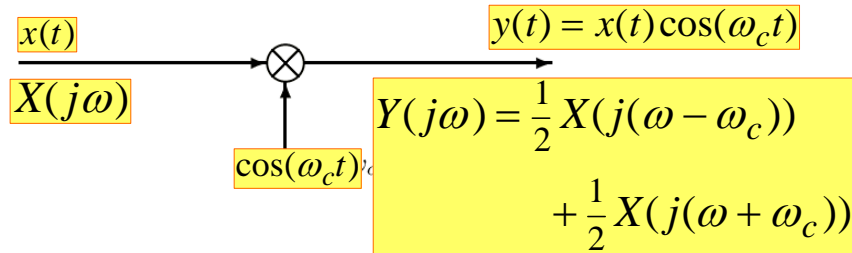
$$Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

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# Amplitude Modulator



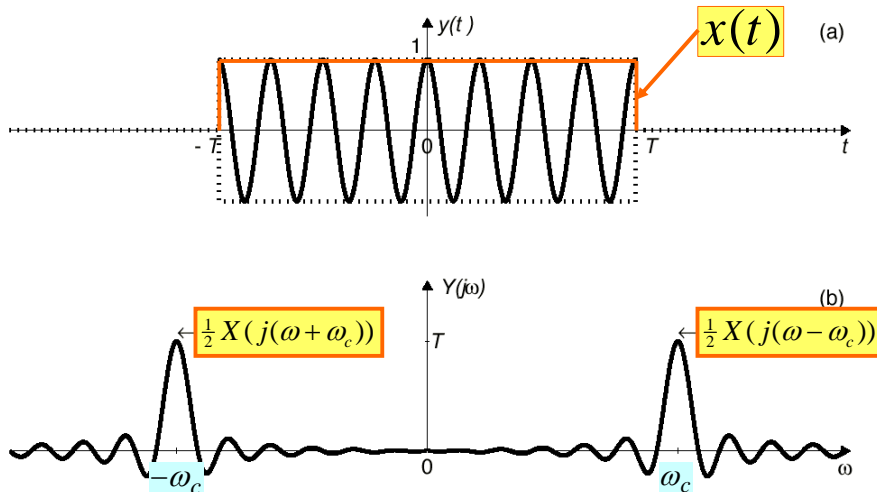
- $x(t)$  **modulates** the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of  $X(j\omega)$ .

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

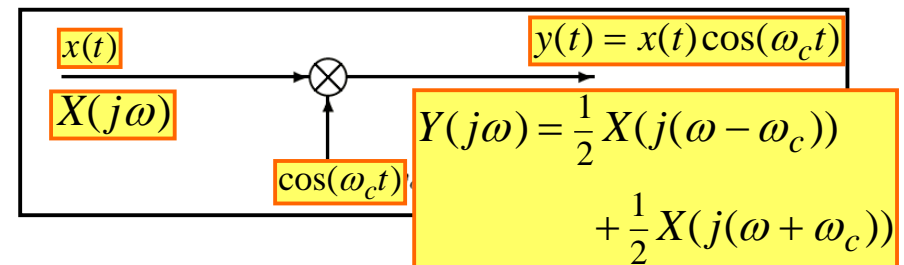
$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2 \frac{\sin(\omega T)}{\omega}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{\sin((\omega - \omega_c)T)}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c)T)}{(\omega + \omega_c)}$$

$$y(t) = x(t) \cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$



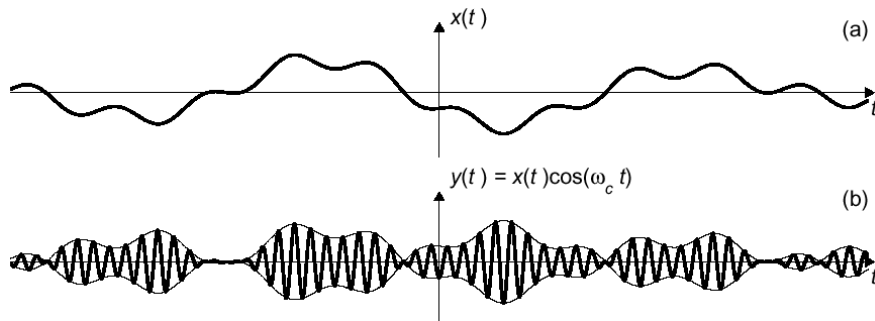
# DSBAM Modulator



- If  $X(j\omega)=0$  for  $|\omega| > \omega_b$  and  $\omega_c > \omega_b$ , the result in the frequency-domain is two shifted and scaled **exact copies** of  $X(j\omega)$ .

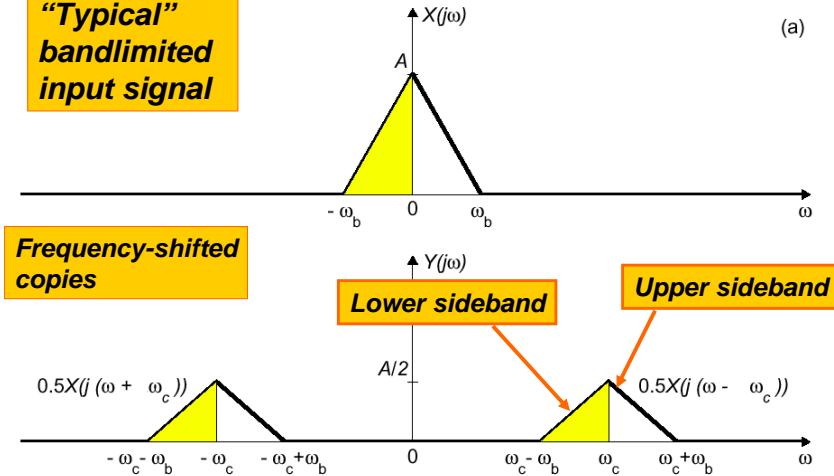
# DSBAM Waveform

- In the time-domain, the “envelope” of sine-wave peaks follows  $|x(t)|$

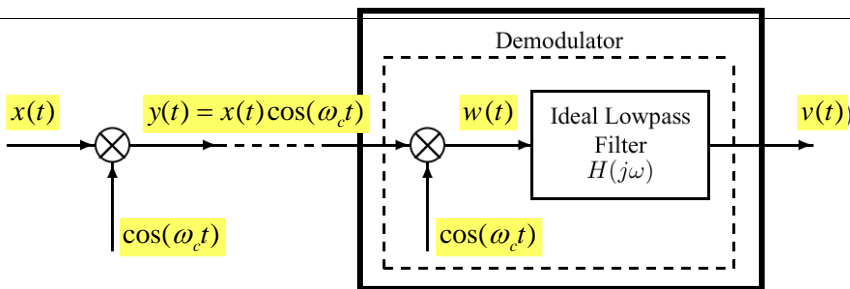


# Double Sideband AM (DSBAM)

“Typical” bandlimited input signal



# DSBAM DEmodulator

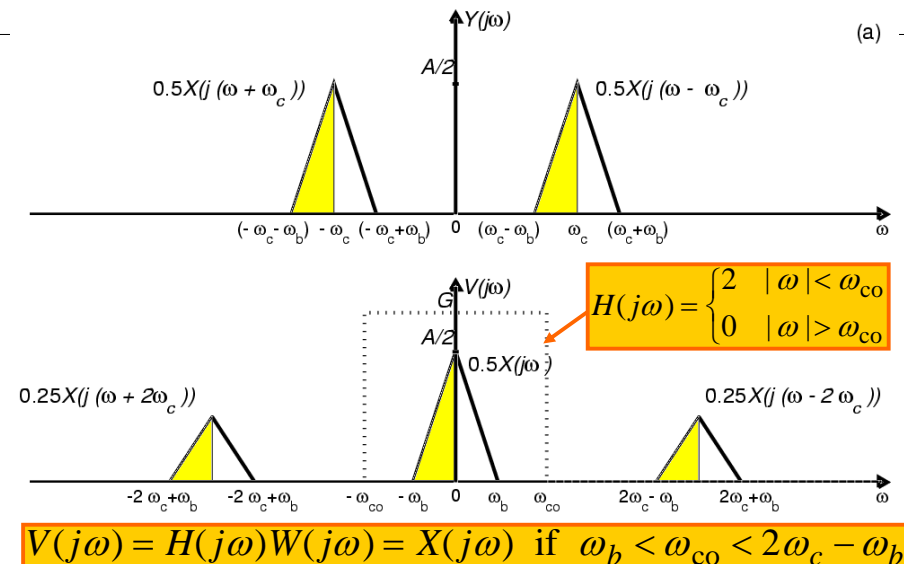


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

# DSBAM Demodulation



$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega)$  if  $\omega_b < \omega_{co} < 2\omega_c - \omega_b$

# Frequency-Division Multiplexing (FDM)

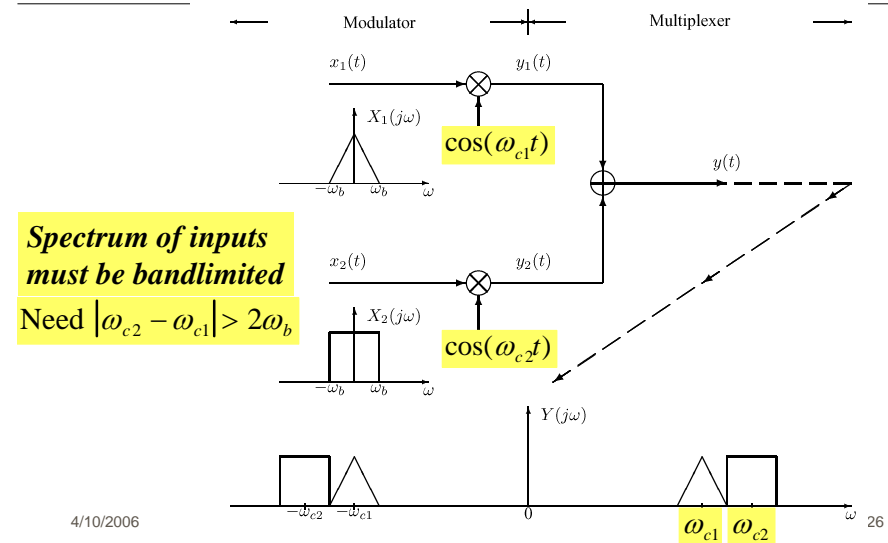
- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple **bandlimited** signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

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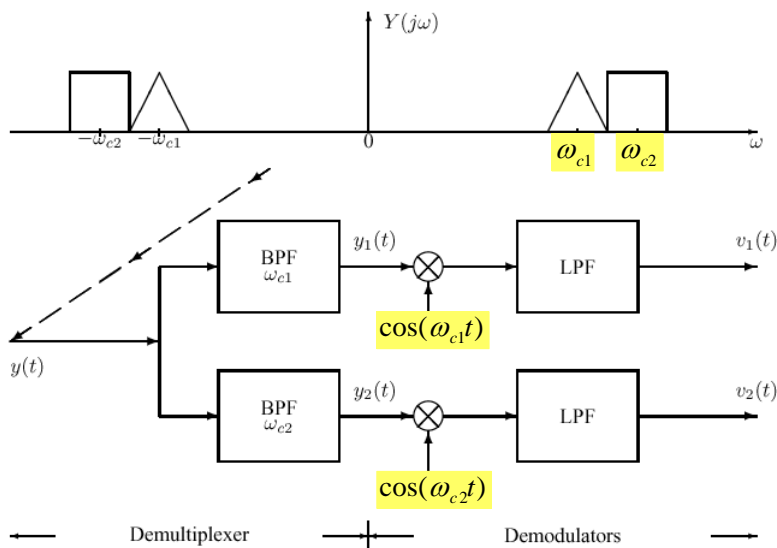
# FDM Block Diagram (Xmitter)



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# Frequency-Division De-Mux



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# Bandpass Filters for De-Mux

