GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #2

DATE: 17-Mar-06 COURSE: ECE-2025

NAME: ______________________________ GT #: ________________

LAST, FIRST (ex: gtz123q)

3 points 3 points 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-Noon (Juang)		L06:Thur-Noon (Verriest)
	L07:Tues-1:30pm (Juang)		
L01:M-3pm (McClellan)	L09:Tues-3pm (Chang)	L02:W-3pm (Zhou)	L10:Thur-3pm (Taylor)
L03:M-4:30pm (Fekri)	L11:Tues-4:30pm (Chang)	L04:W-4:30pm (Zhou)	

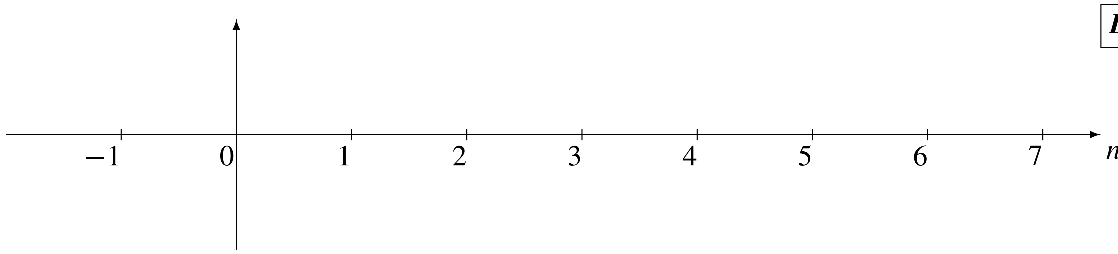
- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- **JUSTIFY** your reasoning clearly to receive partial credit.
  Explanations are also required to receive **FULL** credit for any answer.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	-3	

PROBLEM Spring-06-Q.2.1:

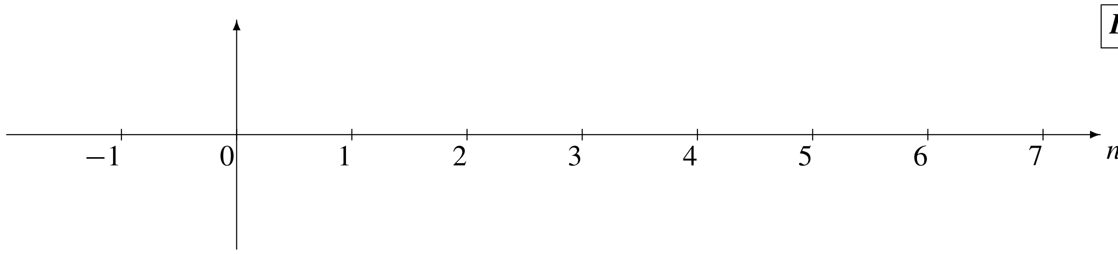
(a) Evaluate the convolution: `yn = conv( [10 10 10 10 10], [6 -3 -2] );`
Note: a MATLAB vector implicitly defines a signal to have its starting point at $n = 0$.
Give your answer as a ***stem plot.***

Label all points



(b) Make a ***stem plot*** of the signal $s[n] = 7\,(u[n-3] - u[n-6])$.

Label all points



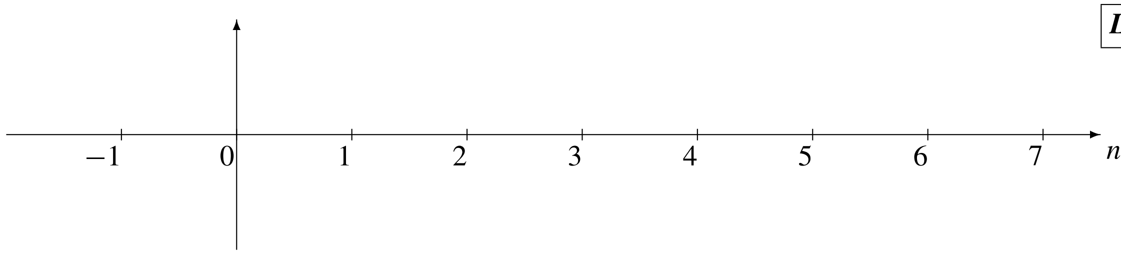
PROBLEM Spring-06-Q.2.2:

(a) Determine the impulse response of the system:

$$y[n] = 88x[n-3] + 88x[n-6]$$

Give your answer as a ***stem plot.***

Label all points



(b) Determine the frequency response of the FIR system:

$$y[n] = 88x[n-3] + 88x[n-6]$$

Give your answer as a formula ***in the following form:*** $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A \cos(\mu\hat{\omega})$ by finding numerical values for A, β and μ.

$A =$	$\beta =$	$\mu =$

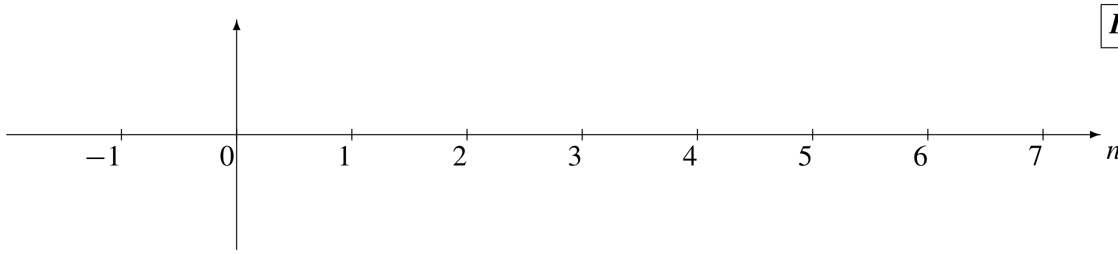
PROBLEM Spring-06-Q.2.3:

Suppose that the system function of an FIR filter is

$$H(z) = 70z^{-4}\left(1 - e^{j5\pi/6}z^{-1}\right)\left(1 - e^{-j5\pi/6}z^{-1}\right)$$

(a) Determine the impulse response, $h[n]$, of the FIR filter. Give your answer as a ***stem plot.***

Label all points



(b) Evaluate the magnitude and phase of the frequency response of the FIR filter at $\hat{\omega} = \pi/6$.

Magnitude =

Phase =

PROBLEM Spring-06-Q.2.4:

Here are some operations that are often done in MATLAB. In each case, you should analyze the code and determine the answer via mathematics.

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:4480099;
zz = (2 - j*5) * exp(j*(1.9*pi*nn + 3.33));
soundsc( real(zz), 500 )
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ =	Hz

Explain your reasoning.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/72000:10000;
xx = cos( 2*pi*9000*tt + pi/3 );
soundsc( xx, fsamp );
```

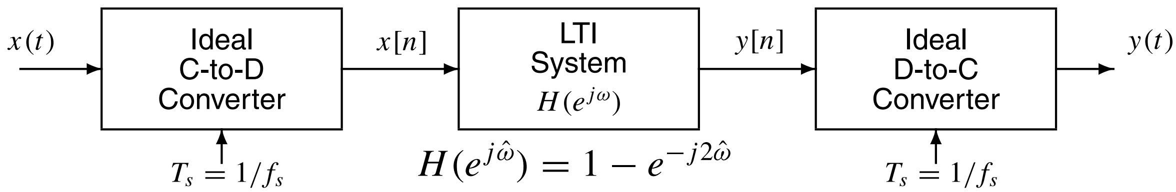
Although the sinusoid was not written to have a frequency of 1100 Hz, it is possible to play out the vector xx so that it sounds like a 1100 Hz tone. Determine the value of **fsamp** that should be used to play the vector xx as a 1100 Hz tone.

fsamp =	Hz

Explain your reasoning.

PROBLEM Spring-06-Q.2.5:

Consider the following system for discrete-time filtering of a continuous-time signal:



For all parts below, the input to the ideal C-to-D converter is $x(t) = 60\cos(2000\pi t)$.

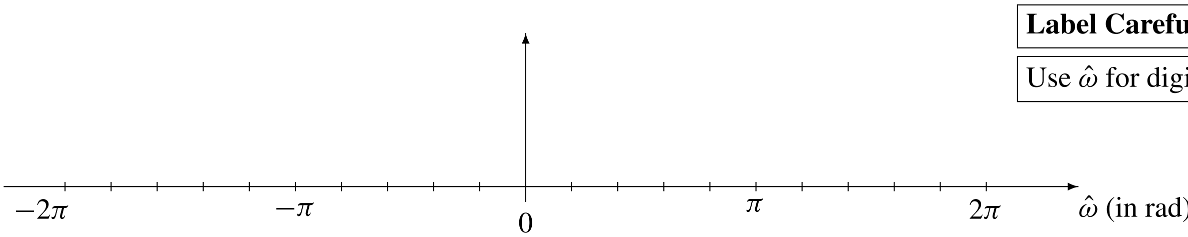
(a) Determine the Nyquist rate (in hertz) for sampling the input signal $x(t)$. Explain.

$f_{\text{Nyquist}} =$ ☐ Hz

(b) If the sampling rate of the C-to-D converter is $f_s = 3000$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \leq \hat{\omega} \leq 2\pi$. Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

Label Carefully

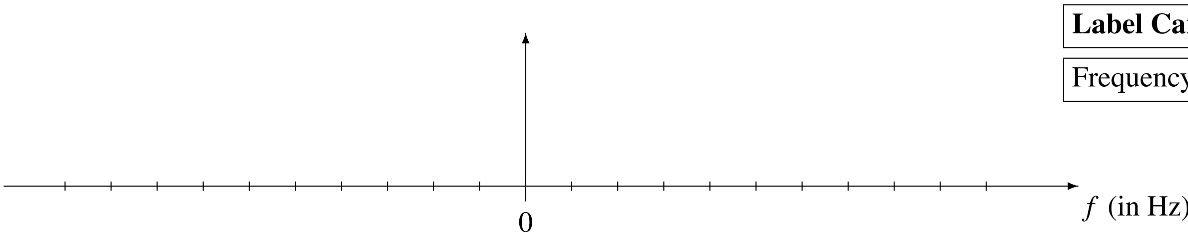
Use $\hat{\omega}$ for digital freq.



(c) If the sampling rate of the the ideal D-to-C converter is $f_s = 3000$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x(t)$ defined above.

Label Carefully

Frequency in ***Hz***



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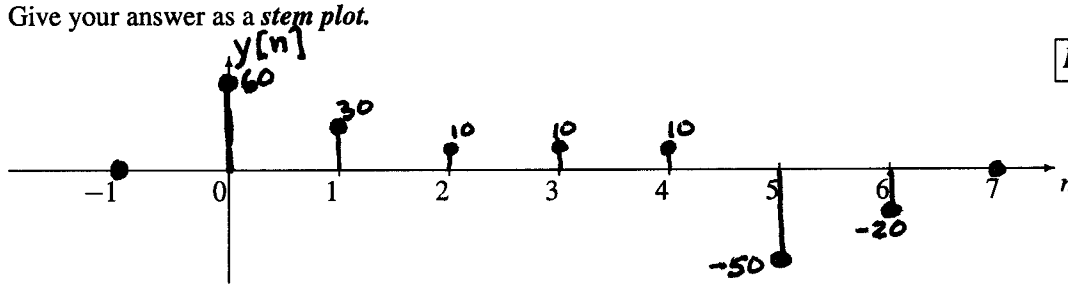
PROBLEM Spring-06-Q.2.1:

(a) Evaluate the convolution: `yn = conv( [10 10 10 10 10], [6 -3 -2] );`
Note: a MATLAB vector implicitly defines a signal to have its starting point at $n = 0$.
Give your answer as a ***stem plot.***

Label all points

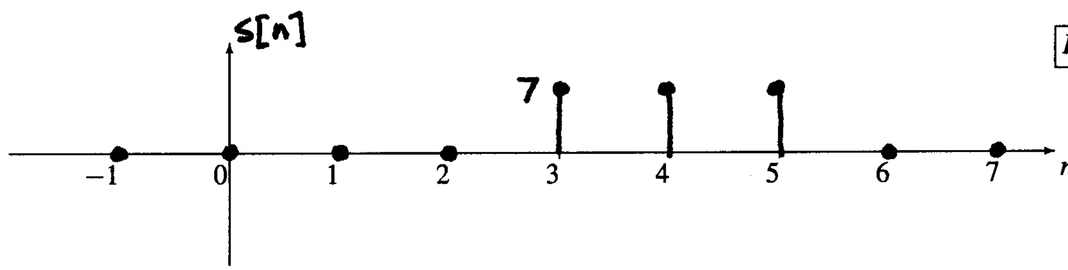


The convolution table is:

```
yn = [60   60   60   60   60    0    0 ]
   + [ 0  -30  -30  -30  -30  -30   0 ]
   + [ 0    0  -20  -20  -20  -20  -20 ]
yn = [60   30   10   10   10  -50  -20 ]
       ↗                        ↗
     n=0                       n=5
```

(b) Make a ***stem plot*** of the signal $s[n] = 7\,(u[n-3] - u[n-6])$.

Label all points



The difference of two unit-step signals is a pulse. The unit-step $u[n-3]$ goes "up" at $n = 3$; the unit-step $-u[n-6]$ goes "down" at $n = 3$, so the final result has two cases:

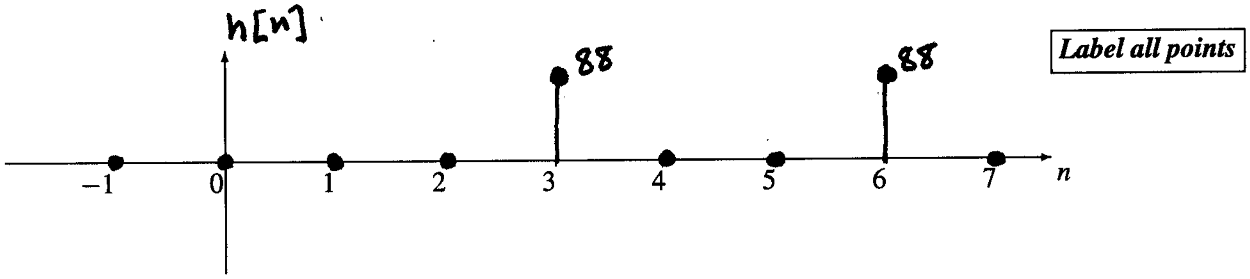
$$7\,(u[n-3] - u[n-6]) = \begin{cases} 7 & 3 \le n < 6 \\ 0 & \text{elsewhere} \end{cases}$$

PROBLEM Spring-06-Q.2.2:

(a) Determine the impulse response of the system:

$$y[n] = 88x[n-3] + 88x[n-6]$$

Give your answer as a ***stem plot.***



When $x[n] = \delta[n]$, you get the impulse response, $h[n]$. Thus,

$$h[n] = 88\delta[n-3] + 88\delta[n-6]$$

(b) Determine the frequency response of the FIR system:

$$y[n] = 88x[n-3] + 88x[n-6]$$

Give your answer as a formula ***in the following form:*** $H(e^{j\hat{\omega}}) = e^{-j\beta\hat{\omega}} A\cos(\mu\hat{\omega})$ by finding numerical values for A, β and μ.

$A = 176$	$\beta = 4.5$	$\mu = 1.5$

The frequency response is $H(e^{j\hat{\omega}}) = \sum_{n=0}^{M} h[n]e^{-jk\hat{\omega}}$. Thus,

$$H(e^{j\hat{\omega}}) = 88e^{-j3\hat{\omega}} + 88e^{-j6\hat{\omega}} = 88e^{-j4.5\hat{\omega}}\left(e^{j1.5\hat{\omega}} + e^{-j1.5\hat{\omega}}\right)$$

$$H(e^{j\hat{\omega}}) = 176e^{-j4.5\hat{\omega}}\cos(1.5\hat{\omega})$$

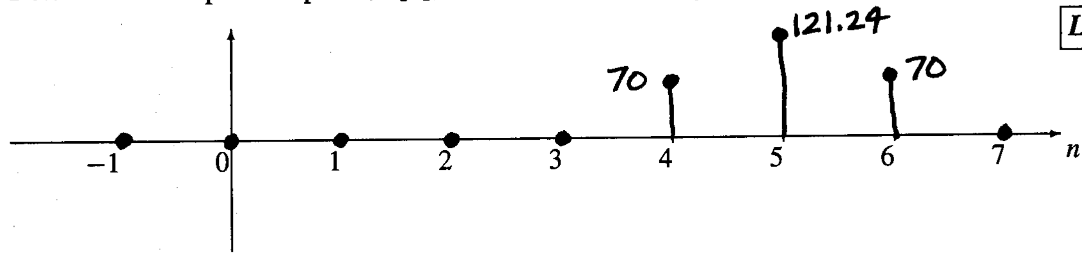
PROBLEM Spring-06-Q.2.3:

Suppose that the system function of an FIR filter is

$$H(z) = 70z^{-4}\left(1 - e^{j5\pi/6}z^{-1}\right)\left(1 - e^{-j5\pi/6}z^{-1}\right)$$

(a) Determine the impulse response, $h[n]$, of the FIR filter. Give your answer as a ***stem plot.***

Label all points



Multiply out the factors and then use the definition of the z-Transform, $H(z) = \sum_{n=0}^{M} h[n]z^{-n}$, to get the impulse response from the coefficients of the z-Transform polymonial:

$$H(z) = 70z^{-4}\left(1 - 2\cos(5\pi/6)z^{-1} + z^{-2}\right) = 70z^{-4} - 121.24z^{-5} + 70z^{-6}$$

$$\Longrightarrow h[n] = 70\delta[n-4] + 121.24\delta[n-5] + 70\delta[n-6]$$

(b) Evaluate the magnitude and phase of the frequency response of the FIR filter at $\hat{\omega} = \pi/6$.

Magnitude = 242.5

Phase = -0.833π = -2.618 radians

Evaluating $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/6$ is the same as evaluating $H(z)$ at $z = e^{j\pi/6}$, so we obtain

$$H(e^{j\pi/6}) = 70e^{-j4\pi/6}\left(1 - e^{j5\pi/6}e^{-j\pi/6}\right)\left(1 - e^{-j5\pi/6}e^{-j\pi/6}\right)$$

which reduces to

$$H(e^{j\pi/6}) = 70\left(e^{-j2\pi/3} - e^{-j0} - e^{-j5\pi/3} + e^{-j\pi}\right)$$
$$= 242.5e^{-j0.833\pi} = 242.5e^{-j5\pi/6} = 242.5e^{-j2.618}$$

PROBLEM Spring-06-Q.2.4:

Here are some operations that are often done in MATLAB. In each case, you should analyze the code and determine the answer via mathematics.

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:4480099;
zz = (2 - j*5) * exp(j*(1.9*pi*nn + 3.33) );
soundsc( real(zz), 500 )
```

Determine the analog frequency (in Hertz) that will be heard. (There might be folding or aliasing!)

FREQ = **25**	Hz

The discrete-time frequency of $\hat{\omega} = \pm 1.9\pi$ will alias to $\hat{\omega} = \mp 0.1\pi = \mp 2\pi(0.05)$, so the analog frequency of the output is $f_{\text{out}} = \frac{\hat{\omega}}{2\pi} f_s = (0.05)(500) = 25$ Hz.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/72000:10000;
xx = cos( 2*pi*9000*tt + pi/3 );
soundsc( xx, fsamp );
```

Although the sinusoid was not written to have a frequency of 1100 Hz, it is possible to play out the vector xx so that it sounds like a 1100 Hz tone. Determine the value of **fsamp** that should be used to play the vector xx as a 1100 Hz tone.

fsamp = **8800**	Hz

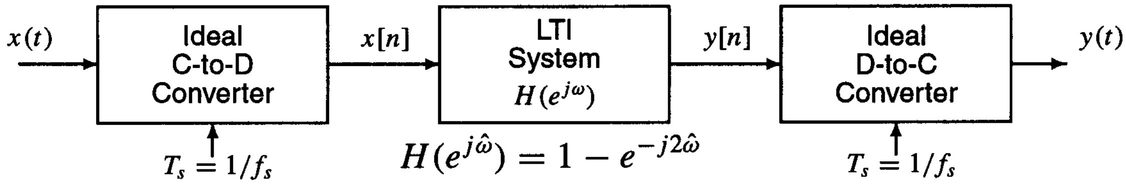
The xx signal is actually a discrete-time signal that came from a sinusoid of frequency 9000 Hz, sampled at 72000 Hz. Thus the discrete-time frequency is $\hat{\omega} = 2\pi(9000/72000) = 2\pi(0.125)$ which does **not** alias. In order to get a specified output frequency of 1100 Hz, we must solve for f_s in

$$f_{\text{out}} = \frac{\hat{\omega}}{2\pi} f_s \qquad \Rightarrow \qquad 1100 = \frac{2\pi(1/8)}{2\pi} f_s$$

Thus, the sampling frequency should be $f_s = 8(1100) = 8800$ Hz.

PROBLEM Spring-06-Q.2.5:

Consider the following system for discrete-time filtering of a continuous-time signal:



For all parts below, the input to the ideal C/D converter is $x(t) = 60\cos(2000\pi t)$.

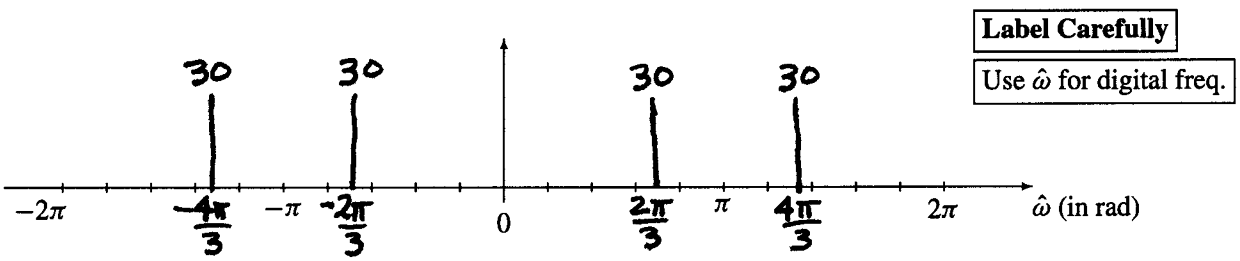
(a) Determine the Nyquist rate (in hertz) for sampling the input signal $x(t)$. Explain.

$f_{\text{Nyquist}} = \boxed{2000}$ Hz

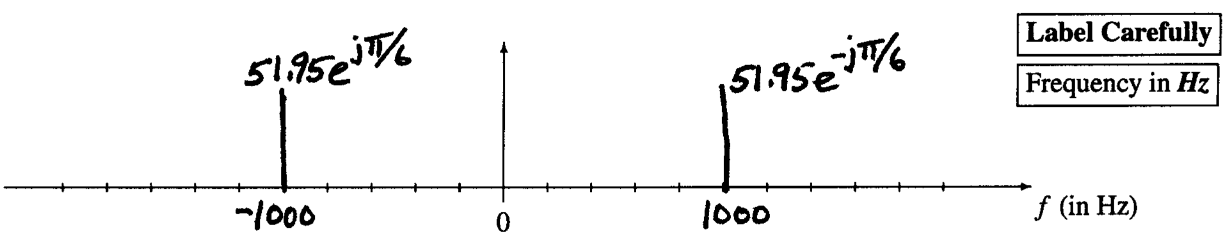
The Nyquist Rate is two times the highest frequency in $x(t)$, whose only frequency is 1000 Hz. The Sampling Theorem says that you must sample at greater than the Nyquist Rate to be able to recover the signal.

(b) If the sampling rate of the C-to-D converter is $f_s = 3000$ samples/sec, make a plot of the spectrum of the discrete-time signal $x[n]$ over the range of frequencies $-2\pi \le \hat{\omega} \le 2\pi$. Make sure to show **all spectrum lines** and label the frequency, amplitude and phase of each spectral component.

The signal $x[n]$ has a discrete-time frequency of $\hat{\omega} = 2000\pi/3000 = 2\pi/3$, so there will be spectral lines at $\hat{\omega} = \pm 2\pi/3 \pm 2\pi$. In the figure the spectrum lines will be at $\hat{\omega} = \{2\pi/3, -4\pi/3, -2\pi/3, 4\pi/3\}$.



(c) If the sampling rate of the the ideal D-to-C converter is $f_s = 3000$ samples/sec, draw the spectrum for the continuous-time output signal, $y(t)$. Use $x[n]$ determined in the previous part.



The frequency response at $\hat{\omega} = 2\pi/3$ is

$$H(e^{j\hat{\omega}}) = 1 - e^{-j4\pi/3} = 1.732e^{-j\pi/6}$$

Therefore, the output of the filter is $y[n] = 103.9\cos(0.8\pi n - \pi/6)$, and converting from $\hat{\omega}$ to the output frequency ($\omega_{\text{out}} = \hat{\omega} f_s$) gives

$$y(t) = 103.9\cos(2000\pi t - \pi/6)$$

Thus the complex amplitudes are $51.95e^{\mp j\pi/6}$ at $f = \pm 1000$ Hz.