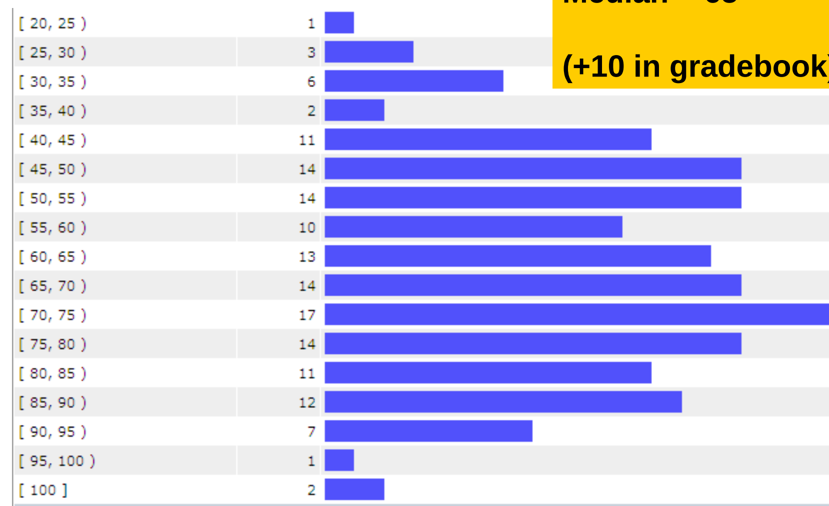
ECE-2025 Spring-2006

Lecture 25
Review: Digital Filtering of Continuous-Time Signals
24-Apr-06

Quiz #3 Results

Average = 64
Median = 65

(+10 in gradebook)



FINAL EXAM

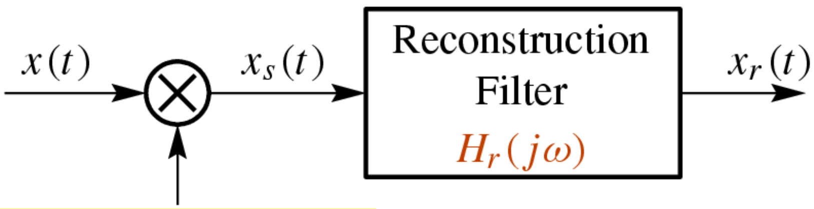
- **Final Exam: Friday @ 2:50pm, 5-May**
    - Review Session on Thursday, 4-May (**6:00pm**)
  - *Bring ID (Buzz Card) to the Final Exam*
  - FORMULA PAGE: **ONE** page **HAND-WRITTEN**
    - **Tables 11.2 and 11.3 will be supplied with the exam**
    - **Z-transform tables also**
- COVERAGE / EMPHASIS?
  - **Fourier Transform**
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Sampling & Aliasing
  - Problems from Quizzes
  - Concepts from Labs #9, #10, #11 and #12
  - **Homework** & Old Quizzes

Lecture

LECTURE OBJECTIVES

- Sampling Theorem Revisited
  - GENERAL: in the FREQUENCY DOMAIN
  - Fourier transform of sampled signal
  - Reconstruction from samples
- **Effective Frequency Response**
- Important FT properties
  - Convolution ←→ multiplication
  - Frequency shifting

Sampling: Freq. Domain



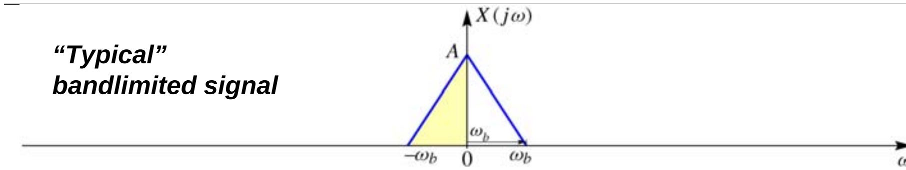
$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

EXPECT FREQUENCY SHIFTING !!!

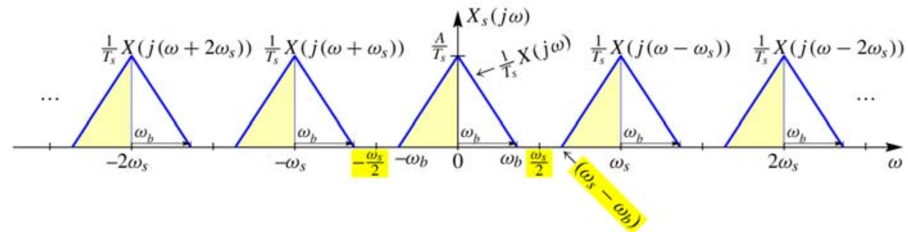
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_s t}$$

Frequency-Domain Representation of Sampling

"Typical" bandlimited signal

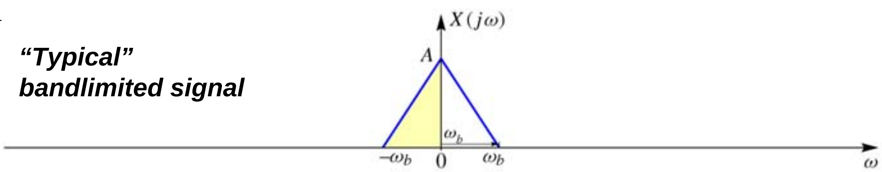


$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

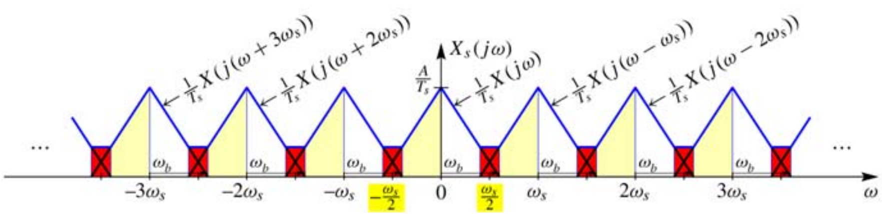


Aliasing Distortion

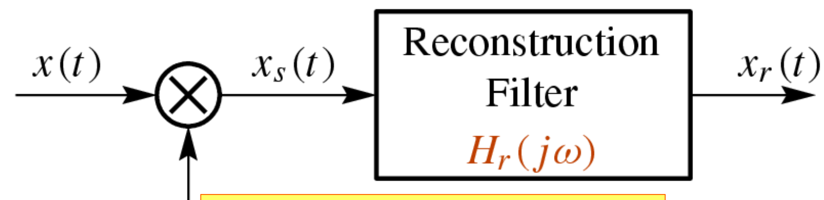
"Typical" bandlimited signal



- If $\omega_s < 2\omega_b$, the copies of $X(j\omega)$ overlap, and we have aliasing distortion.



Reconstruction of $x(t)$

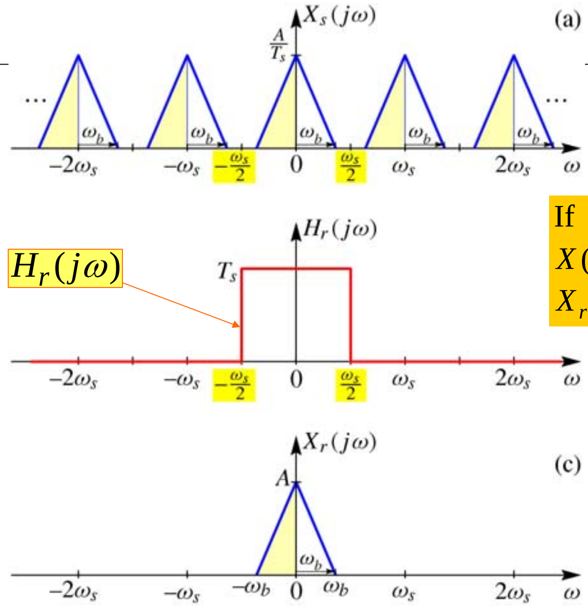


$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega) X_s(j\omega)$$

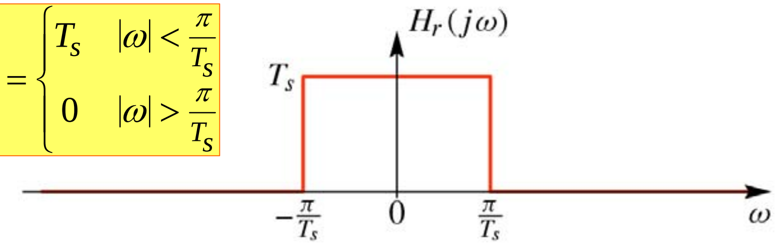
Reconstruction: Frequency-Domain



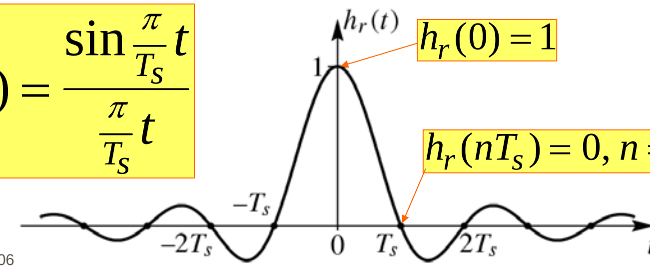
If $\omega_s > 2\omega_b$, the copies of $X(j\omega)$ do not overlap, so $X_r(j\omega) = H_r(j\omega)X_s(j\omega)$

Ideal Reconstruction Filter

$$H_r(j\omega) = \begin{cases} T_s & |\omega| < \frac{\pi}{T_s} \\ 0 & |\omega| > \frac{\pi}{T_s} \end{cases}$$



$$h_r(t) = \frac{\sin\frac{\pi}{T_s}t}{\frac{\pi}{T_s}t}$$



Signal Reconstruction

$$x_r(t) = h_r(t) * x_s(t) = h_r(t) * \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)h_r(t - nT_s)$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

Ideal bandlimited interpolation formula

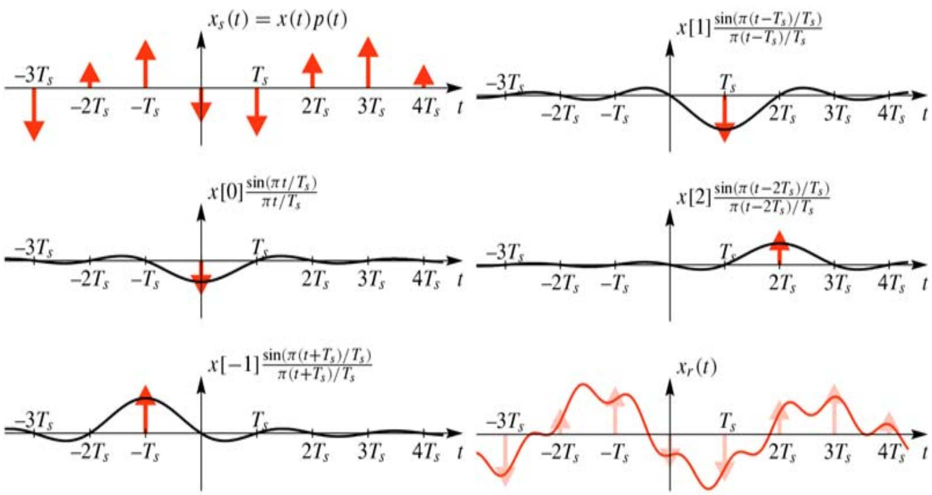
Shannon Sampling Theorem

- **"SINC" Interpolation** is the ideal
  - PERFECT RECONSTRUCTION
  - of BANDLIMITED SIGNALS

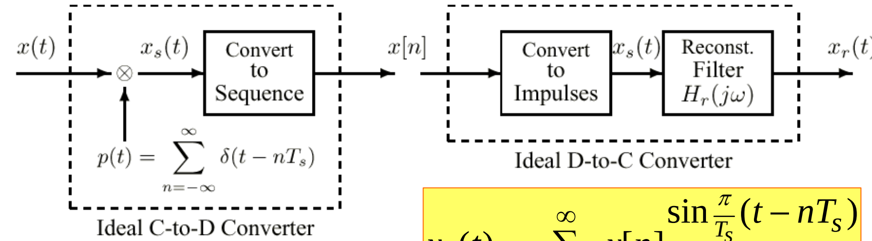
A signal $x(t)$ with bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ can be reconstructed exactly from samples taken with sampling rate $\omega_s = 2\pi/T_s \geq 2\omega_b$ using the following bandlimited interpolation formula:

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\left[\frac{\pi}{T_s}(t - nT_s)\right]}{\frac{\pi}{T_s}(t - nT_s)}.$$

Reconstruction in Time-Domain



Ideal C-to-D and D-to-C



$x[n] = x(nT_s)$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin \frac{\pi}{T_s}(t - nT_s)}{\frac{\pi}{T_s}(t - nT_s)}$$

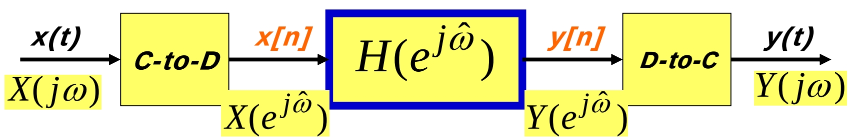
Ideal Sampler

Ideal bandlimited interpolator

$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$X_r(j\omega) = H_r(j\omega) X_s(j\omega)$$

DT Filtering of CT Signals

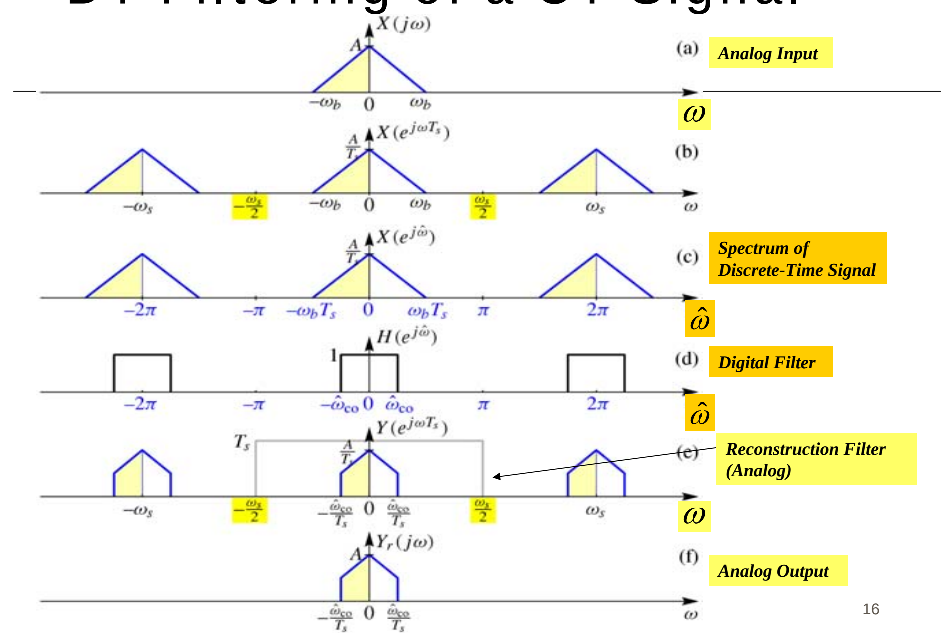


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega) X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ \textbf{UNDEFINED NOT LTI} & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

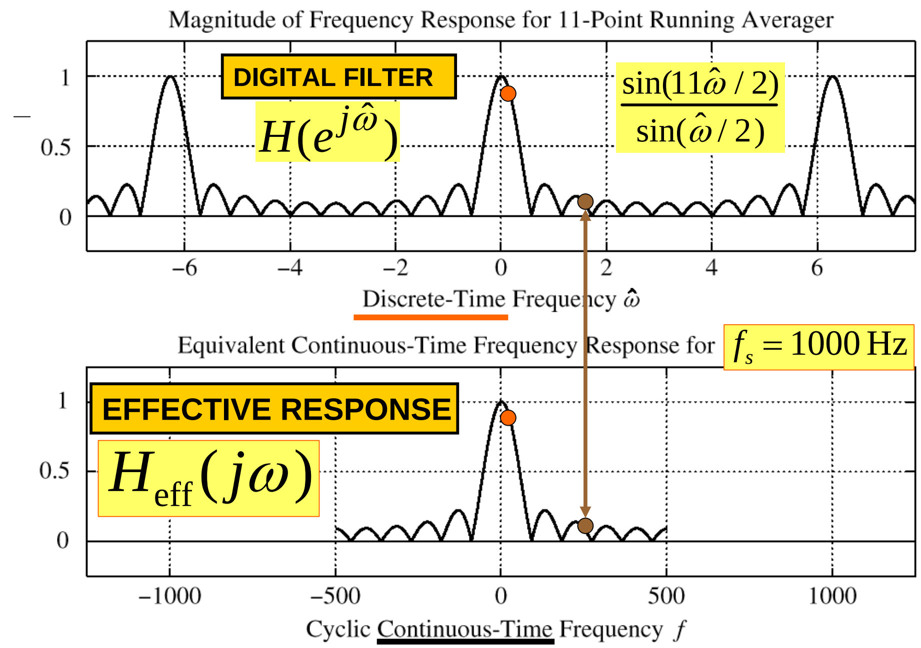
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

  - So, we can plot:
  - Scaled Freq. Axis

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

DIGITAL FILTER

ANALOG FREQUENCY



H_{eff} for 11-pt Averager

- Frequency Response for Discrete-time

$$H(e^{j\hat{\omega}}) = \frac{\sin(11\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

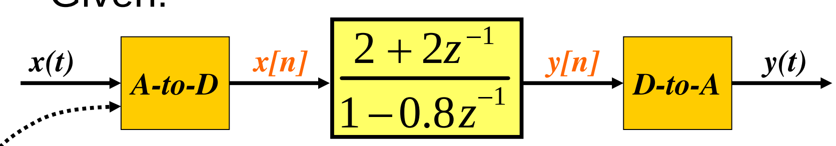
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s} = \frac{\omega}{1000}$$

- Analog Frequency Response

$$H_{\text{eff}}(j\omega) = \frac{\sin(11\omega/2000)}{\sin(\omega/2000)}$$

POP QUIZ

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(2\pi(1000)t)$

$$f_s = 5000\,\text{Hz}$$

Effective Frequency Response

- Given: $H(z)=\dfrac{2+2z^{-1}}{1-0.8z^{-1}}$

NO Aliasing, Because 2(1000) < 5000

- The discrete-time frequency response is

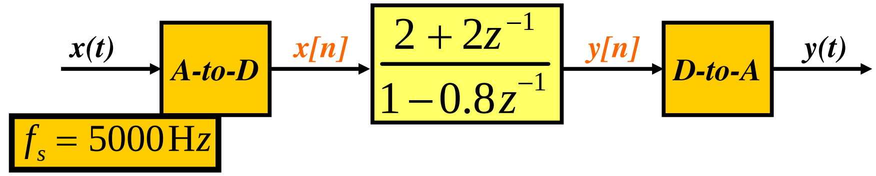
$$H(e^{j\hat{\omega}})=\frac{2+2e^{-j\hat{\omega}}}{1-0.8e^{-j\hat{\omega}}}$$

- Then the Effective Frequency Response is

$$H(j\omega)=\frac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$$

Equivalent Systems

- Given:



- "Effective Analog System" for $\omega<(2\pi f_s)/2$

$x(t)$ → $$H(j\omega)=\frac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$$ → $y(t)$

POP QUIZ ANSWER

$f_s = 5000\,\text{Hz}$

- Given:

$x(t)=\cos(2000\pi t)$ → $$H(j\omega)=\frac{2+2e^{-j\omega/5000}}{1-0.8e^{-j\omega/5000}}$$ → $y(t)$

$$H(j\omega)\Big|_{\omega=2\pi(1000)}=\frac{2+2e^{-j2000\pi/5000}}{1-0.8e^{-j2000\pi/5000}}$$

$$=3.02e^{-j0.452\pi}$$

- The output is

$$y(t)=3.02\cos(2000\pi t-0.452\pi)$$