GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 10-Feb-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ________________

LAST, FIRST (ex: gtz123z)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Juang)		L06:Thur-Noon (Verriest)
	L07:Tues-1:30pm (Juang)		
L01:M-3pm (McClellan)	L09:Tues-3pm (Chang)	L02:W-3pm (Zhou)	L10:Thur-3pm (Taylor)
L03:M-4:30pm (Fekri)	L11:Tues-4:30pm (Chang)	L04:W-4:30pm (Zhou)	

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	-3	

PROBLEM s-06-Q.1.1:

The sum of two sinusoids is another sinusoid:

$$A\cos(\omega t + \varphi) = 200\cos(\tfrac{1}{3}\pi(t + 13)) + 300\cos(\tfrac{1}{3}\pi t - 5\pi/6)$$

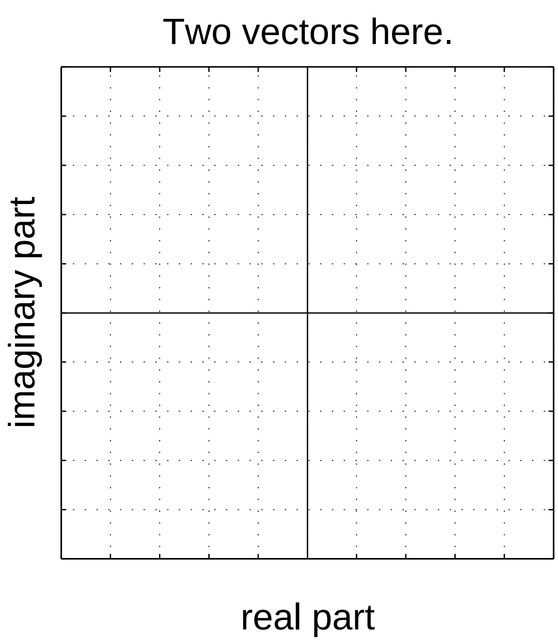
(a) Determine the the numerical values of A and φ, as well as ω (give the correct units).

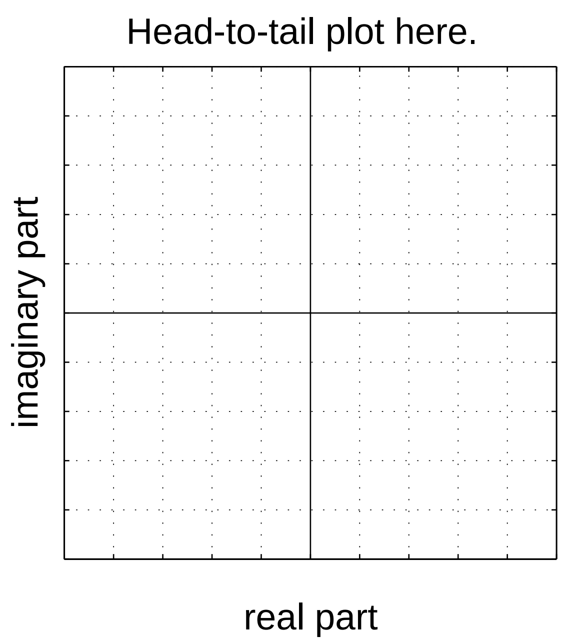
$A =$ ________

$\varphi =$ ________

$\omega =$ ________

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were combined to solve part (a). On the first plot, show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign; on the second plot, show a "head-to-tail" vector plot of those same two complex amplitudes plus the resultant vector that gives the solution. *Use an appropriate scale on the grids below.*





PROBLEM s-06-Q.1.2:

The signal $x(t)$ is defined by complex exponentials and complex amplitudes:

$$x(t) = 50e^{j\pi/3}e^{j7t} + 50e^{-j\pi/3}e^{-j7t} + 77e^{-j\pi}$$

(a) Write the formula for $x(t)$ as a sum of real-valued sinusoids.

(b) Define a new signal $y(t)$ to be the derivative of $x(t)$, i.e., $y(t) = \dfrac{dx(t)}{dt}$.

Make a (well-labeled) sketch of the spectrum of the signal $y(t)$. Simplify the numerical values for the complex amplitudes, so that the values of magnitude and phase are obvious.

PROBLEM s-06-Q.1.3:
Two questions about sinusoids, $A\cos(\omega t+\varphi)$.

(a) The following MATLAB code makes a plot of a sinusoid:

```
tt = 0:0.0001:1;
znum = exp(j*8*pi*tt)-j*exp(j*8*pi*tt);
zden = 3*j*exp(-j*8*pi*tt)+4*j*exp(-j*8*pi*tt);
xx = real(znum./zden);
plot(tt,xx), grid on, shg
```

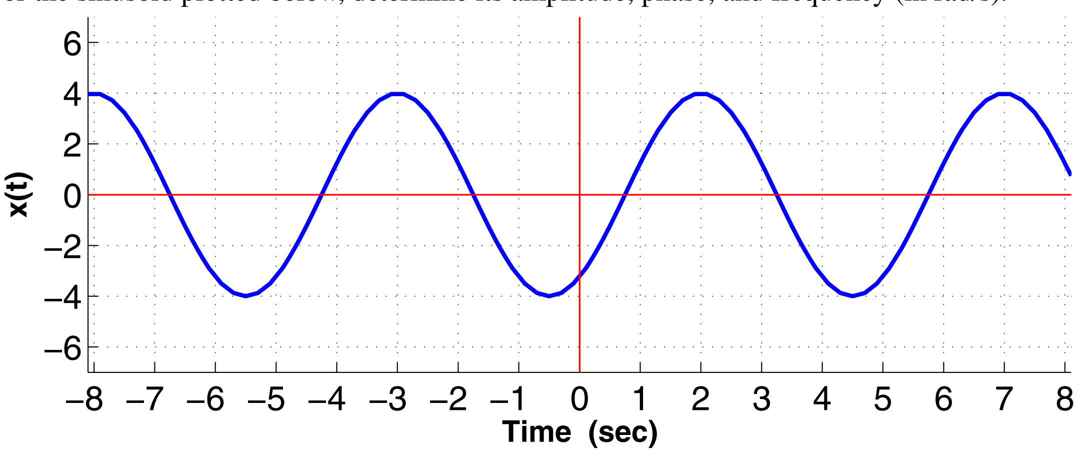
Determine the mathematical formula by giving numerical values for A, φ, and ω (in rad/s).

$A =$ ________

$\varphi =$ ________

$\omega =$ ________

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s).



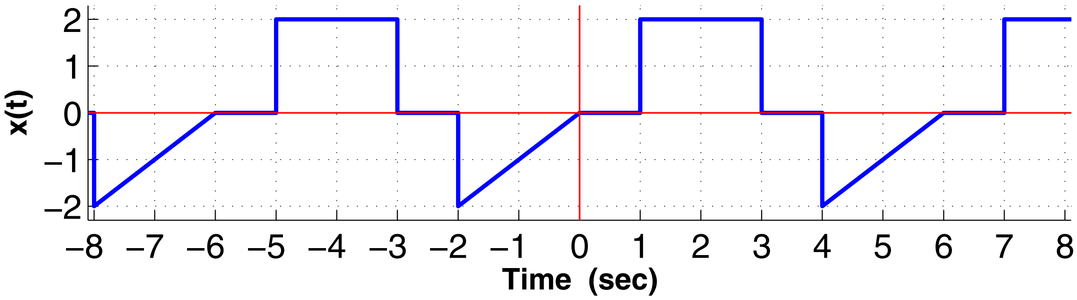
$A =$ ________

$\varphi =$ ________

$\omega =$ ________

PROBLEM s-06-Q.1.4:

Suppose that a periodic signal $x(t)$ is defined by the plot below (only the section $-8 \le t \le 8$ is shown):



(a) Determine the $\boxed{\text{fundamental frequency}}$ of $x(t)$ in Hz.

(b) Determine the $\boxed{\text{DC value}}$ of $x(t)$.

(c) Write the $\boxed{\text{Fourier integral}}$ expression for the coefficient a_6 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral(s) should have numeric values. NOTE: more than one integral might be needed.*

PROBLEM s-06-Q.1.5:

For the FM signal $x(t)$ defined as:

$$x(t) = \Re e\left\{\exp\{jCe^{\lambda t}\}\right\}$$

we will denote its instantaneous frequency (in Hz) as $f_x(t)$.

(a) For $C = -150\pi$ and $\lambda = -0.5$, make a *carefully labeled* plot of the instantaneous frequency $f_x(t)$ over the time interval $0 \leq t \leq 2$ secs. *Note: the frequency should be in hertz (Hz).*

(b) Evaluate $\displaystyle\int_{-2}^{2} \left|7j\exp\{j\pi e^{-0.5t}\}\right|^2 dt$. Since the integral is a definite integral, give a numerical answer.

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QUIZ #1

DATE: 10-Feb-06   COURSE: ECE-2025

NAME: Answer Key
LAST, FIRST

GT #: Version-4
(ex: gtz123z)

3 points   3 points   3 points

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No/Wrong Rec	−3	

PROBLEM s-06-Q.1.1:

The sum of two sinusoids is another sinusoid:

$\frac{13\pi}{3}$ same as $\pi/3$

$$A\cos(\omega t+\varphi) = 200\cos(\tfrac{1}{3}\pi(t+13)) + 300\cos(\tfrac{1}{3}\pi t - 5\pi/6)$$

$200e^{j13\pi/3}$ $\qquad$ $300e^{-j5\pi/6}$

(a) Determine the the numerical values of A and φ, as well as ω (give the correct units).

$A =$ 161.5

$\varphi =$ 3 rads

$\omega = \pi/3$ rad/s

Phasor Addition:

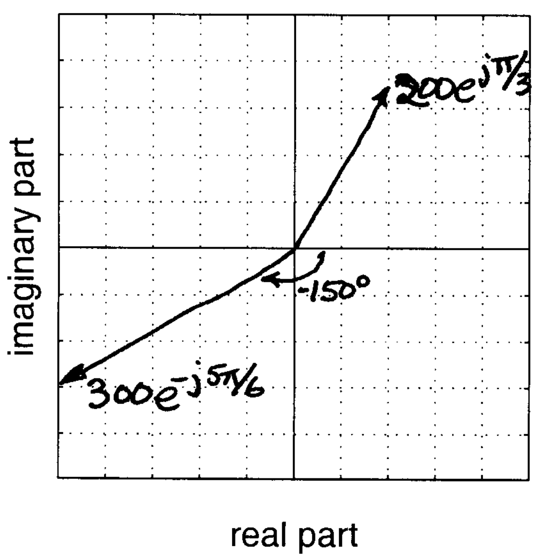
$$200e^{j\pi/3} + 300e^{-j5\pi/6}$$

$$= 161.5e^{j3}$$

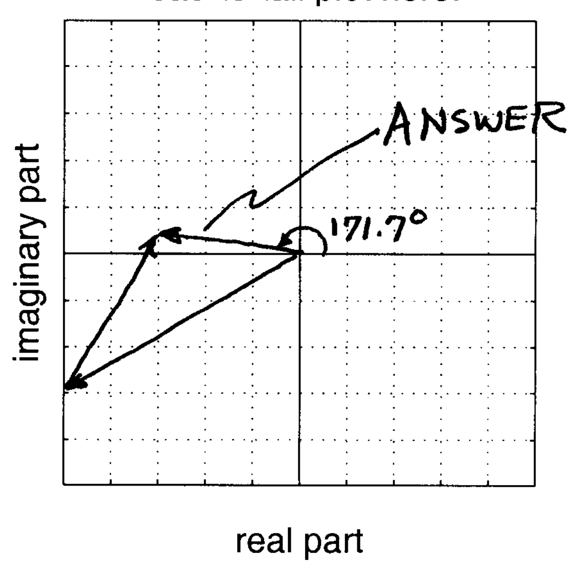
$= 171.7°$ or $.95\pi$ rads

(b) Make two complex plane plots to illustrate how complex amplitudes (phasors) were combined to solve part (a). On the first plot, show a vector plot of the two complex amplitudes whose values are given by the sinusoids on the **right** hand side of the equal sign; on the second plot, show a "head-to-tail" vector plot of those same two complex amplitudes plus the resultant vector that gives the solution. *Use an appropriate scale on the grids below.*

Two vectors here.



Head-to-tail plot here.



PROBLEM s-06-Q.1.2:

The signal $x(t)$ is defined by complex exponentials and complex amplitudes:

$$x(t) = \underbrace{50e^{j\pi/3}e^{j7t} + 50e^{-j\pi/3}e^{-j7t}}_{\text{Cosine}} + 77\underbrace{e^{-j\pi}}_{e^{-j\pi}=-1}$$

(a) Write the formula for $x(t)$ as a sum of real-valued sinusoids.

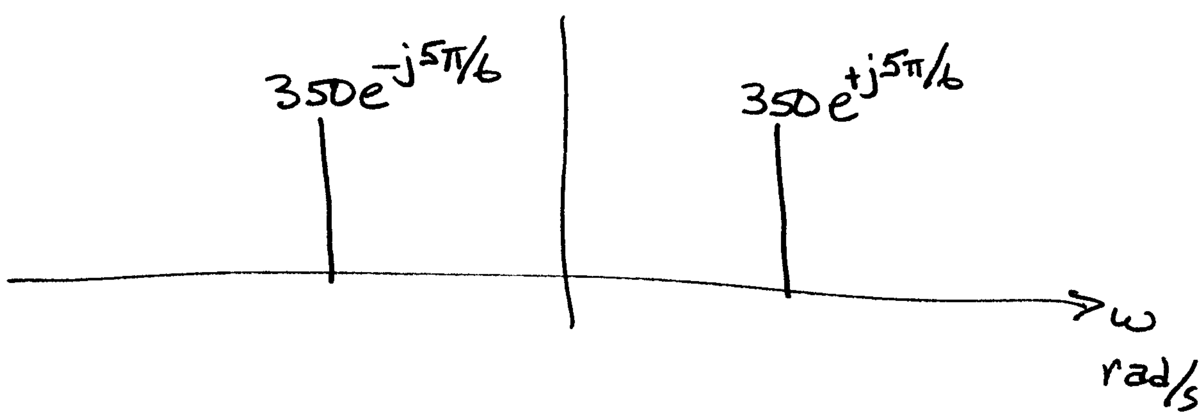
$$x(t) = 100\cos(7t + \pi/3) - 77$$

(b) Define a new signal $y(t)$ to be the derivative of $x(t)$, i.e., $y(t) = \dfrac{dx(t)}{dt}$.

Make a (well-labeled) sketch of the spectrum of the signal $y(t)$. Simplify the numerical values for the complex amplitudes, so that the values of magnitude and phase are obvious.

$$y(t) = (j7)50e^{j\pi/3}e^{j7t} + (-j7)50e^{-j\pi/3}e^{-j7t} + 0$$

$$= 350\underbrace{e^{j\pi/2}e^{j\pi/3}}_{e^{j5\pi/6}}e^{j7t} + 350\underbrace{e^{-j\pi/2}e^{-j\pi/3}}_{e^{-j5\pi/6}}e^{-j7t}$$



PROBLEM s-06-Q.1.3:

Two questions about sinusoids, $A\cos(\omega t + \varphi)$.

(a) The following MATLAB code makes a plot of a sinusoid:

```
tt = 0:0.0001:1;
znum = exp(j*8*pi*tt)-j*exp(j*8*pi*tt);
zden = 3*j*exp(-j*8*pi*tt)+4*j*exp(-j*8*pi*tt);
xx = real(znum./zden);
plot(tt,xx), grid on, shg
```

Determine the mathematical formula by giving numerical values for A, φ, and ω (in rad/s).

$A = 0.202 = \sqrt{2}/7$

$\varphi = -3\pi/4$ rad

$\omega = 16\pi$ rad/s

$$z(t) = \frac{e^{j8\pi t} - j e^{j8\pi t}}{3j e^{-j8\pi t} + 4j e^{-j8\pi t}}$$

$$= \frac{e^{j8\pi t}(1-j)}{e^{-j8\pi t}(7j)}$$

$1 - j = \sqrt{2} e^{-j\pi/4}$

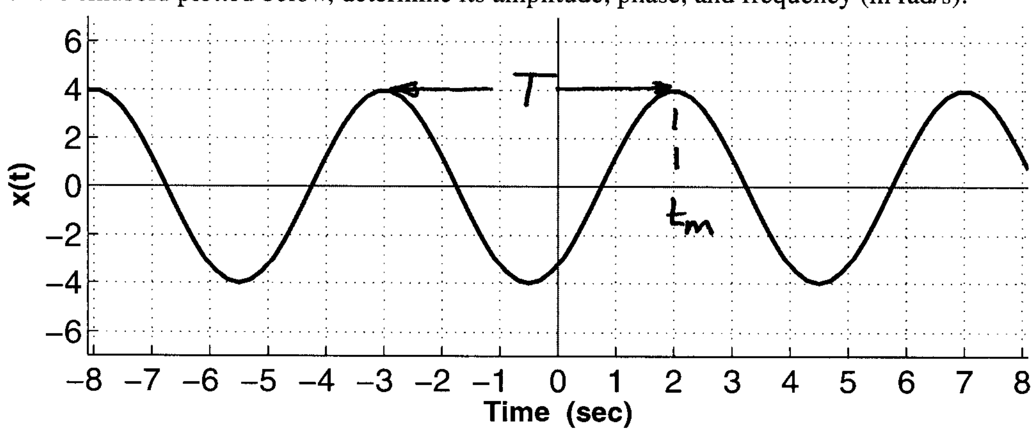
$7j = 7e^{j\pi/2}$

$$z(t) = e^{j16\pi t}\left(\frac{\sqrt{2}}{7}\right)e^{-j3\pi/4}$$

$= 0.202$

$$x(t) = \Re e\{z(t)\} = \frac{\sqrt{2}}{7}\cos(16\pi t - 3\pi/4)$$

(b) For the sinusoid plotted below, determine its amplitude, phase, and frequency (in rad/s).



$T = 2 - (-3) = 5$ secs $\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{5}$

$\varphi = -\omega t_m = -\left(\frac{2\pi}{5}\right)(2) = -4\pi/5$

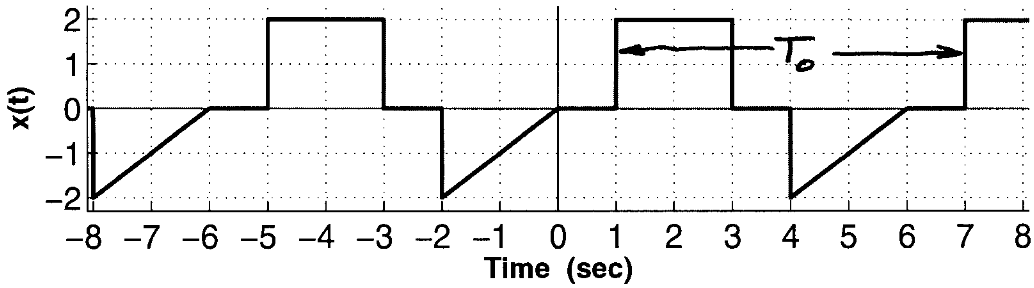
$A = 4$

$\varphi = -4\pi/5$ rad.

$\omega = 2\pi/5$ rad/s

PROBLEM s-06-Q.1.4:

Suppose that a periodic signal $x(t)$ is defined by the plot below:



(a) Determine the $\boxed{\text{fundamental frequency}}$ of $x(t)$ in Hz.

$$T_0 = 7 - 1 = 6 \text{ secs} \implies f_0 = \frac{1}{T_0} = \frac{1}{6} \text{ Hz}$$

(b) Determine the $\boxed{\text{DC value}}$ of $x(t)$.

$$DC = a_0 = \frac{1}{T} \cdot \text{Area}$$

$$a_0 = \frac{1}{6}\left\{(2)(2) + \frac{1}{2}(2)(-2)\right\} = \frac{2}{6} = \frac{1}{3}$$

(c) Write the $\boxed{\text{Fourier integral}}$ expression for the coefficient a_6 in terms of the specific signal $x(t)$ defined above. *Set up all the specifics of the integral (e.g., limits of integration), but do not evaluate the integral. All parameters in the integral(s) should have numeric values. NOTE: more than one integral might be needed.*

$a_6 = a_k$ for $k = 6$

Integrate over one period from -2 to $+4$

Four cases for $x(t)$

$$x(t) = \begin{cases} t & -2 \le t \le 0 \\ 0 & 0 \le t \le 1 \\ 2 & 1 \le t \le 3 \\ 0 & 3 \le t \le 4 \end{cases}$$

$$a_6 = \frac{1}{6}\int_{-2}^{0} t e^{-j\frac{2\pi}{6}\cdot 6t}\,dt + \frac{1}{6}\int_{1}^{3} 2e^{-j\frac{2\pi}{6}\cdot 6t}\,dt$$

$$= \frac{1}{6}\int_{-2}^{0} t e^{-j2\pi t}\,dt + \frac{1}{6}\int_{1}^{3} 2e^{-j2\pi t}\,dt$$

PROBLEM s-06-Q.1.5:

For the FM signal $x(t)$ defined as:

$$x(t) = \Re e\left\{\exp\{jCe^{\lambda t}\}\right\} = \Re e\left\{e^{j\psi(t)}\right\}$$

we will denote its instantaneous frequency in Hz as $f_x(t)$.

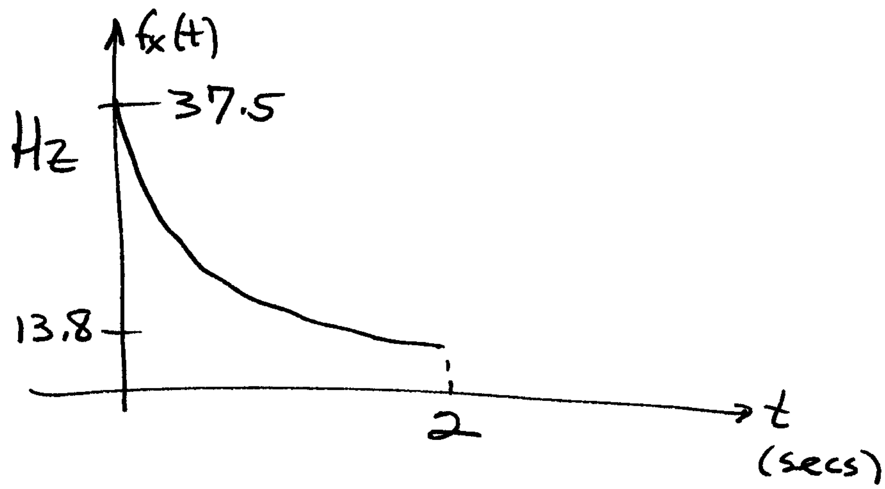
(a) For $C = -150\pi$ and $\lambda = -0.5$, make a *carefully labeled* plot of the instantaneous frequency $f_x(t)$ over the range $0 \le t \le 2$.

$$x(t) = \cos(Ce^{\lambda t}) \Rightarrow \psi(t) = Ce^{\lambda t} = -150\pi e^{-.5t}$$

$$f_x(t) = \frac{1}{2\pi}\frac{d}{dt}\psi(t) = \frac{1}{2\pi}(-150\pi)(-0.5)e^{-0.5t} \text{ Hz}$$

$$= 37.5e^{-0.5t} \text{ Hz}$$

$$f_x(0) = 37.5 \text{ Hz} \qquad f_x(2) = 13.8 \text{ Hz}$$



(b) Evaluate $\int_{-2}^{2} \left|7j\exp\{j\pi e^{-0.5t}\}\right|^2 dt$. Since the integral is a definite integral, give a numerical answer.

$$\left|7j\, e^{j\pi e^{-0.5t}}\right|^2 = 49$$

$$|j| = 1$$

$$|e^{j\theta}| = 1$$

$$\int_{-2}^{2} 49\, dt = 49(4) = 196$$