GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2006

Lab #11: GUIs for Continuous-Time Signals & Systems

Date: 12–18 April 2006

You should read the Pre-Lab section of the lab and do all the exercises in the Pre-Lab section before your assigned lab time.

The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. After completing the warm-up section, turn in the verification sheet to your TA.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss lab exercises with other students and you are allowed to consult old lab reports, but you cannot give or receive written material or electronic files. Your submitted work should be original and it should be your own work.

The lab report for this week will be an **Informal Lab Report.** It is only necessary to turn in Section 4 as this week's lab report. The report will **due the next time your lab meets.**

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1 Introduction

This lab concentrates on the use of three MATLAB GUIs for convolution:

1. **`cconvdemo:`** GUI for continuous-time convolution.

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \qquad (1)$$

2. **`CLTIdemo:`** GUI for continuous-time filtering of sinusoids with a linear, time-invariant (LTI) system

$$y(t) = A|H(j\omega_1)|\cos(\omega_1 t + \phi + \angle H(j\omega_1))$$

   when the input signal is a sinusoid, $x(t) = A\cos(\omega_1 t + \phi)$.

3. **`FseriesDemo:`** GUI for continuous-time Fourier Series.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

   where $x(t)$ is a periodic signal and ω_0 is the fundamental frequency, $\omega_0 = 2\pi/T$.

Each one of these demos illustrates an important point about the behavior of a continuous-time signals and systems. They also provide a convenient way to visualize the output of a continuous-time LTI systems.

All of these demos are available in the *SP-First* Toolbox, or they can be downloaded from the following web page:

`http://users.ece.gatech.edu/mcclella/matlabGUIs/index.html`

2 Pre-Lab: Run the GUIs

Several GUIs have been introduced during lectures over the past few weeks. The first objective of this lab is to demonstrate usage of several GUIs.

2.1 Continuous-Time Convolution Demo

In this demo, you can select an input signal $x(t)$, as well as the impulse response of an **ANALOG** filter $h(t)$. Then the demo shows the "flipping and shifting" used when a convolution integral is performed. Figure 1 shows the interface for the `cconvdemo` GUI.

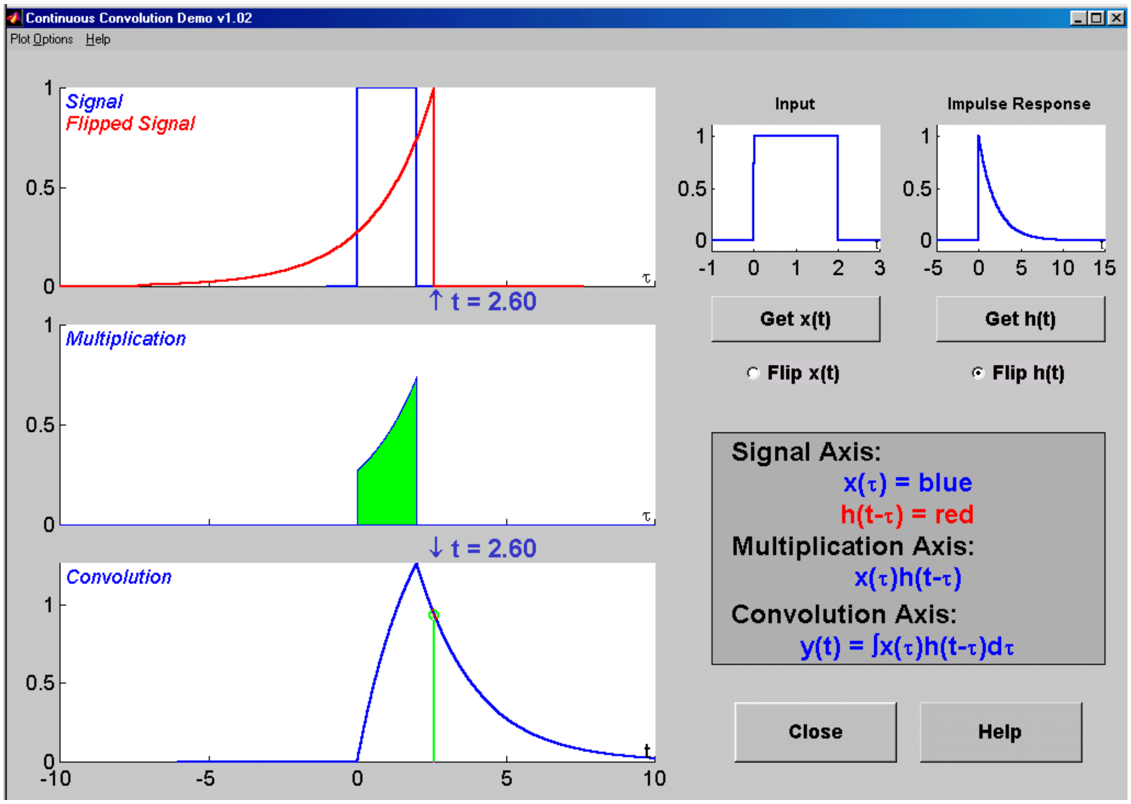

Figure 1: Interface for the continuous-time convolution GUI **`cconvdemo`**.

In the Pre-Lab, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a 4-second pulse $x(t) = u(t) - u(t-4)$.

(b) Set the filter's impulse response to a shifted impulse, i.e., $h(t) = \delta(t-3)$.

(c) Use the GUI to produce the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution.

(d) Set the input to a different shifted impulse, i.e., $x(t) = \delta(t-2)$. Use the GUI to produce the output signal. Notice that when flipping and sliding that there is only one time where the signals overlap.

(e) Compare the outputs from parts (c) and (d). Use properties of the impulse signal to explain the different outputs.

2.2 Sinusoidal Response (CLTIdemo)

In this demo, you can select an input signal that is a sinusoid, and see the change created by the frequency response. This demo reinforces the concept that "sinusoid in gives sinusoid out." Figure 2 shows the interface for the `CLTIdemo`. We know that if the input to an LTI continuous-time system is a sinusoid of

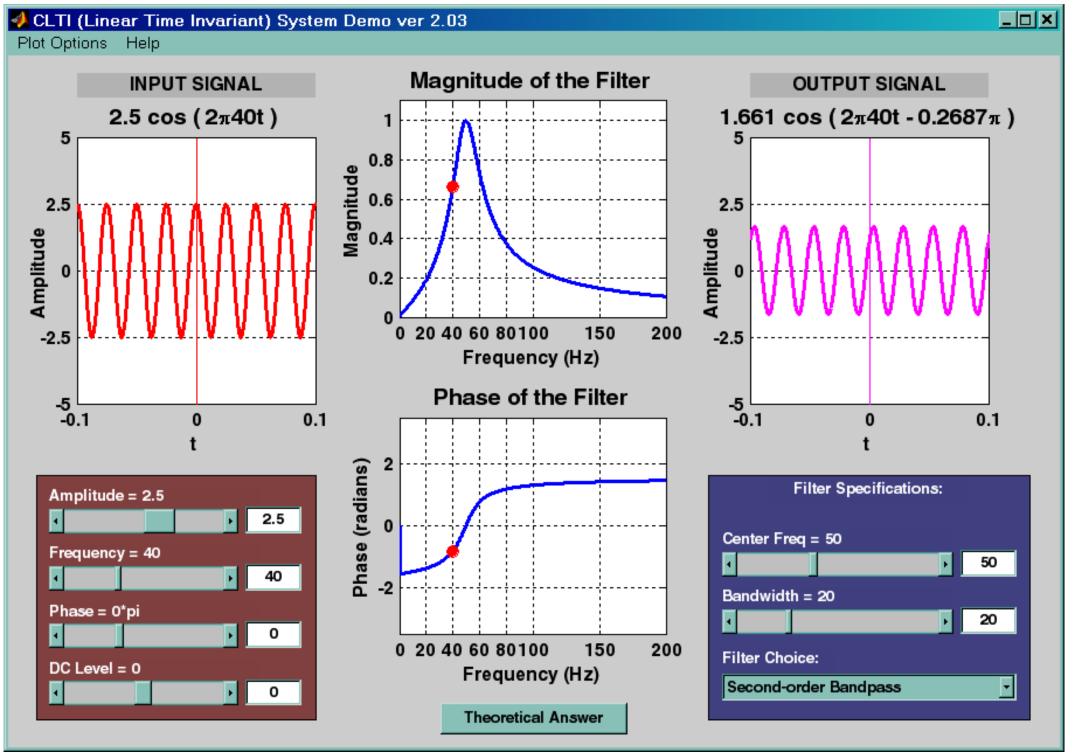

Figure 2: `CLTIdemo` interface for continuous-time frequency response.

the form

$$x(t) = A + B\cos(\omega_1 t + \phi) \qquad -\infty < t < \infty \tag{2}$$

then the corresponding output is also a sinusoid:

$$y(t) = AH(j0) + B|H(j\omega_1)|\cos(\omega_1 t + \phi + \angle H(j\omega_1)) \qquad -\infty < t < \infty, \tag{3}$$

where $H(j\omega)$ is the *frequency response* of the continuous-time LTI system. The **`CLTIdemo`** GUI illustrates this for a variety of simple analog filters.

(a) Use the `CLTIdemo` GUI to find the output of a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to 30 Hz. Recall that the frequency response of this lowpass filter is

$$H(j\omega) = \frac{1}{j\omega + a} \tag{4}$$

where a is the cutoff frequency in rads/sec.

(b) Set the input to

$$x(t) = 1.0 + \cos(20\pi t).$$

Look at the output and compare its amplitude and phase to the input amplitude and phase. Click the box labeled "Theoretical Answer" to see a formula for the output $y(t)$.
Note: The GUI input frequencies are in hertz, which is $f = \omega/(2\pi)$; ω would have units of rad/s.

(c) Keeping the DC level and the amplitude of the cosine the same, use the slider to increase the input frequency and observe the change in the output. Keep increasing the slider until the frequency is $\omega = 80\pi$ rad/s (or $f = 40$ Hz). Compare the output in this case to the output at the original frequency of $\omega = 20\pi$. If you were to describe the output as having a "ripple", does the ripple increase or decrease as ω increases?

(d) Repeat the previous part with the filter set to "Ideal Lowpass" with a cutoff frequency of 30 Hz. Start with the input signal from part (a).

(e) Set the frequency of the input back to $\omega = 20\pi$ and change the filter to "First-Order Highpass" with a cutoff frequency of 30 Hz. Observe the output as the frequency is increased. What is the DC component of the output? Does the amplitude of the output sinusoid get bigger or smaller as the frequency is increased?

(f) Convince yourself that the following frequency response is a first-order HPF:

$$H(j\omega) = \frac{j\omega}{j\omega + b}$$

where the parameter b is the cutoff frequency of the HPF in rad/s.

2.3 Spectrum from Fourier Series

Use the FSeriesDemo GUI (Fig. 3) to show the spectrum for a 50% duty cycle square wave. Notice that the GUI also shows the resynthesized signal (in red) for a finite number of coefficients.

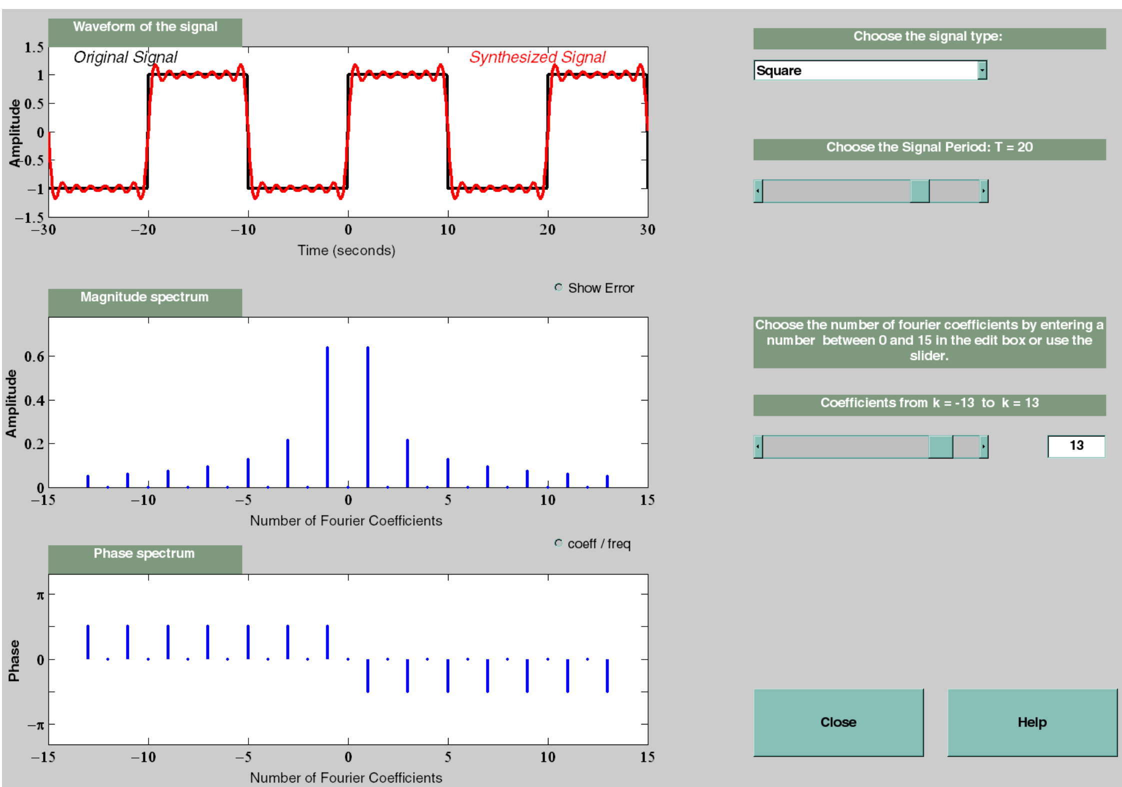

Figure 3: FSeriesDemo interface for Fourier Series synthesis. Moving the mouse-pointer over the spectrum lines will display the Magnitude or Phase of the corresponding harmonic.

3 Warm-up: Compute Outputs with the GUIs

The objective of the warm-up in this lab is to use the GUIs to study how systems process different kinds of signals. Write down your observations on the *Verification Sheet.*

3.1 Convolving Continuous-Time Signals

In the `cconvdemo` GUI, you can select an input signal $x(t)$, as well as the impulse response of an ***analog*** filter $h(t)$. Then the demo shows the "flipping and shifting" used when a convolution integral is performed. Figure 1 shows the interface for the `cconvdemo` GUI.

3.1.1 Convolving Rectangles

In the Warm-up, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a 4-second pulse $x(t) = u(t) - u(t-4)$.

(b) Set the filter's impulse response to a 2-second pulse with amplitude $\frac{1}{2}$, i.e., $h(t) = \frac{1}{2}\{u(t) - u(t-2)\}$.

(c) Use the GUI to produce the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution.

(d) Set the filter's impulse response to a 4-second pulse with amplitude $\frac{1}{4}$, i.e., $h(t) = \frac{1}{4}\{u(t) - u(t-4)\}$. Use the GUI to produce the output signal.

(e) Set the filter's impulse response to a shifted impulse, i.e., $h(t) = \delta(t-3)$. Use the GUI to produce the output signal.

(f) Compare the outputs from parts (c), (d) and (e). Notice the different shapes (triangle, rectangle or trapezoid), the different maximum values, and the different lengths of the outputs. Be prepared to explain these differences.
If the duration of $x(t)$ is T_x and the duration of $h(t)$ is T_h, what will the duration of $y(t)$ be?

Instructor Verification (separate page)

3.1.2 Convolving Exponentials

In the warm-up, you should perform the following steps with the `cconvdemo` GUI.

(a) Set the input to a rectangular pulse: $x(t) = \{u(t) - u(t-3)\}$.

(b) Set the filter's impulse response to an exponential: $h(t) = e^{-0.25t}\,\{u(t) - u(t-7)\}$.

(c) Use the GUI to produce a plot of the output signal. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution. Note: if you move the hand tool past the end of the plot, the plot will automatically scroll in that direction.

(d) The top panel is a plot of $x(\tau)$, overlayed with the "flipped" impulse response $h(t-\tau)$ used to produce the "flip and slide" effect of convolution. The middle panel shows the integrand which is the product of $x(\tau)$ and $h(t-\tau)$. The top two plots are functions of τ, while the bottom plot of $y(t)$ is a function of t. Observe that the output $y(t)$ is composed of five distinct regions: no overlap (on the left side), partial overlap (on the left side), complete overlap, partial overlap (on the right side), and no overlap (on the right side). If you substitute $x(t)$ and $h(t)$ from from parts (a) and (b) into Eq. (1), you can show that the output is given by the piecewise equation

$$y(t) = \begin{cases} 0 & t < T_0 & \text{Region } 1 \\ \int_{L_1}^{L_2} e^{-0.25(t-\tau)} d\tau & T_1 \le t < T_2 & \text{Region } 2 \\ \int_{L_3}^{L_4} e^{-0.25(t-\tau)} d\tau & T_3 \le t < T_4 & \text{Region } 3 \\ \int_{L_5}^{L_6} e^{-0.25(t-\tau)} d\tau & T_5 \le t < T_6 & \text{Region } 4 \\ 0 & T_7 \le t & \text{Region } 5 \end{cases} \tag{5}$$

Use the GUI to observe that $y(t)$ does indeed have five distinct regions, and use it to confirm that

$$T_0 = 0,\ T_1 = 0,\ T_2 = 3,\ T_3 = 3,\ T_4 = 7,\ T_5 = 7,\ T_6 = 10,\ \text{and}\ T_7 = 10$$

are the correct values for the boundaries of the regions. Then determine the limits of integration for each integral above. Make sure that you are flipping and sliding $h(t)$. *Also, notice that the limits of integration might depend on the variable t.*

Instructor Verification (separate page)

3.2 Frequency Response of an Analog Filter

In this section of the warm-up, we will investigate the following frequency response:

$$H(j\omega) = \frac{jb\omega}{(\omega_c^2 - \omega^2) + jb\omega} \tag{6}$$

where ω_c is the center frequency, and the parameter b controls the bandwidth of the filter. This is a second-order frequency response that will be used in the lab project for filtering continuous-time signals.

(a) Make a plot of the magnitude and phase of $H(j\omega)$ versus ω in rad/s. Pick the parameters of the frequency response to be $\omega_c = 40\pi$, and $b = 20\pi$. In order to get enough values for a smooth plot, you should evaluate the $H(j\omega)$ formula directly for a dense grid of frequencies. Use a range of frequencies that extends from -500 rad/s to $+500$ rad/s.[1] From the plot of $|H(j\omega)|$ versus ω, determine what kind of filter $H(j\omega)$ is.

(b) Determine the peak value of the magnitude (frequency) response and the location of the peak. Use the algebraic form of the frequency response formula $H(j\omega)$ to explain why the peak value, and location, are correct.

Instructor Verification (separate page)

4 Lab Exercises

In each of the following exercises, you should make a screen shot of the final picture produced by the GUI to validate that you were able to do the implementation. In all cases, you will have to do some mathematical calculations to verify that the MATLAB GUI result is correct.

[1] You can plot the frequency response versus frequency in hertz or radians/sec. Either way is acceptable, but make sure that you label the horizontal axis.

4.1 Convolving Finite-Duration Signals

In the first exercise, you should perform the following steps with the **cconvdemo** GUI.

(a) Set the input signal to a rectangular pulse; $x(t) = \{u(t) - u(t-6)\}$.

(b) Set the filter's impulse response to a rectangular pulse: $h(t) = \{u(t) - u(t-4)\}$.

(c) Use the GUI to produce a plot of the output signal over its entire duration, i.e., determine the starting and ending times of $y(t)$. Use the *sliding hand tool* to grab the time marker and move it to produce the flip-and-slide effect of convolution. Note: if you move the hand tool past the end of the plot, the plot will automatically scroll in that direction.

(d) The top panel is a plot of $x(\tau)$, overlayed with the "flipped" impulse response $h(t-\tau)$ used to produce the "flip and slide" effect of convolution. The middle panel shows the integrand, $J(\tau, t)$, which is the product of $x(\tau)$ and $h(t-\tau)$. The top two plots are functions of τ, while the bottom plot of $y(t)$ is a function of t. Observe that the output $y(t)$ is composed of five distinct regions: no overlap (on the left side), partial overlap (on the left side), complete overlap, partial overlap (on the right side), and no overlap (on the right side). If you substitute $x(t)$ and $h(t)$ from from parts (a) and (b) into Eq. (1), you can show that the output is given by the piecewise equation

$$y(t) = \begin{cases} 0 & t < T_0 & \textit{Region } 1 \\ \displaystyle\int_{L_1}^{L_2} J(\tau, t)d\tau & T_1 \le t < T_2 & \textit{Region } 2 \\ \displaystyle\int_{L_3}^{L_4} J(\tau, t)d\tau & T_3 \le t < T_4 & \textit{Region } 3 \\ \displaystyle\int_{L_5}^{L_6} J(\tau, t)d\tau & T_5 \le t < T_6 & \textit{Region } 4 \\ 0 & T_7 \le t & \textit{Region } 5 \end{cases} \tag{7}$$

Use the GUI to observe that $y(t)$ does indeed have five distinct regions, and use it to help you figure out the values for the boundaries of the regions and the limits of integration for each part. Determine whether you are flipping $x(t)$ or $h(t)$. *Hint: The limits of integration might depend on the variable t.*

Use the plots of $x(\tau)$ and $h(t-\tau)$ together with the corresponding plot of $y(t)$ to complete the following table with the correct values for the time region boundaries and integral limits in Eq. (7).

	$T_0 =$			Region 1
$T_1 =$	$T_2 =$	$L_1 =$	$L_2 =$	Region 2
$T_3 =$	$T_4 =$	$L_3 =$	$L_4 =$	Region 3
$T_5 =$	$T_6 =$	$L_5 =$	$L_6 =$	Region 4
	$T_7 =$			Region 5

(e) Write down the expression for the integrand which was denoted by $J(\tau, t)$ above.

(f) Finally, determine the mathematical formula for $y(t)$ in each of the five regions. Use the GUI to help in setting up the integrals, but carry out the mathematics of the integrations by hand.

4.2 Fourier Series Through a LPF

The **CLTIdemo** GUI can implement a first-order LPF

$$H(j\omega) = \frac{2\pi f_1}{2\pi f_1 + j\omega}$$

if you choose the filter named "First-Order Lowpass." For the frequencies to be used below, define $f_1 = (23 - d_1 - 0.1d_2)$ Hz and $f_2 = (37 + d_3)$ Hz, where d_1, d_2, and d_3 are the first three digits in your `gtxxxx` login ID. For example, `gtg273x` would use $f_1 = 20.3$ Hz and $f_2 = 40$ Hz.

(a) Use the `CLTIdemo` GUI to create a first-order lowpass filter by selecting "First-Order Lowpass" from the menu and setting the cutoff frequency to f_1 Hz.[2]

(b) Set the input signal to $x(t) = 1.0 + \cos(2\pi f_2 t)$. Look at the output and compare its amplitudes and phases to the input amplitudes and phases. Click Theoretical Answer and record the result.

(c) Now change the input signal to $x(t) = \cos(4\pi f_2 t)$, and record the numerical values of the output signal's amplitude and phase.

(d) Now we would like to change the input to be a half-wave rectified sine wave with a period of $1/f_2$ secs. But, we cannot work this in the time domain directly, so we have to use our knowledge of Fourier Series and frequency response to get the output.[3] The Fourier Series coefficients $\{a_k\}$ for the half-wave rectified sine are:

$$a_k = \begin{cases} \dfrac{\cos(\pi k/2)}{\pi(1-k^2)}(-j)^k & k \neq \pm 1 \\ 0.25(-j)^k & k = \pm 1 \end{cases} \tag{8}$$

The **FseriesDemo** GUI in Section 2.3 can be used to make a plot of the spectrum as $\{a_k\}$ vs. k.

(e) When the input to the LPF is the rectified sine wave, use the $\{a_k\}$ coefficients in (8) and the frequency response of the LPF to determine the Fourier Series coefficients $\{b_k\}$ for the output $y(t)$ which can be expressed as a sum of cosines:

$$y(t) = B_0 + B_1 \cos(\omega_0 t + \psi_1) + B_2 \cos(2\omega_0 t + \psi_2) + \ldots \tag{9}$$

Use the values of the frequency response from the **CLTIdemo** GUI and the $\{a_k\}$ coefficients in (8) to fill in the table below, i.e., determine the numerical values for ω_0, $H(jk\omega_0)$, b_k, B_0, B_1, B_2, ψ_1, and ψ_2. Write all complex number in polar form. *Note:* $B_k \neq b_k$.

(f) Plot $x(t)$ and $y(t)$ for $0 \leq t \leq 3/f_2$. Use the approximation in (9) up to the 2nd harmonic for $y(t)$.

The calculation above is an analysis of how you can "filter" the periodic input signal through a continuous-time LTI system whose frequency response is given in Eq. (6) of Section 3.2. Since this is an analog system, we cannot do the actual filtering in MATLAB; instead, we can only calculate what the output signal would be by finding the Fourier Series of the output, and expressing the output as a sum of sinusoids as in (9).

	$\omega_0 =$	$H(jk\omega_0)$ values			
$k = 0$	$a_0 =$		$b_0 =$	$B_0 =$	
$k = 1$	$a_1 =$		$b_1 =$	$B_1 =$	$\psi_1 =$
$k = 2$	$a_2 =$		$b_2 =$	$B_2 =$	$\psi_2 =$

[2] The `CLTIdemo` GUI will convert the frequencies from hertz to rad/s.

[3] Refer to section 10-2.4, Fig. 10-5 in the *SP-First* text.

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INSTRUCTOR VERIFICATION PAGE

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: ______________________________ Date of Lab: __________

Part 3.1.1: Demonstrate that you can run the continuous-time convolution demo. Explain the duration of the output signal in terms of the duration of the input signal and the duration of the impulse response.

Verified:____________________ Date/Time:__________

Part 3.1.2: Demonstrate that you can run the continuous-time convolution demo. Explain how to find the FIVE regions for this convolution integral. Use the plots of $x(\tau)$ and $h(t-\tau)$ together with the corresponding plot of $y(t)$ to complete the following table with the correct values for the integral limits in Eq. (5).

	$T_0 = 0$			Region 1
$T_1 = 0$	$T_2 = 3$	$L_1 =$	$L_2 =$	Region 2
$T_3 = 3$	$T_4 = 7$	$L_3 =$	$L_4 =$	Region 3
$T_5 = 7$	$T_6 = 10$	$L_5 =$	$L_6 =$	Region 4
	$T_7 = 10$			Region 5

Note that the area under the curve in the middle plot is shaded green. When you set the time indicator to $t = 5$, how is the shaded area related to $y(5)$? How does the shaded area tell you the limits of integration?

Explain the above answers to your TA.

Verified:____________________ Date/Time:__________

Part 3.2: Frequency Response of an analog bandpass filter. Explain why the formula gives the peak location at ω_c.

Verified:____________________ Date/Time:__________