GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

FINAL EXAM

DATE: 5-May-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ________________
LAST, FIRST (ex: gtz129a)

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

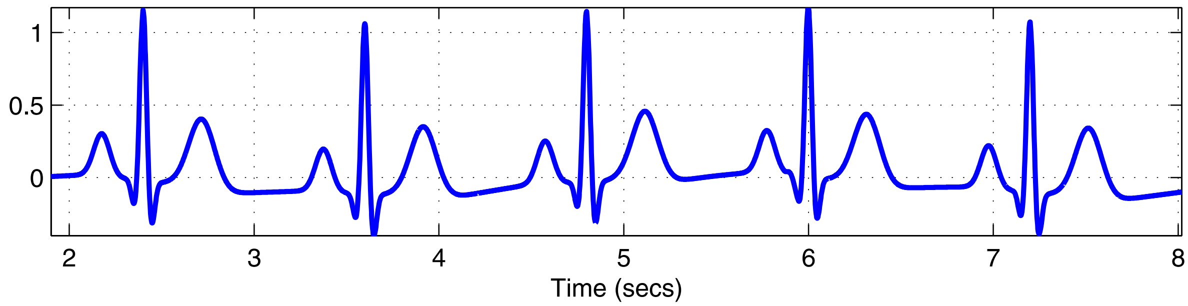
	L05:Tues-Noon (Juang)		L06:Thur-Noon (Verriest)
	L07:Tues-1:30pm (Juang)		
L01:M-3pm (McClellan)	L09:Tues-3pm (Chang)	L02:W-3pm (Zhou)	L10:Thur-3pm (Taylor)
L03:M-4:30pm (Fekri)	L11:Tues-4:30pm (Chang)	L04:W-4:30pm (Zhou)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly. to receive partial credit.
  Explanations are also required. to receive full credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	30	
2	30	
3	30	
4	30	
5	30	
6	30	
7	30	

PROBLEM s-06-F.1:

(a) An ECG signal is (more or less) periodic. If we ***ignore small variations and assume*** that the ECG signal is exactly periodic, then it could be represented as a Fourier Series. For the ECG signal below, determine the ***fundamental frequency(in Hz)*** that would be used in the Fourier Series. Make ***accurate estimates*** from the plot.



$f_0 =$ Hz

(b) Suppose that a signal $s(t)$ is periodic and is represented by the following Fourier Series:

$$s(t) = \sum_{k=-7}^{7} a_k e^{j200\pi kt}$$

i.e., the sum contains a *finite* number of terms. Determine the ***Nyquist rate (in Hz)*** for sampling $s(t)$.

Nyquist rate = Hz

(c) Suppose that the signal $x(t)$ is an FM signal defined via:

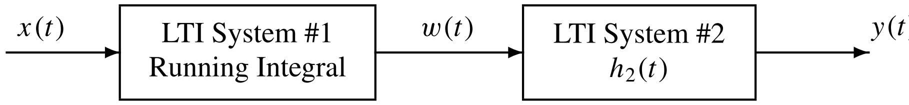
$$x(t) = \Re e\left\{\exp\{jCe^{\lambda t}\}\right\}$$

over the time interval $0 \le t \le 6$ secs. Let the parameters be $C = -150\pi$ and $\lambda = -0.2$. Use the instantaneous frequency, $f_x(t)$ (in Hz), to determine the ***Nyquist rate (in Hz)*** for sampling $x(t)$.

Nyquist rate = Hz

PROBLEM s-06-F.2:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$$w(t) = \int_{-\infty}^{t-7} x(\tau)d\tau$$

$$h_2(t) = \pi\delta(t-6) - \pi\delta(t-14)$$

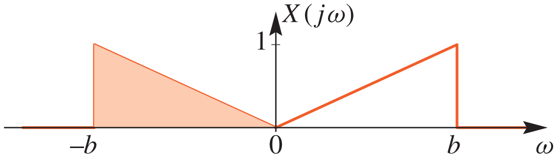
(a) Determine the impulse response of the first system.

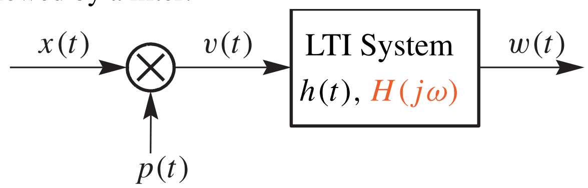
(b) Determine the overall impulse response for this cascade of two systems. Give your answer in the ***simplest possible form.***

(c) The overall frequency response of this system, $H(j\omega)$, is zero for infinitely many values of ω. Derive a general formula that gives **all** the zeros of $H(j\omega)$. ***Explain.***

PROBLEM s-06-F.3:

The transmitter system below involves a multiplier followed by a filter:

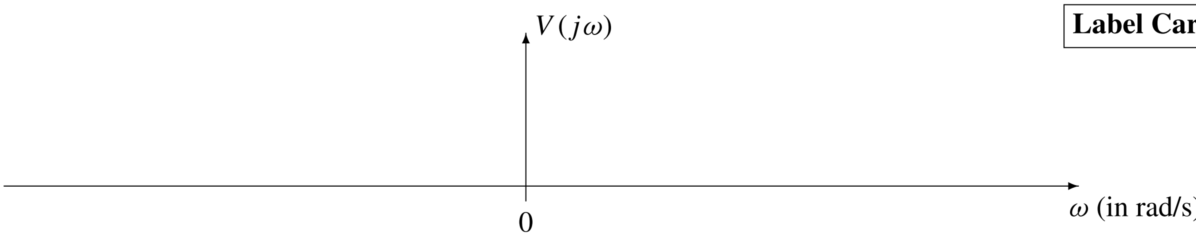




The Fourier transform of the input is $X(j\omega)$. In all parts, assume that $p(t) = \cos(50\pi t)$, and $b = 20\pi$.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, when the input is $X(j\omega)$ shown above.

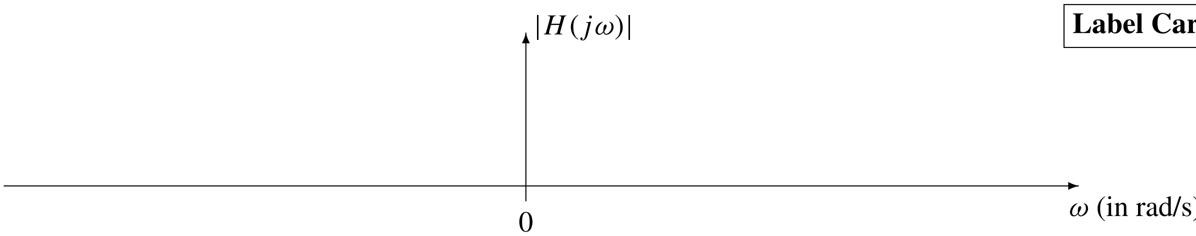
Label Carefully



(b) If the filter is an ideal filter defined by $H(j\omega) = \begin{cases} 0 & |\omega| < 60\pi \\ e^{-j\omega/180} & 60\pi \le |\omega| \end{cases}$

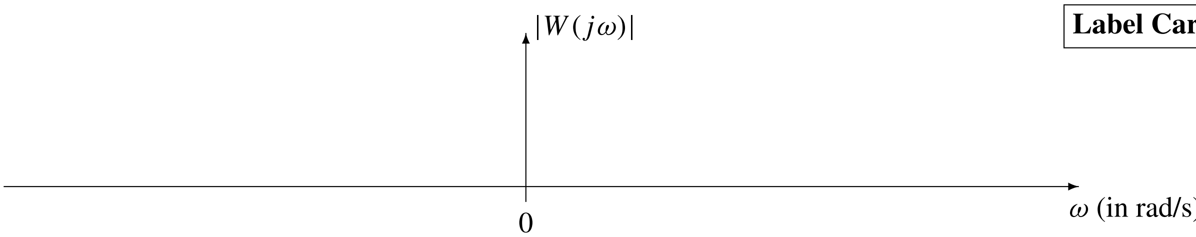
Make a sketch of $|H(j\omega)|$, the magnitude of the Fourier transform of the filter.

Label Carefully



(c) Using the filter from part (b), make a sketch of $|W(j\omega)|$, the magnitude of the Fourier transform of the output $w(t)$, when the input is $X(j\omega)$ shown above.

Label Carefully



PROBLEM s-06-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

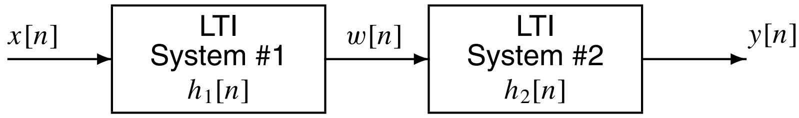


Figure 1: Cascade connection of two discrete-time LTI systems.

In all parts, assume that System #1 is an IIR filter described by the system function: $H_1(z) = \dfrac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}$

(a) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $w[n]$.

(b) If the difference equation of the second system is

$$y[n] = y[n-1] - 5w[n]$$

Determine the overall system function, $H(z)$, for the cascade. ***Simplify the expression for*** $H(z)$ ***to have the lowest degree polynomials in the numerator and denominator.***

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-5]$$

From this information, determine the system function $H_2(z)$ for the second system. Simplify your answer. *Note:* $H_2(z)$ obtained in this part will be different from part (b).

PROBLEM s-06-F.5:

Pick the correct frequency response characteristic and enter the number in the answer box:

Difference Equation or Impulse Response

(a) $y[n] = x[n-1] + 2x[n-3] + x[n-5]$

ANS =

(b) $h[n] = \sum_{k=0}^{3} \delta[n-k]$

ANS =

(c) $y[n] = -\frac{1}{2}y[n-1] + 2x[n] + x[n-1]$

ANS =

(d) $h[n] = (-\frac{1}{2})^n u[n]$

ANS =

(e) `filter(1,[1,-0.5],xn)`

ANS =

(f) `conv(ones(1,3),xn)`

ANS =

Frequency Response

1. $H(e^{j\hat{\omega}}) = \dfrac{\sin\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}$

2. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$

3. $H(e^{j\hat{\omega}}) = 1 + \frac{1}{2}e^{-j\hat{\omega}}$

4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$

5. $H(e^{j\hat{\omega}}) = \dfrac{\sin 2\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})} e^{-j1.5\hat{\omega}}$

6. $|H(e^{j\hat{\omega}})| = 2$

7. $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$

8. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$

PROBLEM s-06-F.6:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes next to the questions.*** (The operator $*$ denotes convolution.)

(a) ☐ $x(t) = \delta(t+2) * e^{-t+1}u(t-1) * \delta(t-1)$

(b) ☐ $x(t) = e^{-(t-4)} \int_{-\infty}^{0} \delta(\tau - 4)d\tau$

(c) ☐ $x(t) = u(t-3) - u(t-5)$

(d) ☐ $x(t) = -e^{-t}u(t) + \delta(t)$

(e) ☐ $x(t) = \delta(t) - \delta(t-8)$

(f) ☐ $x(t) = \cos(\pi t)\delta(t-4)$

(g) ☐ $x(t) = \int_{-\infty}^{t} e^{-t+\tau}\delta(\tau - 4)d\tau$

Each of the time signals above has a Fourier transform that can be found in the list below.

[1] $X(j\omega) = 2e^{-j4\omega} \dfrac{\sin(\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{\sin(4\omega)}{\omega/2}$

[3] $X(j\omega) = e^{-j4\omega}\left[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)\right]$

[4] $X(j\omega) = e^{-j4\omega}\left[u(\omega) - u(\omega-8)\right]$

[5] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[6] $X(j\omega) = \dfrac{1}{1+j\omega}$

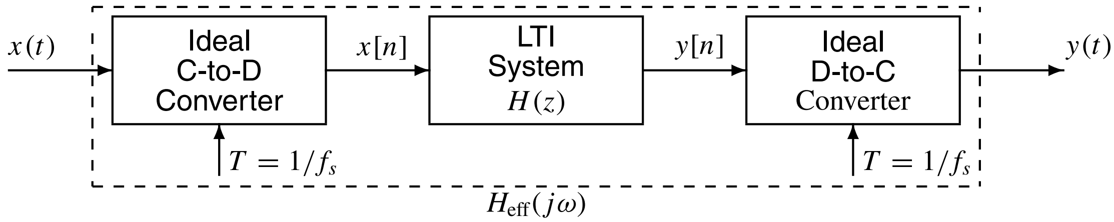
[7] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[8] $X(j\omega) = e^{-j4\omega}$

[9] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[10] $X(j\omega) = 0$

PROBLEM s-06-F.7:



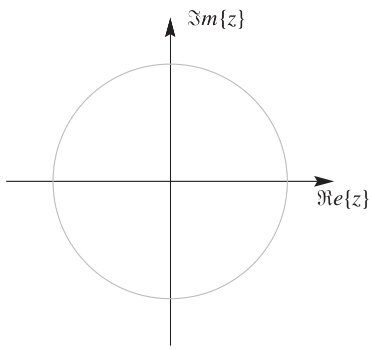
In all parts below, the sampling rates of both converters is **equal to** $f_s = 72$ **samples/sec,** the LTI system is an IIR notch filter described by the difference equation:

$$y[n] = 1.2728y[n-1] - 0.81y[n-2] + 3x[n] - 4.2426x[n-1] + 3x[n-2]$$

and the input signal $x(t)$ is a periodic signal whose Fourier Series is

$$x(t) = \sum_{k=-6}^{6} a_k e^{j9\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1-2k^2} & k \neq 0 \\ 0.2 & k = 0 \end{cases}$$

(a) Determine the poles and zeros of the LTI system, and give your answer as a plot in the z-plane.



(b) Using the periodic input signal given above, determine the DC value of the output signal, $y(t)$.

DC value =

(c) The filter, $H_{\text{eff}}(j\omega)$, defined above is a notch filter—like the one used to remove sinusoidal interference from an ECG signal. For the periodic input signal $x(t)$ given above via its Fourier Series, determine which terms in the Fourier Series will be removed completely, i.e., nulled, by the notch filter. ***Explain.***

Indices of terms removed, $k =$

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NAME: **ANSWER KEY** GT #: **VERSION #4**

LAST, FIRST (ex: gtz129a)

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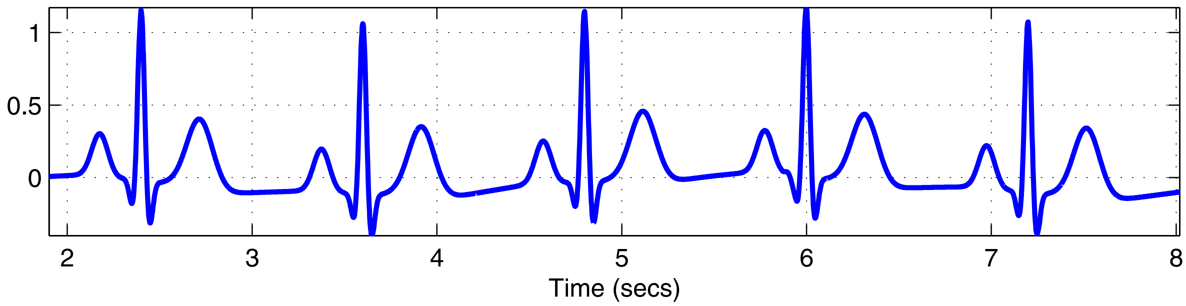
PROBLEM s-06-F.1:

(a) An ECG signal is (more or less) periodic. If we ***ignore small variations and assume*** that the ECG signal is exactly periodic, then it could be represented as a Fourier Series. For the ECG signal below, determine the ***fundamental frequency(in Hz)*** that would be used in the Fourier Series. Make ***accurate estimates*** from the plot.



If we estimate the period as the time between the peaks at $t = 4.8$ secs and $t = 6$ secs, then the fundamental frequency in Hz is $f_0 = 1/1.2 = 5/6 = 0.8333$ Hz.

(b) Suppose that a signal $s(t)$ is periodic and is represented by the following Fourier Series:

$$s(t) = \sum_{k=-7}^{7} a_k e^{j200\pi kt}$$

i.e., the sum contains a *finite* number of terms. Determine the ***Nyquist rate (in Hz)*** for sampling $s(t)$.

The highest frequency term in the summation is $\omega_{\max} = (7)(200\pi) = 2\pi(700)$ when $k = 7$. The Nyquist rate is two times the highest frequency, so $f_{\text{Nyquist}} = 2(700) = 1400$ Hz.

(c) Suppose that the signal $x(t)$ is an FM signal defined via:

$$x(t) = \Re e\left\{\exp\{jCe^{\lambda t}\}\right\}$$

over the time interval $0 \le t \le 6$ secs. Let the parameters be $C = -150\pi$ and $\lambda = -0.2$. Use the instantaneous frequency, $f_x(t)$ (in Hz), to determine the ***Nyquist rate (in Hz)*** for sampling $x(t)$.

The instantaneous frequency is the derivative of $\psi(t) = Ce^{\lambda t}$, so

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\left\{-150\pi e^{-0.2t}\right\} = \frac{1}{2\pi}\left((-0.2)(-150\pi)e^{-0.2t}\right) \text{ Hz}$$

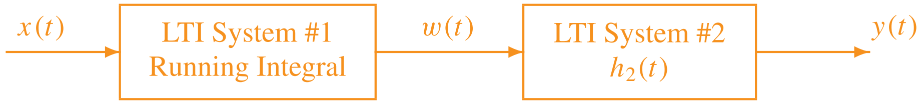
The instantaneous frequency, $f_i(t)$, is an exponential that is *decreasing*. Therefore, over the time interval, $t \in [0, 6]$, the highest frequency occurs at $t = 0$, and we get

$$f_i(0) = \frac{(-150\pi)(-0.2)}{2\pi} e^{-0.2(0)} = 15 \text{ Hz}$$

The Nyquist rate is two times the highest frequency, so $f_{\text{Nyquist}} = 30$ Hz.

PROBLEM s-06-F.2:

A cascade of linear time-invariant systems is depicted by the following block diagram:



$$w(t) = \int_{-\infty}^{t-7} x(\tau)d\tau \qquad h_2(t) = \pi\delta(t-6) - \pi\delta(t-14)$$

(a) Determine the impulse response of the first system.

The impulse response response of a running integral is a unit-step signal; in this case, the upper limit of the running integral causes a delay of 7 secs, so

$$h_1(t) = \int_{-\infty}^{t-7} \delta(\tau)d\tau = u(t-7)$$

(b) Determine the overall impulse response for this cascade of two systems. Give your answer in the ***simplest possible form.***

Since the first system does the running integral, and we are given the impulse response of the second system, we can invoke the LTI property that says we can swap the ordering of the cascade. Then we can do the running integral of $h_2(t)$.

$$h(t) = \int_{-\infty}^{t-7} h_2(\tau)d\tau = \int_{-\infty}^{t-7} (\pi\delta(\tau-6) - \pi\delta(\tau-14))\, d\tau = \pi u(t-13) - \pi u(t-21)$$

Note: the integral of the unit-impulse signal, $\delta(t)$, is the unit-step, $u(t)$.

(c) The overall frequency response of this system, $H(j\omega)$, is zero for infinitely many values of ω. Derive a general formula that gives **all** the zeros of $H(j\omega)$. ***Explain.***

In this cascade, the overall frequency response is the Fourier transform of $h(t)$, which is a rectangular pulse that starts at $t = 13$ and ends at $t = 21$. The center of the rectangular pulse is at $t = 17$, so the frequency response is a "sinc" function multiplied by a complex exponential:

$$H(j\omega) = \pi \frac{\sin(8\omega/2)}{\omega/2} e^{-j17\omega}$$

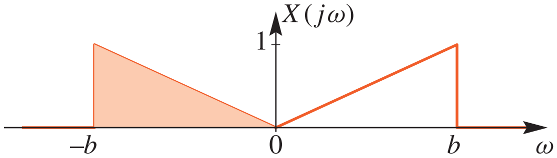
The zeros of $H(j\omega)$ are the zeros of a "sinc" function, so we get zeros of $H(j\omega)$ when the sine function in the numerator is zero:

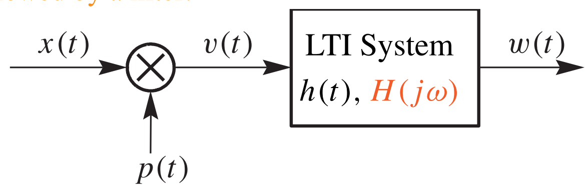
$$8\omega/2 = \pi k \qquad \Longrightarrow \qquad \omega = \frac{2\pi k}{8}, \quad \underline{\textbf{except for}}\ k = 0.$$

i.e., $\omega = 0$ is NOT one of the zeros because the denominator also zero at $\omega = 0$.

PROBLEM s-06-F.3:

The transmitter system below involves a multiplier followed by a filter:

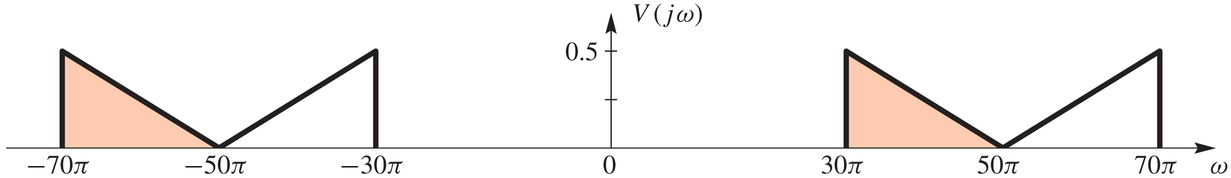




The Fourier transform of the input is $X(j\omega)$. In all parts, assume that $p(t) = \cos(50\pi t)$, and $b = 20\pi$.

(a) Make a sketch of $V(j\omega)$, the Fourier transform of $v(t)$, when the input is $X(j\omega)$ shown above.

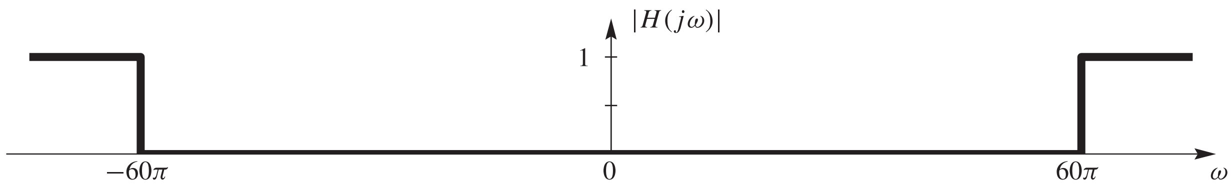
Frequency shifting: $V(j\omega) = \frac{1}{2}X(j(\omega + 50\pi)) + \frac{1}{2}X(j(\omega - 50\pi))$



(b) If the filter is an ideal filter defined by $H(j\omega) = \begin{cases} 0 & |\omega| < 60\pi \\ e^{-j\omega/180} & 60\pi \le |\omega| \end{cases}$

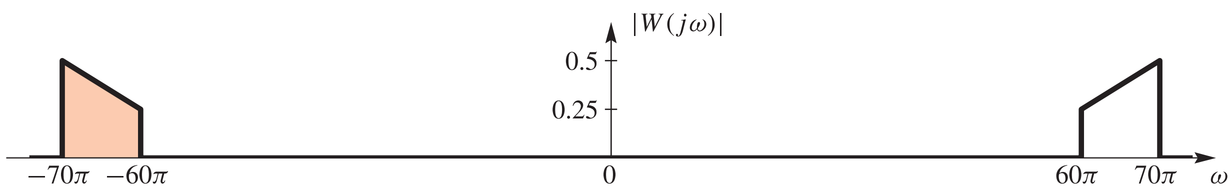
Make a sketch of $|H(j\omega)|$, the magnitude of the Fourier transform of the filter.

The magnitude of $e^{-j\omega/180}$ is one.



(c) Using the filter from part (b), make a sketch of $|W(j\omega)|$, the magnitude of the Fourier transform of the output $w(t)$, when the input is $X(j\omega)$ shown above.

Filtering: $W(j\omega) = H(j\omega)V(j\omega)$ implies that $|W(j\omega)| = |H(j\omega)|\,|V(j\omega)|$



PROBLEM s-06-F.4:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems, i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

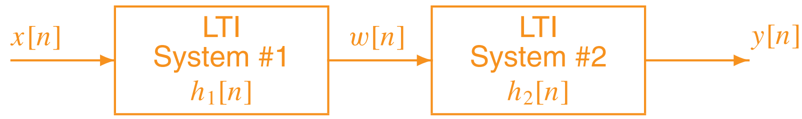


Figure 1: Cascade connection of two discrete-time LTI systems.

In all parts, assume that System #1 is an IIR filter described by the system function: $H_1(z) = \dfrac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}$

(a) When the input signal $x[n]$ is a ***unit-step*** signal, determine the output of the first system, $w[n]$.

If the input is a unit-step signal, then its z-transform is $X(z) = \dfrac{1}{1 - z^{-1}}$. The output z-transform is

$$W(z) = H_1(z)X(z) = \frac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}\left(\frac{1}{1 - z^{-1}}\right) = \frac{5z^{-3}}{1 - 3z^{-1}}$$

Then the inverse z-transform gives $w[n] = 5(3)^{n-3}u[n-3]$.

(b) If the difference equation of the second system is

$$y[n] = y[n-1] - 5w[n]$$

Determine the overall system function, $H(z)$, for the cascade. ***Simplify the expression for $H(z)$ to have the lowest degree polynomials in the numerator and denominator.***

The overall z-transform is the product $H(z) = H_1(z)H_2(z)$, and the z-transform of the second system is $H_2(z) = \dfrac{-5}{1 - z^{-1}}$, so we get

$$H(z) = H_1(z)H_2(z) = \frac{5z^{-3} - 5z^{-4}}{1 - 3z^{-1}}\left(\frac{-5}{1 - z^{-1}}\right) = \frac{-25z^{-3}}{1 - 3z^{-1}}$$

(c) When the input signal $x[n]$ is a ***unit-impulse*** signal, the output $y[n]$ of the overall cascaded system is:

$$y[n] = \delta[n-5]$$

From this information, determine the system function $H_2(z)$ for the second system. Simplify your answer. *Note:* $H_2(z)$ obtained in this part will be different from part (b).

In order to find $H_2(z)$ we use the z-transform and divide. Starting with $Y(z) = H_1(z)H_2(z)X(z)$, and the facts that the z-transform of $x[n] = \delta[n]$ is $X(z) = 1$ and $y[n] = \delta[n-5]$ is $Y(z) = z^{-5}$, we get

$$H_2(z) = \frac{Y(z)}{H_1(z)X(z)} = \frac{z^{-5}(1 - 3z^{-1})}{5z^{-3} - 5z^{-4}} = \frac{z^{-2}(1 - 3z^{-1})}{5(1 - z^{-1})}$$

PROBLEM s-06-F.5:

Pick the correct frequency response characteristic and enter the number in the answer box:

Difference Equation or Impulse Response	Frequency Response
(a) $y[n] = x[n-1] + 2x[n-3] + x[n-5]$ **ANS = 7**	1. $H(e^{j\hat{\omega}}) = \dfrac{\sin\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}$
(b) $h[n] = \sum_{k=0}^{3} \delta[n-k]$ **ANS = 5**	2. $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$
(c) $y[n] = -\frac{1}{2}y[n-1] + 2x[n] + x[n-1]$ **ANS = 6**	3. $H(e^{j\hat{\omega}}) = 1 + \frac{1}{2}e^{-j\hat{\omega}}$
(d) $h[n] = (-\frac{1}{2})^n u[n]$ **ANS = 4**	4. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 + \frac{1}{2}e^{-j\hat{\omega}}}$
(e) `filter(1,[1,-0.5],xn)` **ANS = 8**	5. $H(e^{j\hat{\omega}}) = \dfrac{\sin 2\hat{\omega}}{\sin(\frac{1}{2}\hat{\omega})}e^{-j1.5\hat{\omega}}$
(f) `conv(ones(1,3),xn)` **ANS = 2**	6. $\lvert H(e^{j\hat{\omega}})\rvert = 2$
	7. $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$
	8. $H(e^{j\hat{\omega}}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\hat{\omega}}}$

- The MATLAB code `filter(1,[1,-0.5],xn)` defines a filter whose system function is $H(z) = \dfrac{1}{1-0.5z^{-1}}$ which then gives the frequency response $H(e^{j\omega}) = \dfrac{1}{1-0.5e^{-j\hat{\omega}}}$
- $h[n] = (-\frac{1}{2})^n u[n]$ has a z-transform equal to $H(z) = \dfrac{1}{1+0.5z^{-1}}$ which then gives the frequency response $H(e^{j\omega}) = \dfrac{1}{1+0.5e^{-j\hat{\omega}}}$
- $y[n] = -\frac{1}{2}y[n-1] + x[n] + 2x[n-1]$ has a frequency response equal to $H(e^{j\omega}) = \dfrac{1+2e^{-j\hat{\omega}}}{1+0.5e^{-j\hat{\omega}}}$ which leads to $\lvert H(e^{j\omega})\rvert^2 = H(e^{j\omega})H^*(e^{j\omega}) = \dfrac{1+4+4\cos(\hat{\omega})}{1+0.25+\cos(\hat{\omega})} = 4$, i.e., it's an all-pass filter.
- `conv(ones(1,3),xn)` defines a 3-point summing filter so its frequency response can be written as a Dirichlet form, $\sin(L/\hat{\omega}/2)/\sin(\hat{\omega}/2)$, times an $e^{j\text{phase}}$ term: $H(e^{j\omega}) = \dfrac{\sin(3\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j\hat{\omega}}$
- $h[n] = \sum_{k=0}^{3}\delta[n-k]$ defines a 4-point summing filter so $H(e^{j\omega}) = \dfrac{\sin(4\hat{\omega}/2)}{\sin(\hat{\omega}/2)}e^{-j1.5\hat{\omega}}$
- $y[n] = x[n-1] + 2x[n-3] + x[n-5]$ has a frequency response $H(e^{j\omega}) = e^{-j\hat{\omega}} + 2e^{-j3\hat{\omega}} + e^{-j5\hat{\omega}}$ which can be rewritten as $e^{-j3\hat{\omega}}(2 + 2\cos(2\hat{\omega}))$; thus, the phase is $\angle H(e^{j\omega}) = -3\hat{\omega}$.

PROBLEM s-06-F.6:

For each of the following time-domain signals, select the correct match from the list of Fourier transforms below. ***Write your answers in the boxes next to the questions.*** (The operator $*$ denotes convolution.)

(a) **6** $x(t) = \delta(t+2) * e^{-t+1}u(t-1) * \delta(t-1) \quad \Rightarrow \quad X(j\omega) = \dfrac{1}{1+j\omega}$

(b) **10** $x(t) = e^{-(t-4)} \displaystyle\int_{-\infty}^{0} \delta(\tau-4)d\tau \quad \Rightarrow \quad X(j\omega) = 0$

(c) **1** $x(t) = u(t-3) - u(t-5) \quad \Rightarrow \quad X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

(d) **5** $x(t) = -e^{-t}u(t) + \delta(t) \quad \Rightarrow \quad X(j\omega) = \dfrac{j\omega}{1+j\omega}$

(e) **9** $x(t) = \delta(t) - \delta(t-8) \quad \Rightarrow \quad X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

(f) **8** $x(t) = \cos(\pi t)\delta(t-4) \quad \Rightarrow \quad X(j\omega) = e^{-j4\omega}$

(g) **7** $x(t) = \displaystyle\int_{-\infty}^{t} e^{-t+\tau}\delta(\tau-4)d\tau \quad \Rightarrow \quad X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

Each of the time signals above has a Fourier transform that can be found in the list below.

[1] $X(j\omega) = 2e^{-j4\omega}\dfrac{\sin(\omega)}{\omega}$

[2] $X(j\omega) = \dfrac{\sin(4\omega)}{\omega/2}$

[3] $X(j\omega) = e^{-j4\omega}[\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$

[4] $X(j\omega) = e^{-j4\omega}[u(\omega) - u(\omega-8)]$

[5] $X(j\omega) = \dfrac{j\omega}{1+j\omega}$

[6] $X(j\omega) = \dfrac{1}{1+j\omega}$

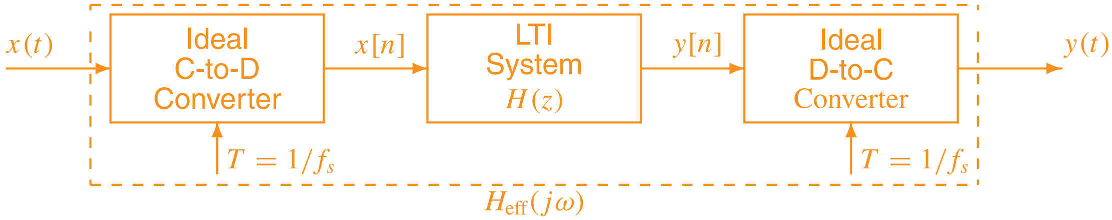
[7] $X(j\omega) = \dfrac{e^{-j4\omega}}{1+j\omega}$

[8] $X(j\omega) = e^{-j4\omega}$

[9] $X(j\omega) = j2e^{-j4\omega}\sin(4\omega)$

[10] $X(j\omega) = 0$

PROBLEM s-06-F.7:



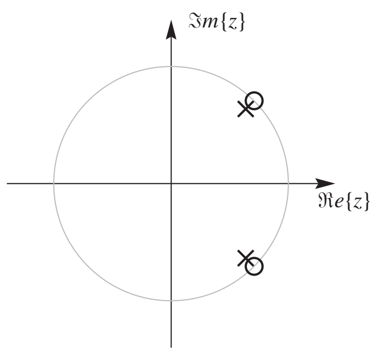
In all parts below, the sampling rates of both converters is **equal to** $f_s = 72$ **samples/sec,** the LTI system is an IIR notch filter described by the difference equation:

$$y[n] = 1.2728y[n-1] - 0.81y[n-2] + 3x[n] - 4.2426x[n-1] + 3x[n-2]$$

and the input signal $x(t)$ is a periodic signal whose Fourier Series is

$$x(t) = \sum_{k=-6}^{6} a_k e^{j9\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1/\pi}{1-2k^2} & k \neq 0 \\ 0.2 & k = 0 \end{cases}$$

(a) Determine the poles and zeros of the LTI system, and give your answer as a plot in the z-plane.



The difference equation can be converted to the z-transform system function:

$$H(z) = \frac{3 - 4.2426z^{-1} + 3z^{-2}}{1 - 1.2728z^{-1} + 0.81z^{-2}}$$

from which the poles and zeros are obtained:

$$\text{poles} = 0.9e^{\pm j\pi/4} \qquad \text{zeros} = e^{\pm j\pi/4}$$

(b) Using the periodic input signal given above, determine the DC value of the output signal, $y(t)$.

The DC value of the input signal is the value of the Fourier Series coefficient a_0, which is $a_0 = 0.2$. When filtered by $H(z)$, the DC value is multiplied by the frequency response at $\hat{\omega} = 0$, or equivalently, by $H(z)$ at $z = 1$. If we denote the DC value of the output signal by b_0 because $y(t)$ also has a Fourier Series representation, then we get

$$b_0 = a_0 H(z)\big|_{z=1} = (0.2)\left(\frac{3 - 4.2426 + 3}{1 - 1.2728 + 0.81}\right) = \frac{(0.2)(1.7574)}{0.5272} = 0.6543$$

(c) The filter, $H_{\text{eff}}(j\omega)$, defined above is a notch filter—like the one used to remove sinusoidal interference from an ECG signal. For the periodic input signal $x(t)$ given above via its Fourier Series, determine which terms in the Fourier Series will be removed completely, i.e., nulled, by the notch filter. ***Explain.***

The null is determined by the fact that the system function $H(z)$ has its zeros exactly on the unit circle. The angles of the zeros ($\pm\pi/4$) give the locations of the nulls in the frequency domain at $\hat{\omega} = \pm\pi/4$. We must then convert the "digital frequency" $\hat{\omega}$ to "analog frequency" via

$$\omega = \hat{\omega} f_s \quad \text{rad/s}$$

Thus the notch filter will completely remove the analog frequencies $\omega = \pm 18\pi$ rad/s. Since the fundamental frequency is $\omega_0 = 9\pi$, the terms removed from the Fourier Series are the ones at the indices $k = \pm 18\pi/9\pi = \pm 2$.