ECE-2025 Spring-2006

Lecture 26
Review
28-Apr-06

FINAL EXAM

- **Final Exam: Friday @ 2:50pm, 5-May**
    - Review Session on Thursday, 4-May (**6:00pm**)
  - *Bring ID (Buzz Card) to the Final Exam*
  - FORMULA PAGE: **ONE** page **HAND-WRITTEN**
    - **Tables 11.2 and 11.3 will be supplied with the exam**
    - **Z-transform tables also**
- COVERAGE / EMPHASIS?
  - **Fourier Transform**
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Sampling & Aliasing
  - Problems from Quizzes
  - Concepts from Labs #9, #10, #11 and #12
  - **Homework** & Old Quizzes

Lecture

Senior Design Course(s)

- Graduation requires
  - ECE-4000 *Project Engineering*
  - ECE-4006 *Design Project*
    - Can specialize in different areas, e.g., DSP
    - Real-Time DSP Projects
- DSP concentration
    - ECE-3075 *Random Signals*
    - ECE-4270 *DSP*
    - ECE-4271 *Applications of DSP*
    - ECE-4273 *ASICs for DSP*

Lecture

LECTURE OBJECTIVES

- Review
  - Fourier Theory and Frequency Content
  - Spectrograms
  - Filtering (LTI systems)
  - Sampling (and Aliasing)
  - Digital Filters: h[n], H(z), Frequency Response

Mathematical Elegance

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

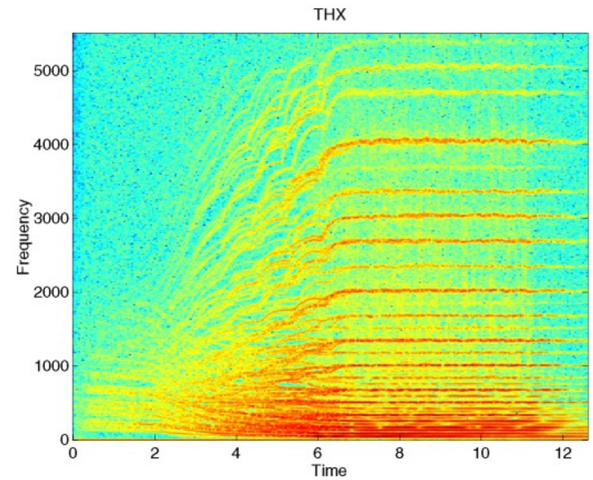
Fourier S
(Inverse

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier Analysis
(Forward Transform)

$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$

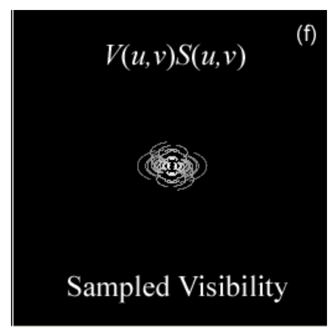
THX SPECTROGRAM



Radio Astronomy - What

Images courtesy NRAO/AUI
(www.vla.nrao.edu)

Radio Astronomy - How



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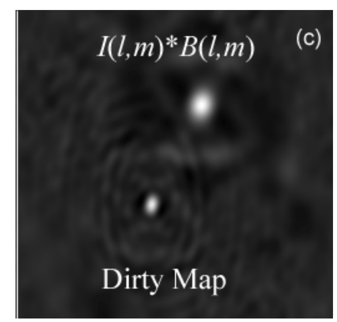

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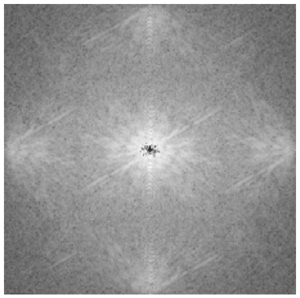
Desired Image

Images by **Dale E. Gary**
From physics.njit.edu/~dgary/728/Lecture6.html

Magnetic Resonance Imaging



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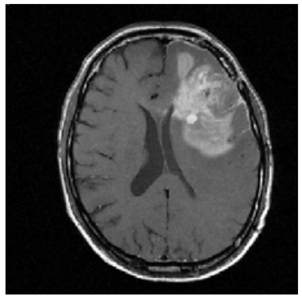

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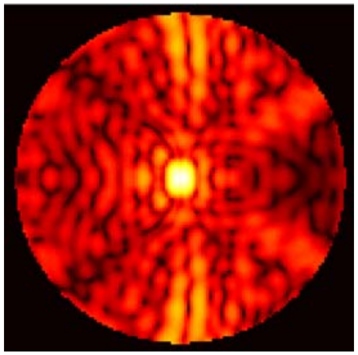
Images by **Cynthia B. Paschal** from www.bme.vanderbilt.edu/Paschal

Radar Imaging



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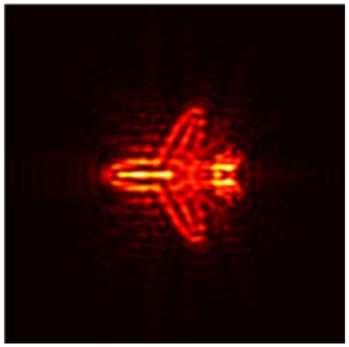

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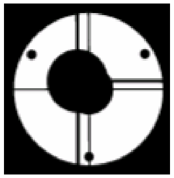
Images from Aaron Lanterman's research

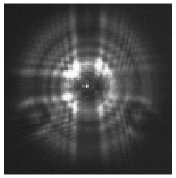
Yes, FTs show up in optics

- Characterize how an optical system blurs images you're trying to make, like on the Hubble Space Telescope

Aperture function



FT

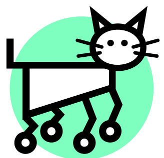


2-D Impulse response

Images by **James R. Fienup** from http://cfao.ucolick.org/presentations/springretreat2003/SRO3_Fienup_Hubble.pdf

Other places FTs show up

- Fourier theory tells how to do X-ray Computer Aided Tomography (CAT scans)
  - "Projection Slice Theorem"



- Under some approximations, FTs characterize antenna patterns (so yes, they show up in electromagnetics!)
- Determining protein shapes with X-ray crystallography and NMR

IMPORTANT CONCEPTS

- ALL Signals have Frequency Content
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- FILTERS alter the Frequency Content
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- 3 Domains for Analysis

SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if} \quad x[n] = e^{j\hat{\omega}n} \quad \text{then}$$

$$y[n] = H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$$

$$\text{where} \quad H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

THREE DOMAINS

$$H(z) = \frac{\sum b_k z^{-k}}{1 - \sum a_\ell z^{-\ell}}$$

$$z = e^{j\hat{\omega}}$$

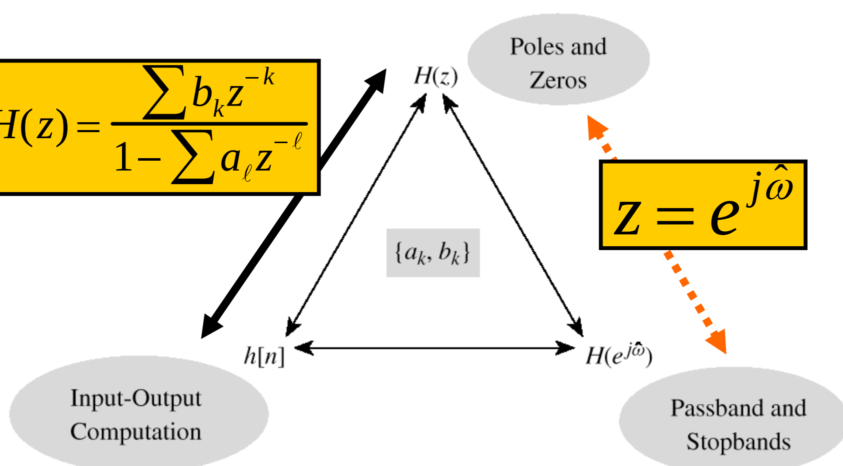
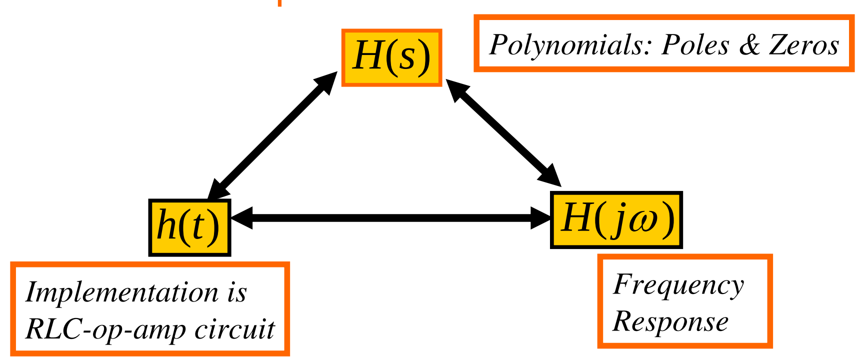


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

THE FUTURE

- Circuits & Laplace Transforms



Shannon Sampling Theorem

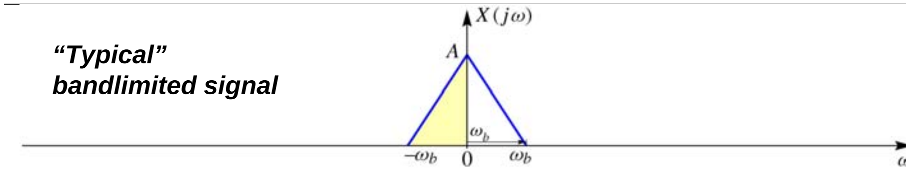
- **"SINC" Interpolation** is the ideal
  - PERFECT RECONSTRUCTION
  - of BANDLIMITED SIGNALS

A signal $x(t)$ with bandlimited Fourier transform such that $X(j\omega) = 0$ for $|\omega| \geq \omega_b$ can be reconstructed exactly from samples taken with sampling rate $\omega_s = 2\pi/T_s \geq 2\omega_b$ using the following bandlimited interpolation formula:

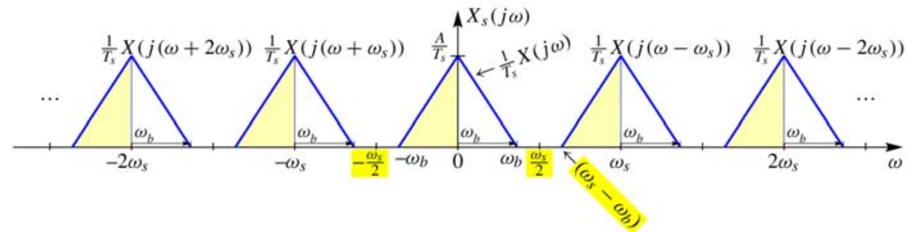
$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin\left[\frac{\pi}{T_s}(t - nT_s)\right]}{\frac{\pi}{T_s}(t - nT_s)}.$$

Frequency-Domain Representation of Sampling

"Typical" bandlimited signal



$$X_s(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$



INSTANTANEOUS FREQ of the Chirp

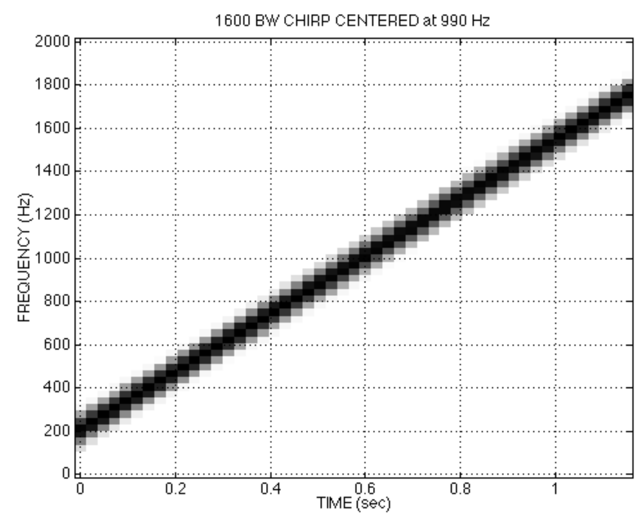
- Chirp Signals have Quadratic phase
- Freq will change LINEARLY vs. time

$$x(t) = A\cos(\alpha t^2 + \beta t + \varphi)$$

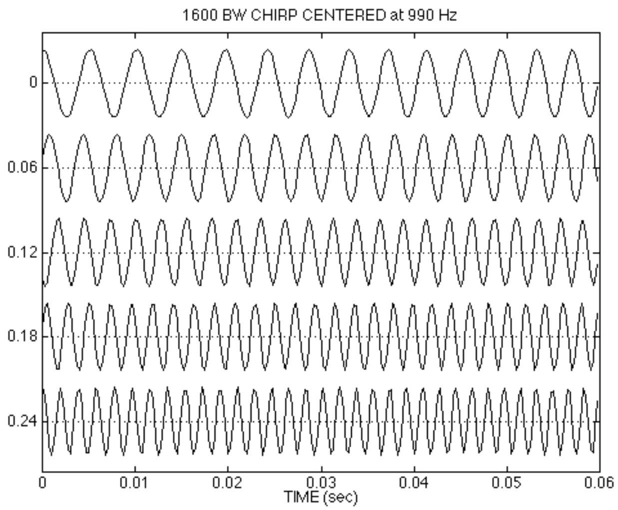
$$\Rightarrow \psi(t) = \alpha t^2 + \beta t + \varphi$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t) = 2\alpha t + \beta$$

CHIRP SPECTROGRAM



CHIRP WAVEFORM



OTHER CHIRPS

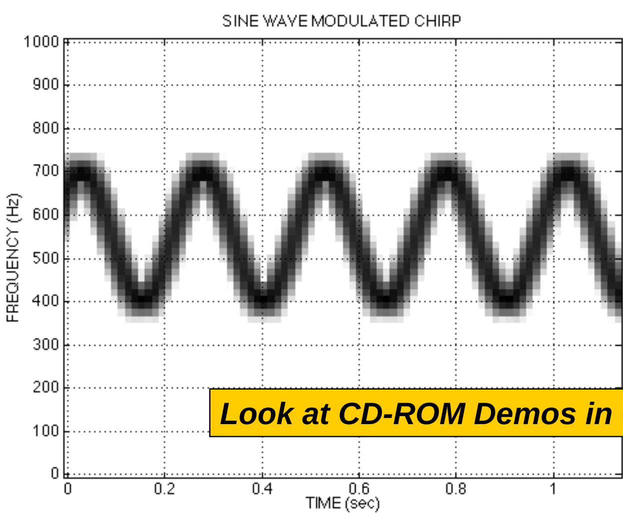
- $\psi(t)$ can be anything:

$$x(t) = A\cos(\alpha\cos(\beta t) + \varphi)$$

$$\Rightarrow \omega_i(t) = \frac{d}{dt}\psi(t) = -\alpha\sin(\beta t)$$

- $\psi(t)$ could be speech or music:
  - FM radio broadcast

SINE-WAVE FREQUENCY MODULATION (FM)



Look at CD-ROM Demos in Ch 3

The END

- Education is what survives when what has been learned has been forgotten
  - B.F. Skinner